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# Maintenance optimization incorporating lumen degradation failure for energy-efficient lighting retrofit projects☆

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## HIGHLIGHTS

- Lamp failure model based on lumen degradation.
- Energy savings model considering the surviving population.
- Maintenance optimization model based on lumen degradation failure.
- Optimal maintenance model optimizes maintenance costs and energy savings.

## ARTICLE INFO

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## ABSTRACT

This study presents an optimal lighting maintenance plan that takes into account lumen degradation failure. In lighting retrofit projects, retrofitted lights fail over time mainly owing to burnout and lumen degradation failures. These failures result in a reduced illumination level and lower project savings if proper maintenance is not performed. Previous studies developed lighting maintenance plans by modeling lamp population decay due to burnout failure. In this study, we present an optimal lighting maintenance plan based on lumen degradation failure. The lumen degradation failure is modeled based on the statistical properties of degradation rates. By using the Kaplan–Meier method, the formulated lumen degradation failure is used to model the surviving population. The surviving population model is used to design an optimal lighting maintenance plan, which maximizes energy savings and minimizes maintenance costs. The effectiveness of the formulated maintenance plan is demonstrated by an actual residential energy-efficient lighting retrofit project implemented in South Africa. Results show that the proposed maintenance plan is more cost-effective than full maintenance.

## 1. Introduction

Energy-efficient (EE) lighting retrofit projects are promoted in various incentives energy efficiency programs, such as clean development mechanism (CDM) [1], demand-side management [2], and white tradable certificate schemes [3]. Over time, the number of retrofitted lights decreases owing to lumen degradation and burnout failures. These failures lead to reductions in illumination levels and energy savings if proper maintenance is not performed. To control the lamp population decay in the EE lighting retrofit projects promoted under incentive programs, some project guidelines suggest that no rebate will be given to the implemented project if 50% of the initial population has failed during the project crediting period [4]. In consideration of lumen degradation, the Alliance for Solid-State Illumination Systems and Technologies recommends 70% lumen (30% degradation from the initial lumen output) threshold to determine the useful life of LEDs for general lighting applications [5]. Therefore, for LED-based lighting retrofit projects, a 70% lumen degradation criterion should be considered to maintain both the illumination levels and energy savings, since burnout failure is not significant in LEDs because of their longevity.

Burnout failure in lighting devices is mostly generated by defective materials, deviations in the manufacturing process or incorrect handling and operation [6]. When burnout failure occurs, the light suddenly goes off. The degradation is due to increased wear and aging of the material [7]. The most common degradation in electric lights is lumen degradation. The causes of lumen degradation differ from one lighting technology to another. Lumen degradation in LEDs varies, depending

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on the package and system design [8].

In the literature, various models of lamp population decay have been proposed. The AMS-II.J CDM guideline suggests linear population decay for compact fluorescent lamps [9]. The CDM model formulates linear decay over the rated lifetime of lamps. This model has been applied in [10] for optimal sampling design for measurement and verification (M&V). The Polish efficient lighting project (PELP), conducted by the World Bank through an international finance corporation [11], presented a logistic curve for a population of 1.2 million lamps. References [12,13] proposed a model derived from existing biological population growth equations for modeling the decay of compact fluorescent lamps over time. This model has been applied in [14] for optimal maintenance planning of the EE lighting retrofit project. Reference [15] introduces LEDs' failure model based on lumen degradation, and this model is used to design an optimal maintenance plan of an LED-based lighting retrofit project.

Based on when it is performed (before or after failure), maintenance can be classified into two main categories [16]: corrective maintenance (CM), and preventive maintenance (PM). CM is performed after a failure has occurred, while PM is performed before a failure has occurred. PM is usually performed according to a schedule and can be based on the usage of the product (e.g.: every 1000 h, 10,000 h, etc.), time-based (every week, every month, every year, etc.), or condition-based [17]. For lighting systems, the preventive maintenance scheduled at fixed time intervals is the most efficient [15]. PM maintains the efficiency of the equipment and keeps it running more efficiently. However, the implementation of PM can be time-consuming and costly because of the tasks performed, such as continuous monitoring of the working conditions of the item. Therefore, effective PM should be prioritized to balance maintenance costs and additional benefits.

The current work continues in the line of study [15] by considering large-scale lighting retrofit projects. In large-scale lighting retrofit projects, monitoring when each lamp will fail can be time-consuming and costly. In this work, the degradation failure model is developed by analyzing the statistical properties of sample lumen degradation rates. This model considers the variations in lumen degradation rates which may be caused by manufacturing materials and processes, or different handling conditions. The main contribution of this study is the formulation of optimal lighting maintenance that takes into account lumen degradation failure based on the statistical properties of the lumen degradation rates.

In this study, lumen degradation failure is modeled by analyzing the statistical properties of the degradation rates of LUXEON rebel, Lumileds Philips LEDs. Fifty-seven probability distributions are tested to find the best distribution fit of the samples' degradation rates. Kolmogorov Smirnov, Anderson Darling, and Chi-Squared goodness fit tests are used to determine the best fit of samples' degradation rates. The generalized extreme value (GEV) distribution is the best fit for the tested degradation rates and is used to calculate the cumulative probability of failure distribution and model the lumen degradation failure. By using the Kaplan–Meier method, the failure model is used to estimate the surviving population. Thereafter, the surviving population model developed is used to design an optimal maintenance plan for an LED-based lighting retrofit project. The optimal maintenance plan is formulated to maximize energy savings and minimize maintenance costs. The optimization problem is solved using the Mixed Integer Distributed Ant Colony Optimization (MIDACO) solver in MATLAB. Failed lamps are optimally replaced based on a minimum number of surviving lamps required to maintain adequate illumination level, and maintenance budget limit. The effectiveness of the proposed maintenance plan is illustrated by a case study of an actual LED-based lighting retrofit project for residential buildings implemented in South Africa.

The organization of the paper is as follows: The problem formulation and modeling are given in Section 2. The case study is given in Section 3. Simulation results are discussed in Section 4, followed by conclusion in Section 5.

## 2. Problem formulation and modeling

For LED lighting retrofit projects registered under incentive energy efficiency programs, energy saving rebates can only be awarded if the lighting population is greater than or equal to 70% of the initial population. At the beginning of the installation, all installed light bulbs will function properly. However, as time goes by, the number of operational light bulbs will decrease as well as the energy savings and illuminance levels, because of the lumen degradation. Proper maintenance is required to sustain the project performance and rebates. However, maintenance activities can be costly. This study, therefore, proposes an optimal lighting maintenance plan that maximizes energy savings and minimizes maintenance cost, while taking into account lumen degradation failure.

We start with the lamp failure modeling for the formulation of the optimization problem.

### 2.1. Lamp failure modeling

#### 2.1.1. Burnout failure models

In the literature, different models have been developed to predict the number of lamps that survived over time based on burnout failure. The burnout failure models developed in the existing studies are discussed below.

(i) Exponential decay model

The exponential decay model quantifies growth or decay at a rate proportional to the population size [18].

$$\frac{dN(t)}{dt} = kN(t), \tag{1}$$

where $N(t)$ is the lamp population size at time $t$. Eq. (1) describes the law of natural growth if $k > 0$, and the law of natural decay if $k < 0$. The solution to Eq. (1) is an exponential function given as

$$N(t) = N(0)e^{kt}, \tag{2}$$

where $N(0)$ is the size of the initial population.

(ii) Linear population model

This model is suggested in the AMS-II.J CDM guideline [19].

$$N(t) = \begin{cases} N(0) - t \times H \times \frac{100-y}{100 \times L}, & \text{for} \quad t \times H < L, \\ 0, & \text{for} \quad t \times H \geqslant L, \end{cases} \tag{3}$$

where $L$ is the average lifetime of lamps (in h), $H$ is the number of operating hours per year (in h), and $y$ is the percentage of surviving population at the end of the average lifetime.

(iii) Regression model

A regression model has been proposed to fit the PELP data [11].

$$\mathbf{\Phi}(t) = \frac{1}{1 + e^{t-L}}, \tag{4}$$

where $\mathbf{\Phi}(t)$ is the proportion of the population surviving at time $t$.

(iv) Improved model to Eq. (4)

Previous studies [12,13], proposed an improvement to Eq. (4), given as

$$\mathbf{\Phi}(t) = \frac{1}{c + e^{bt-L}}, \tag{5}$$

where $c$ and $b$ are the initial value and slope parameters, respectively. The discrete dynamic form of Eq. (5) is given as [12]

$$\mathbf{\Phi}(t+1) = bc\mathbf{\Phi}(t)^2 - b\mathbf{\Phi}(t) + \mathbf{\Phi}(t). \tag{6}$$

#### 2.1.2. Lumen degradation failure modeling

LED lamp failure based on lumen degradation is modeled to predict the surviving population in LED-based lighting retrofit projects. Previous studies [20,21] analyzed the lumen degradation of LEDs and recommended the exponential decay model (7) as an appropriate empirical model to describe the lumen degradation of LEDs.

$$\phi(t) = \phi(0)e^{-\beta t}, \tag{7}$$

where $\phi(t)$ is the luminous flux (in lm) at time $t$, $\phi(0)$ is the initial luminous flux (in lm), and $\beta$ is the degradation rate.

In model (2), the population failure rate $k$ is assumed to be constant. However, the failure rate may vary among LEDs owing to variations in lumen degradation rates. The lumen degradation rate may vary among LEDs owing to variations in materials, different manufacturing processes, or different handling conditions. For this reason, in this study LED failure is modeled by analyzing the statistical properties of the lumen degradation rates. The modeling process is detailed as follows:

*i). Lumen measurement*

Currently, there is no universal standard for measuring the photometric properties of LEDs. The Illumination Engineers Society (IES) released documents (IES LM-80-08 and IES LM-IES TM-21-11) describing standards for testing LEDs [22]. These standards appear to be the frontrunner in becoming the benchmark standards in testing photometric properties and have been adopted by most top-tier manufacturers such as Osram and Philips. The IES standards recommend a minimum of 20 samples to project 6 times of test duration and 10 to 19 samples to project 5.5 times test of duration. For luminous flux data collection, sequential measurements after the initial 1 000 h at intervals less than 1 000 h are encouraged. For curve fit, at least 5 000 h of data for test duration greater than or equal to 6 000 h, or at least 50% of the total test duration for a test duration greater than 10 000 h is recommended.

Because of their longevity, up to now, no actual measurement data of lumen degradation of LEDs have been collected over their complete lifetime. In this study, the data used to model lumen degradation failure were obtained from the Illuminating Engineering Society of North America (IESNA) LM-80 test report of Philips Lumileds [23]. It reports on the lumen degradation of 25 LUXEON Rebel, Lumileds Philips being tested in compliance with the LM-80-08 standard. Luminous flux data of tested units were collected every 1 000 h over an evaluation period of 7 000 h. All collected luminous flux data were normalized to 1 at the original test point. Sample test units were tested in homogeneous operating and environmental conditions. The lumen degradation threshold $Lp_{th}$ (30% degradation from the initial lumen output) commonly used in the general lighting application was considered. When the lumen degradation of a unit at any time $t$ is below $Lp_{th}$, the unit is deemed to have failed.

*ii). Estimation of degradation rates and statistical property analysis*

The degradation rate of each test unit is estimated using regression analysis. Regression analysis examines the relationship between time and lumen degradation. Each test unit is represented by time and lumen degradation data points, $(t_1, Lp_1)$, $\cdots$,$(t_K, Lp_K)$. The model function of each unit is given as

$$Lp_i = f(t, \beta_i), \qquad i = 1, 2, \cdots, n, \qquad t = 1, 2, \cdots, K, \tag{8}$$

where $Lp_i$ and $\beta_i$ are the lumen measurement and degradation rate of the $i$th unit, respectively. It is found that the test units data are characterised by exponential models with the coefficient of determination ($R^2$) between 0.97 and 0.99. The estimated degradation rate for each unit is given in Table 1.

To analyze the statistical properties of the degradation rates, the probability distribution fitting is used to determine the statistical distribution that best fits the degradation rates. There are different distribution fitting programs, including EasyFit, Matlab, Excel, ExpertFit, and R. In this study, EasyFit[1] is used. Kolmogorov Smirnov, Anderson Darling, and Chi-squared tests are used to test the best fit. Table 2 presents different tested distributions and their statistics. The distribution fits are tested at a 95% confidence level. The results show that the degradation rates fit 40 distributions out of 57 fitted distributions. It has been found that GEV distribution is the best fit, and it is used to estimate the cumulative probability of failure distribution.

**Table 1**
Lumen degradation rates of test units over the evaluation period of 7 000 h.

| Test unit | $\beta$ | Test unit | $\beta$ |
|---|---|---|---|
| 1 | $0.1382 \times 10^{-4}$ | 14 | $0.0908 \times 10^{-4}$ |
| 2 | $0.1211 \times 10^{-4}$ | 15 | $0.1609 \times 10^{-4}$ |
| 3 | $0.1346 \times 10^{-4}$ | 16 | $0.1008 \times 10^{-4}$ |
| 4 | $0.1274 \times 10^{-4}$ | 17 | $0.1097 \times 10^{-4}$ |
| 5 | $0.1001 \times 10^{-4}$ | 18 | $0.1287 \times 10^{-4}$ |
| 6 | $0.1177 \times 10^{-4}$ | 19 | $0.1462 \times 10^{-4}$ |
| 7 | $0.1192 \times 10^{-4}$ | 20 | $0.0902 \times 10^{-4}$ |
| 8 | $0.1142 \times 10^{-4}$ | 21 | $0.1312 \times 10^{-4}$ |
| 9 | $0.1368 \times 10^{-4}$ | 22 | $0.1015 \times 10^{-4}$ |
| 10 | $0.1142 \times 10^{-4}$ | 23 | $0.1508 \times 10^{-4}$ |
| 11 | $0.1428 \times 10^{-4}$ | 24 | $0.1053 \times 10^{-4}$ |
| 12 | $0.1269 \times 10^{-4}$ | 25 | $0.1206 \times 10^{-4}$ |
| 13 | $0.1386 \times 10^{-4}$ | | |

*iii). Model prediction*

The probability that a brand new LED will fail at or before a specified time is represented by a cumulative distribution function $F(t)$. $F(t)$ is expressed as [24]

$$F(t) = P[\epsilon \leqslant t], \tag{9}$$

where $\epsilon$ is the lifetime of LED (hours of use before the LED fails). Assuming $\beta$ of the LED is known and $Lp_{th}$ is set, $\epsilon$ can be expressed as [21]

$$\epsilon = \frac{\ln[Lp_{th}]}{-\beta}. \tag{10}$$

Substituting (10) in (9) we obtain

$$F(t) = P\left[\frac{\ln(Lp_{th})}{-\beta} \leqslant t\right]. \tag{11}$$

$\beta_i$ follows the GEV distribution with location parameter $\iota$, scale parameter $\nu$, and shape parameter $\kappa$. $\beta$~GEV($\iota$, $\nu$, $\kappa$), the GEV cumulative distribution is given as [25].

$$F(t) = P\left[\frac{\ln(Lp_{th})}{-\beta} \leqslant t\right] = exp\left[-\left(1 - \left(\frac{\frac{\ln(Lp_{th})}{-t} - \iota}{\nu}\right) \times \kappa\right)^{\frac{1}{\kappa}}\right]. \tag{12}$$

After modeling the failed proportions ($F(t)$), the surviving population is estimated using the Kaplan–Meier method [26] as

$$N\left(t+1\right) = N(t)\prod_{j=1}^{t}\left(1 - \frac{d(j)}{N(j)}\right), \tag{13}$$

where $d(j) = N(j-1)F(j)$ is the number of LEDs failed at time $j$ (with luminous flux below the threshold).

*iv). Model parameter estimation*

Parameter estimation methods, including the mixed method [27], maximum likelihood method [25], and probability weighted moment (PWM) method [28], have been used to estimate GEV distribution parameters. Reference [28] shows that maximum-likehood estimators

[1] http://www.mathwave.com/easyfit-distribution-fitting.html.

**Table 2**
Tested distributions using goodness fit tests.

| | | Kolmogorov Smirnov | | Anderson Darling | | Chi-Squared | |
|---|---|---|---|---|---|---|---|
| No | Distribution | Statistic | Rank | Statistic | Rank | Statistic | Rank |
| 1 | Beta | 0,13345 | 37 | 4,2804 | 42 | N/A | – |
| 2 | Burr | 0,07417 | 6 | 0,14771 | 6 | 0,40447 | 10 |
| 3 | Burr (4P) | 0,72372 | 53 | 15,225 | 52 | 0,08885 | 5 |
| 4 | Cauchy | | | Reject | | | |
| 5 | Dagum | 0,0989 | 27 | 0,27946 | 27 | 0,91312 | 16 |
| 6 | Dagum (4P) | | | Reject | | | |
| 7 | Erlang | 0,10107 | 28 | 0,21429 | 19 | 1,1869 | 33 |
| 8 | Erlang (3P) | 0,10167 | 29 | 0,21717 | 20 | 1,2339 | 34 |
| 9 | Error | 0,065 | 3 | 0,11954 | 3 | 0,05274 | 4 |
| 10 | Error Function | | | Reject | | | |
| 11 | Exponential | | | Reject | | | |
| 12 | Exponential (2P) | | | Reject | | | |
| 13 | Fatigue Life | 0,09586 | 25 | 0,22367 | 21 | 1,1334 | 29 |
| 14 | Fatigue Life (3P) | 0,07848 | 11 | 0,15991 | 9 | 0,38145 | 9 |
| 15 | Frechet | 0,15555 | 41 | 0,80594 | 35 | 1,9925 | 37 |
| 16 | Frechet (3P) | 0,11238 | 31 | 0,37649 | 30 | 0,86061 | 13 |
| 17 | Gamma | 0,0868 | 16 | 0,1815 | 14 | 1,0919 | 21 |
| 18 | Gamma (3P) | 0,08357 | 15 | 0,17538 | 11 | 1,1005 | 22 |
| 19 | Gen. Extreme Value | 0,06181 | 1 | 0,11934 | 2 | 0,02251 | 1 |
| 20 | Gen. Gamma | 0,08757 | 18 | 0,19079 | 16 | 1,1054 | 24 |
| 21 | Gen.Gamma (4P) | 0,17954 | 42 | 4,7381 | 45 | N/A | – |
| 22 | Gen. Logistic | 0,07572 | 8 | 0,17576 | 12 | 0,89488 | 15 |
| 23 | Gen. Pareto | 0,09732 | 26 | | Reject | | |
| 24 | Gumbel Max | 0,13582 | 38 | 0,69617 | 34 | 1,1247 | 27 |
| 25 | Gumbel Min | 0,12555 | 35 | 0,50988 | 31 | 1,1048 | 23 |
| 26 | Hypersecant | 0,09581 | 24 | 0,33599 | 28 | 2,0534 | 38 |
| 27 | Inv. Gaussian | 0,09461 | 21 | 0,24275 | 25 | 1,1103 | 25 |
| 28 | Inv. Gaussian (3P) | 0,13287 | 36 | 1,0899 | 36 | 1,4291 | 35 |
| 29 | Johnson SB | 0,06343 | 2 | 0,12404 | 4 | 0,04739 | 3 |
| 30 | Kumaraswamy | 0,14292 | 39 | 4,365 | 44 | N/A | – |
| 31 | Laplace | 0,11644 | 32 | 0,54984 | 32 | 1,8644 | 36 |
| 32 | Levy | | | Reject | | | |
| 33 | Levy (2P) | | | Reject | | | |
| 34 | Log–Logistic | 0,12391 | 34 | 0,34163 | 29 | 0,86605 | 14 |
| 35 | Log–Logistic (3P) | 0,07821 | 10 | 0,18234 | 15 | 0,91584 | 17 |
| 36 | Log-Pearson 3 | 0,07743 | 9 | 0,15157 | 7 | 0,34299 | 7 |
| 37 | Logistic | 0,08736 | 17 | 0,23458 | 24 | 1,0031 | 19 |
| 38 | Lognormal | 0,09577 | 23 | 0,22402 | 22 | 1,1293 | 28 |
| 39 | Lognormal (3P) | 0,08127 | 14 | 0,17376 | 10 | 1,1136 | 26 |
| 40 | Nakagami | 0,07875 | 12 | 0,15949 | 8 | 0,36317 | 8 |
| 41 | Normal | 0,07244 | 5 | 0,13822 | 5 | 0,03183 | 2 |
| 42 | Pareto | | | Reject | | | |
| 43 | Pareto 2 | | | Reject | | | |
| 44 | Pearson 5 | 0,10372 | 30 | 0,26755 | 26 | 1,0735 | 20 |
| 45 | Pearson 5 (3P) | 0,09124 | 19 | 0,19634 | 17 | 1,1607 | 32 |
| 46 | Pearson 6 | 0,09168 | 20 | 0,20607 | 18 | 1,1498 | 30 |
| 47 | Pearson 6 (4P) | | | Reject | | | |
| 48 | Pert | | | Reject | | | |
| 49 | Power Function | 0,118 | 33 | Reject | | 1 | 18 |
| 50 | Rayleigh | | | Reject | | | |
| 51 | Rayleigh (2P) | 0,15047 | 40 | 0,64834 | 33 | 0,57672 | 12 |
| 52 | Reciprocal | | | Reject | | | |
| 53 | Rice | | | Reject | | | |
| 54 | Uniform | | | Reject | | | |
| 55 | Wakeby | 0,06932 | 4 | 0,11185 | 1 | 0,13297 | 6 |
| 56 | Weibull | 0,07419 | 7 | 0,23448 | 23 | 0,54344 | 11 |
| 57 | Weibull (3P) | 0,08051 | 13 | 0,1758 | 13 | 1,1583 | 31 |

are unstable for a small samples and recommends PWM estimators. The PWM estimators are equivalent to L-moment estimators [29]. By using the L-moment estimators, the GEV distribution parameters are given as [29]

$$\hat{\iota} = \hat{\lambda}_1 - \frac{\hat{\nu}}{\hat{\kappa}}(1 - \Gamma(1 + \hat{\kappa})), \quad (14)$$

$$\hat{\nu} = \frac{\hat{\lambda}_2 \hat{\kappa}}{(1 - 2^{-\hat{\kappa}})\Gamma(1 + \hat{\kappa})}, \quad (15)$$

$$\hat{\kappa} = 7.8590c + 2.9554c^2, \qquad c = \frac{2}{3 + \hat{\tau}} - \left(\frac{\ln 2}{\ln 3}\right), \quad (16)$$

where $\hat{\tau} = \hat{\lambda}_3/\hat{\lambda}_2$, and $\Gamma(\cdot)$ is the complete gamma function. Parameters $\hat{\lambda}_1$, $\hat{\lambda}_2$, and$\hat{\lambda}_3$ are given as [30]

$$\hat{\lambda}_1 = b_0, \quad (17)$$

$$\hat{\lambda}_2 = 2b_1 - b_0, \quad (18)$$

$$\hat{\lambda}_3 = 6b_2 - 6b_1 + b_0, \quad (19)$$

where $b_0$, $b_1$, $b_2$ are an unbiased estimator calculated using Eq. (20)

[31],

$$b_r = \frac{1}{n}\sum_{j=1}^{n}\left[\frac{(j-1)(j-2)\cdots(j-r)}{(n-1)(n-2)\cdots(n-r)}Lp_{n-j+1:n}\right], \qquad r = 0, 1, 2. \tag{20}$$

In practice, the parameters $\iota$, $\nu$, and $\kappa$ may not be known accurately beforehand. Based on lumen degradation data available at each time $t$, the failed proportions function can be expressed as

$$F(t) = f(t, \iota_t, \nu_t, \kappa_t), \tag{21}$$

where $\iota_t$, $\nu_t$, and $\kappa_t$ can be determined at each time interval using Eq. (20).

*v). Applicability of the model*

The models (12) and (13) are applicable to LUXEON rebel LEDs operating under normal indoor conditions. In this study, these models are used to design an optimal maintenance plan of the LUXEON-based LED lighting retrofit project in residential buildings.

### 2.2. Optimization formulation

A lighting maintenance optimization problem is formulated to maximize energy savings and minimize maintenance cost. The design variables, objective function, and constraints of the optimization problem are given in the following subsections.

#### 2.2.1. Design variables

The design variables are the number of light bulbs to be replaced over the evaluation period. Let $u(t)$ denote the number of light bulbs to be replaced at time $t$. For $t = 1, 2, \cdots, K$, $U$ the design variable set, which characterizes the optimization problem, is given as

$$U = [u(1), u(2), \cdots, u(K)]^T. \tag{22}$$

The design variables are integers bound between 0 and the initial lighting population.

$$L_b = [0, \cdots, 0]^T_{K\times 1}, \quad \text{and} \quad U_b = [N(0), \cdots, N(0)]^T_{K\times 1}, \tag{23}$$

where $L_b$ and $U_b$ are the lower and upper bounds of the design variables, respectively.

#### 2.2.2. Objective function

The objectives of the lighting maintenance optimization problem are to maximize energy savings and minimize maintenance cost, which can be formulated into a multi-objective optimization problem as

$$\begin{cases} \min & -W, \\ \min & M_c. \end{cases} \tag{24}$$

The weighted sum method [32] is used to translate the problem (24) into a single objective optimization problem as

$$\min \Theta = -\omega_1 W + \omega_2 M_c, \tag{25}$$

where $\omega_1$ and $\omega_2$ are the weighting coefficients, which are in the range $[0, 1]$, and $\omega_1 + \omega_2 = 1$. The weighting coefficients are determined by the project developers, based on their preferences.

Maximum values of energy savings ($\overline{W}$) and maintenance cost ($\overline{M_c}$) are used to normalize the objective function (25). Eq. (25) is re-written as

$$\min \quad \Theta = -\omega_1 \frac{W}{\overline{W}} + \omega_2 \frac{M_c}{\overline{M_c}}. \tag{26}$$

The energy savings of the EE lighting retrofit project are estimated as the product of energy savings of each EE lamp and the number of surviving lamps.

$$W = \sum_{t=1}^{K} ES \times N(t). \tag{27}$$

By considering maintenance, the surviving population Eq. (13) is expressed as

$$N\left(t+1\right) = N(t)\prod_{j=1}^{t}\left(1 - \frac{d(j)}{N(j)}\right) + u(t), \tag{28}$$

where $ES$ is the energy savings (in kWh) per retrofit unit. The $ES$ is determined using an M&V approach [33]. M&V approach quantifies the energy consumption before and after energy efficiency measure is implemented to verify and report energy savings achieved. It is characterized by installing and maintaining energy meters, gathering data, developing methods for computation and estimates, using data for computations, and reporting verified results.

Time $t = 0$ is regarded as the installation stage, and the surviving population at $t = 0$ is equal to the initial installed lighting population $N(0)$. The failed lamps will be replaced by the same type of lamps, which will not change the population's degradation distribution. The maintenance cost of replacing the failed light bulbs by new ones is given as

$$M_c = \sum_{t=1}^{K} (\sigma_0 + L_c) \times u(t), \tag{29}$$

where $\sigma_0$ is the price (in R[2]) of each bulb, and $L_c$ is the labor cost (in R) to replace a bulb.

#### 2.2.3. Constraints

The objective function (26) is constrained by economic and population size constraints expressed in the following equations:

$$0.7 \times N(0) \leqslant N(t) \leqslant N(0), \tag{30}$$

$$\sum_{t=1}^{K} (\sigma_0 + L_c) \times u(t) \leqslant \zeta, \tag{31}$$

where $\zeta$ is the initial retrofit investment given as

$$\zeta = (\sigma_0 + L_c) \times N(0). \tag{32}$$

The constraint (30) indicates that the EE lighting retrofit project registered under an incentive energy efficiency program will earn energy saving rebates only if $N(t)$ is greater than or equal to 70% of the initial population. Also, $N(t)$ should not be higher than the initial population. The maintenance budget constraint (31) indicates that the maintenance cost over the evaluation period should not exceed the initial retrofit investment.

### 2.3. Solution methodology

The optimization problem (22)–(32) is an integer programming problem and is solved with the MIDACO solver, which is a numerical high-performance solver for single and multi-objective optimization problems. The MIDACO solver can be applied to continuous, discrete/integer, and mixed-integer problems. It is available for several programming languages including MATLAB, Octave, and Python [34]. In this study, it is used in MATLAB R2017b.

## 3. Case study

The formulated model is used to plan effective strategic maintenance for a large-scale lighting retrofit project. In South Africa, Eskom[3] in its program of residential mass roll-out (RMR) encourages project developers to implement EE lighting retrofit projects [35]. In one of the sub-RMR projects, LED light bulbs are replacing halogen light bulbs in households in different provinces of South Africa. LEDs that are

[2] South African currency (1 Rand = 0.066 USD), as at 19 December 2019.
[3] A South African electricity public utility

**Table 3**
Case study information.

| Parameter | Existing lights (halogen) | LED light bulb |
|---|---|---|
| Power | 50 W | 10 W |
| Light output | 800 lm | 800 lm |
| Lifetime | 1 500 h | 15 000–40 000 h |
| Operating hours/day | 10 h | 10 h |
| Initial population | 207 693 | 207 693 |
| Unit price | R 35 | R 50 |
| Replacement cost/bulb | - | R 7 |

installed have the equivalent lumen output to the replaced halogen. LUXEON-based LED light bulbs with a rated power of 10 W and lumen output of 800 lm are considered to replace halogen light bulbs of the rated power of 50 W and lumen output of 800 lm. The lighting retrofit project is evaluated for the duration of $K = 10$ years, with a sampling interval of one year. The replacement cost $L_c$ refers to the labour cost to change a bulb. This cost is calculated as minutes to change a bulb (task takes an average of eight minutes) divided into 60 min in an hour times hourly rate for lighting maintenance (this value is obtained from South Africa labour cost data 2008–2016). The hourly rate for lighting maintenance used is for indoor lighting applications. Thus, the unit labour cost considered is for indoor lighting retrofits. The case study key information is given in Table 3.

The EE lighting retrofit project entails a large lighting population, thus installing a light meter at each light bulb is not feasible because of the high cost of light meters. A simple random sampling approach is used to determine the required sample of lights for a given population to achieve some confidence and precision. The sample size ($n$) for the lighting population is calculated as [36]

$$n = \frac{z^2 CV^2}{p^2}, \tag{33}$$

where $z$ is the z-score, $p$ is the relative precision, and $CV$ is the standard deviation of the sampling records divided by the mean.

The CDM guidelines of 90% of the confidence interval and 10% of relative precision, and a $CV$ of 0.5 are applied in the calculation. To ensure accurate and representative luminous flux data, the following protocols are applied to measure the luminous flux: (i) only the light provided by the lamp being tested is measured; (ii) luminous flux measurements are recorded at a typical office desk height (0.76 m); (iii) the same type of calibrated lux meters are used; iv) adjacent electric lights are switched off during measurement, and (v) measurements are scheduled and taken every day after sunset to avoid daylight disturbance. The luminous flux collected at each sampling interval is normalized to the initial luminous flux.

## 4. Simulation results

To evaluate the effectiveness of the proposed maintenance plan, the no maintenance and full maintenance scenarios are calculated for comparison purposes.

### 4.1. No maintenance scenario

In the no maintenance scenario, the lighting project in Section 3 is implemented without maintenance. The number of failed and surviving lamps is estimated at each sampling interval using Eqs. (12) and (13). As shown in Fig. 1, the number of surviving lamps decreases over time and reaches the minimum number (70% of N(0)) required for an adequate illumination level after operating for 21 860 h. The total energy savings over 10 years are $177.9 \times 10^3$ MWh.

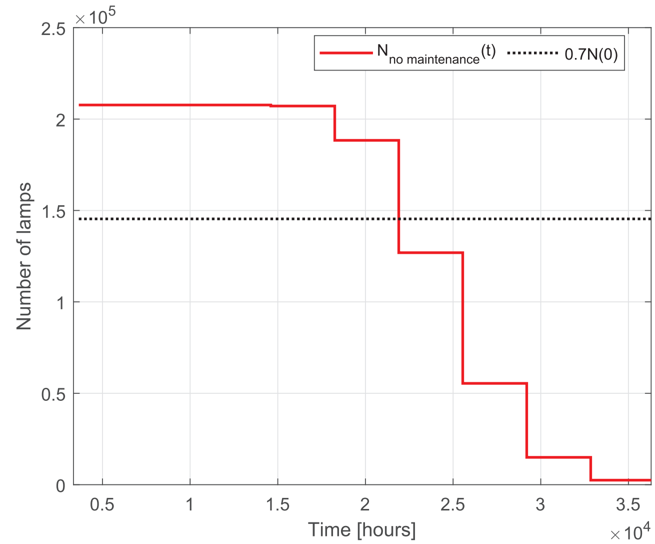


**Fig. 1.** Surviving lamp population under no maintenance and threshold surviving lamps.

### 4.2. Full maintenance scenario

Full maintenance refers to maintenance by which all lamps will be replaced by new ones once the number of surviving lamps has reached 70% of N(0). All lamps will be replaced after every 21 860 h of operation. Full maintenance will be performed once over the evaluation period. The total energy savings under full maintenance are $297.6 \times 10^3$ MWh, and the cost of full maintenance is R11 838 501.

### 4.3. Optimal maintenance scenario

In this scenario, the proposed maintenance plan is applied to the lighting retrofit project in the case study. The optimization problem is solved using the MIDACO solver. The parameters used in the solver are given in Table 4. The objectives in Eq. (26) are treated as equal, thus the weighting coefficients are equal (i.e. $\omega_1 = \omega_2 = 0.5$). At each sampling interval, the luminous flux collected is normalized to the initial luminous flux and used to calculate Eq. (20), then distribution parameters and the number of failed lamps are calculated. The energy savings and surviving population are calculated using Eqs. (27) and (28), respectively. Results show that the first replacement of 7 996 lamps will happen at 18 250 h, and 143 456 failed lamps will be replaced over the evaluation period. The optimal number of lamps to be replaced and the surviving population at each sampling interval are shown in Fig. 2. The total energy savings under the optimal maintenance plan are $282.2 \times 10^3$ MWh, and the maintenance cost is R8 176 992.

Compared to the no maintenance scenario, the optimal maintenance plan increases energy savings by 59%. A full maintenance plan produces more energy savings than optimal maintenance plan because more failed lamps are replaced, but at a higher maintenance cost. Fig. 3 compares energy savings under the no maintenance, full maintenance,

**Table 4**
Solver parameters.

| Parameter name | Value |
|---|---|
| problem.o (number of objectives) | 1 |
| problem.n (number of variables) | 10 |
| problem.ni (number of integer variables) | 10 |
| problem.m (number of constraints in total) | 21 |
| problem.me (number of equality constraints) | 0 |
| problem.xl (lower bound) | 0×ones(1,problem.n) |
| problem.xu (upper bound) | N(0)×ones(1,problem.n) |

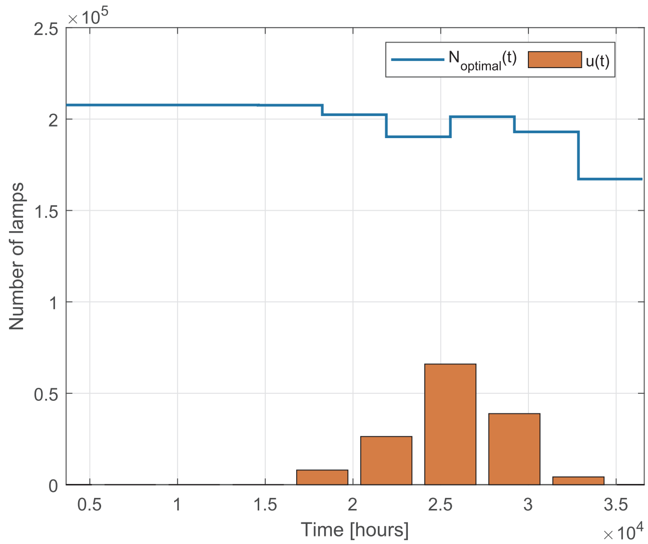


**Fig. 2.** Optimal number of lamps to be replaced and surviving population.

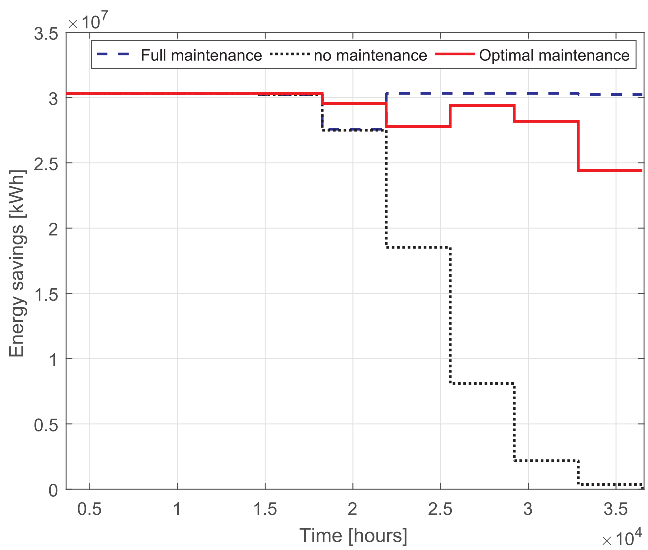


**Fig. 3.** Energy savings under no maintenance, full maintenance, and optimal maintenance scenarios.

**Table 5**
Project key performance factors analysis under full and optimal maintenance plans.

| Factor | Full maintenance (FM) | Optimal maintenance (OM) |
|---|---|---|
| Energy savings (MWh) | $297.6 \times 10^3$ | $282.2 \times 10^3$ |
| Number of replaced lamps | 207 693 | 143 456 |
| Maintenance cost (R) | 11 838 501 | 8 176 992 |
| **Performance indicator (R/MWh)** | **98.9** | **78.3** |

and optimal maintenance scenarios. Both the full maintenance and optimal maintenance plans maintain the lumen degradation within the threshold. Table 5 presents and compares the project key performance factors under the full maintenance and optimal maintenance plans. It is observed that the optimal maintenance plan is more cost-effective (with R78.3 per MWh saved) than a full maintenance plan. The ratio between the maintenance cost and additional energy savings is calculated to indicate the cost-effectiveness of different maintenance plans.

### 4.4. Sensitivity analysis

Uncertainties associated with parameters used in the optimization are discussed below.

#### 4.4.1. Weighting coefficients

The weighting coefficients are selected depending on the project developers' preferences. The higher the weighting coefficient, the more preference is given to the associated objective. Results show that the optimal number of lamps to be replaced varies with the weighting coefficients. For example, for $\omega_1 = \omega_2 = 0.5$, 143 456 failed lamps will be replaced, while for $\omega_1 = 1$, and $\omega_2 = 0$, 166 154 failed lamps will be replaced.

#### 4.4.2. Daily light usage

In the case study, the daily light usage of 10 h is considered, but the daily usage may increase or decrease depending on users' behavior and occupancy patterns. Simulation results show that the optimal number of lamps to be replaced is sensitive to the number of operating hours. When the operating hours increase, the number of lamps to be replaced increases, and when the operating hours decrease, the number of replacements decreases.

#### 4.4.3. Light price

The price of LED lights has dropped significantly over the past few years and the trend is expected to continue owing to the rapid development of LED technology. Results show that the optimal number of lamps to be replaced is sensitive to the unit price of LEDs. When the unit price of the LEDs decreases, the number of lamps to be replaced increases.

#### 4.4.4. Lighting population size

In lighting retrofit projects, the initial population size differs from one project to another. Results show that the optimal number of lamps to be replaced increases when the initial population increases and decreases when the initial population decreases. For example, if the initial population is increased by 25%, the number of lamps to be replaced increases to 173 302 lamps. Table 6 presents the sensitivity of the optimization parameters.

## 5. Conclusion

An optimal maintenance plan for an LED lighting retrofit project is studied. A lumen degradation failure model is developed for LUXEON-based LED lights. Based on the statistical properties of the degradation rates, the cumulative probability of failure distribution and the survival function are modeled. The formulated survival function is incorporated

**Table 6**
Sensitivity analysis of the optimization parameters.

| | Weighting coefficients | | Daily light usage | | Light price | | Initial population | |
|---|---|---|---|---|---|---|---|---|
| Factor | $\omega_1 = \omega_2 = 0.5$ | $\omega_1 = 1, \omega_2 = 0$ | 10 h | 4 h | R 50 | R 40 | 207,693 | 259,617 |
| Energy savings (GWh) | 282.2 | 287.4 | 282.2 | 286.3 | 282.2 | 283.7 | 282.2 | 191.6 |
| Number of replaced lamps | 143,456 | 166,154 | 143,456 | 0 | 143,456 | 145,231 | 143,456 | 173,302 |
| Maintenance cost (R) | 8,176,992 | 9,470,778 | 8,176,992 | 0 | 8,176,992 | 6,825,857 | 8,176,992 | 9,878,214 |

into the lighting maintenance optimization problem to balance energy savings and maintenance costs. A case study carried out shows that, in 10 years, the optimal lighting maintenance plan would save up to 59% of lighting energy consumption with acceptable maintenance costs. It is found that the proposed maintenance plan is more cost-effective than full maintenance. It is concluded that lumen degradation failure should be considered when investigating the performance of lighting retrofit projects, as this may not only affect energy savings but also reduce the level of illumination, which can cause visual discomfort.

## CRediT authorship contribution statement

**Alice Ikuzwe:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - original draft, Writing - review & editing. **Xiaohua Xia:** Supervision. **Xianming Ye:** Supervision.

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## References

[1] Michaelowa A, Jotzo F. Transaction costs, institutional rigidities and the size of the clean development mechanism. Energy Policy 2005;33(4):511–23.
[2] Eskom, The measurement and verification guideline for demand-side management projects., Tech. rep., Eskom; 2011.
[3] Bertoldi P, Rezessy S. Tradable white certificate schemes: fundamental concepts. Energ Effi 2008;1(4):237–55.
[4] UNFCCC, Approved baseline and monitoring methodology, distribution of efficient light bulbs to household, Tech. rep.; 2007.
[5] Van Driel WD, Fan X, Zhang GQ. Solid state lighting reliability Part 2: Components to systems vol. 3. Switzerland: Springer International Publishing; 2017.
[6] Chang M-H, Das D, Varde P, Pecht M. Light emitting diodes reliability review. Microelectron Reliab 2012;52(5):762–82.
[7] Gorjian N, Ma L, Mittinty M, Yarlagadda P, Sun Y. A review on degradation models in reliability analysis. Engineering asset lifecycle management. Springer; 2010. p. 369–84.
[8] Yang S-C, Lin P, Wang C-P, Huang SB, Chen C-L, Chiang P-F, Lee A-T, Chu M-T. Failure and degradation mechanisms of high-power white light emitting diodes. Microelectron Reliab 2010;50(7):959–64.
[9] CDM, AMS-II.J: Demand-side activities for efficieny lighting technologies, Tech. rep. (Version 05.0).
[10] Ye X, Xia X, Zhang J. Optimal sampling plan for clean development mechanism energy efficiency lighting projects. Appl Energy 2013;112:1006–15.
[11] Botha-Moorlach M, Mckuur G. A report on the factors that influence the demand and energy savings for Compact Fluorescent Lamp door-to-door rollouts in South Africa, Eskom Energy Research Centre, Cape Town; 2009.
[12] Carstens H, Xia X, Ye X. Improvements to longitudinal clean development mechanism sampling designs for lighting retrofit projects. Appl Energy 2014;126:256–65.
[13] Carstens H, Xia X, Zhang J, Ye X. Characterising compact fluorescent lamp population decay. In: AFRICON, 2013, IEEE; 2013. p. 1–5.
[14] Ye X, Xia X, Zhang L, Zhu B. Optimal maintenance planning for sustainable energy efficiency lighting retrofit projects by a control system approach. Control Eng Pract 2015;37:1–10.
[15] Ikuzwe A, Ye X, Xia X. Energy-maintenance optimization for retrofitted lighting system incorporating luminous flux degradation to enhance visual comfort. Appl Energy 2020;261:114379.
[16] Wang H. A survey of maintenance policies of deteriorating systems. Eur J Oper Res 2002;139(3):469–89.
[17] Lee H, Cha JH. New stochastic models for preventive maintenance and maintenance optimization. Eur J Oper Res 2016;255(1):80–90.
[18] O'Connor P, Kleyner A. Practical reliability engineering. New York: John Wiley & Sons; 2012.
[19] UNFCCC. Approved small scale methodology AMS II.C.: demand-side energy efficiency activities for specific technologies., Tech. rep. (Version 14.0; 2012b.).
[20] Wang F-K, Lu Y-C. Useful lifetime analysis for high-power white LEDs. Microelectron Reliab 2014;54(6):1307–15.
[21] Van Driel WD, Fan X, Zhang GQ. Solid state lighting reliability Part 2: Components to systems vol. 3. New York: Springer International Publishing; 2017.
[22] Bierman A, Freyssinier JP, Zhu Y, Narendran N. LED Testing Standards Overview, Tech. rep., Lighting Research Center, Rensselaer Polytechnic Institute, Troy, NY, USA; 2012.
[23] Lumileds Philips. IESNA LM-80 Test Report, Tech. rep., Stunning Lighting Technology CO., Ltd; 2012.
[24] De Oliveira VRB, Colosimo EA. Comparison of methods to estimate the time-to-failure distribution in degradation tests. Quality Reliab Eng Int 2004;20(4):363–73.
[25] Martins ES, Stedinger JR. Generalized maximum-likelihood generalized extreme-value quantile estimators for hydrologic data. Water Resour Res 2000;36(3):737–44.
[26] Miller Jr RG. Survival analysis vol. 66. New York: John Wiley & Sons; 2011.
[27] Ailliot P, Thompson C, Thomson P. Mixed methods for fitting the GEV distribution. Water Resources Res. 2011; 47 (5).
[28] Hosking JR, Wallis JR, Wood EF. Estimation of the generalized extreme-value distribution by the method of probability-weighted moments. Technometrics 1985;27(3):251–61.
[29] Hosking JR. L-moments: analysis and estimation of distributions using linear combinations of order statistics. J Roy Stat Soc Ser B (Methodological) 1990:105–24.
[30] Wang Q. Direct sample estimators of L -moments. Water Resour Res 1996;32(12):3617–9.
[31] Hosking J, Wallis J. A comparison of unbiased and plotting-position estimators of L-moments. Water Resour Res 1995;31(8):2019–25.
[32] Marler RT, Arora JS. The weighted sum method for multi-objective optimization: new insights. Struct Multidiscip Optim 2010;41(6):853–62.
[33] Xia X, Zhang J. Mathematical description for the measurement and verification of energy efficiency improvement. Appl Energy 2013;111:247–56.
[34] Schleuter M, Munetomo M. Introduction to MIDACO-SOLVER software, Tech. rep.; 2013.
[35] ASSAF. The state of research, development and innovation of electrical energy efficiency technologies in South Africa, Tech. rep., Academy of Science of South Africa; 2017.
[36] Lynn P. Principles of Sampling. Res Meth Postgraduates 2002;2:185–94.