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Optimal maintenance planning in building retrofitting with interacting energy effects

Bo Wang[1,2] | **Xiaohua Xia**[2] | **Zhongtao Cheng**[1] | **Lei Liu**[1]

[1]Key Laboratory of Image Processing and Intelligent Control, School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan, China

[2]Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria, South Africa

Correspondence
Zhongtao Cheng, Key Laboratory of Ministry of Education for Image Processing and Intelligent Control, Artificial Intelligence and Automation School, Huazhong University of Science and Technology, Wuhan 430074, China.
Email: ztcheng@hust.edu.cn

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Summary
This paper extends a control system framework for the maintenance planning investment decision for building energy retrofitting. The interacting energy and reliability effects that are ignored by previous models are incorporated in the current study. A set of energy efficiency and population decay models with interacting parameters and decision variables are established, and a state-space model with coupled nonlinear equations is obtained. The control objectives are maximizing the energy savings and financial paybacks with limited budget during a finite time period. An model predictive control (MPC) controller is designed for the problem. The interacting effects and effectiveness of the proposed approach are verified by the case study, where improvements from the modeling considering interaction are revealed.

KEYWORDS
building retrofitting, control system modeling, energy efficiency, lighting/air conditioner interactions, maintenance planning

1 | INTRODUCTION

Improving energy efficiency in existing buildings is an important part of today's movements for energy efficiency. The building energy retrofitting[1,2] and accordingly the energy-oriented maintenance plan optimization (MPO)[3-5] are the major approaches with this purpose. The maintenance planning aims at sustaining as much energy performances as possible during operation, given an implemented building energy retrofitting project. Ye et al[3] took into account the MPO in a light retrofitting project for the sustainable energy efficiency and financial performances over the long term. Wang et al[4] extended the MPO problem to a building retrofitting project involving lighting, HVAC, and other office appliances.

In above studies, the MPO problems are actually management level investment decision issues. The term “management level” implies a fact that the operation and management of a single device is not the focus. Instead, the major concern is the resource allocation and planning for an energy efficiency project under the constraints such as limited budget and manpower. For the MPO at management level, the retrofitted items are categorized into several homogeneous item groups. A hypothesis suggests that three kinds of characteristics are employed to categorize the homogeneous item groups: the inherent energy and reliability performance, the operational environment of the items, and the corresponding operating schedules.[4] Consequently, each item group consists of homogeneous items, which can be considered to manifest same energy consumptions and failure rate under the same condition. Thereafter, the aggregated energy and economy performances of a group can be obtained from the performances of one single homogeneous item and the corresponding population. The term "population" is adopted to represent the overall number of functioning retrofitted items from one item group. Such a population is not constant during operation, the retrofitted item failures result in the decrease of the number of available devices in an item group, namely the population decay. The measurement and verification (M&V) principles[6] interpret that a retrofitted item contributes zero energy saving if it becomes malfunctioning. Therefore, the population decay results in the deterioration of the overall performances, for example, the energy savings. Such a population decay can be extracted from the statistical sources or first principle models, which leaves space for further improvements. At the current stage, several methods have been employed to model the population dynamics in literatures. The Clean Development Mechanism (CDM) studies adopted either a linear assumption[7] or an experimental data fitting.[8] Wang et al[4] employed reliability functions for different types of items from reliability engineering[9] to develop the population decay model.

The impacts of maintenance to performances have been extensively studied in past decades.[10-13] From the energy retrofitting perspective, maintenance actions can restore the energy savings via reversing the population decay, that is, restoring a number of failed devices to normal working condition. In this way, the maintenance influences the overall energy savings of a retrofitting project. Such maintenance can be realized by either repair or replacement. An assumption is applied that the performances of a single device will not change after the repair or replacement, that is, the device is restored to an "as good as new" condition. The term "maintenance intensity" is adopted to describe the number of the implemented maintenance actions upon a specific time, namely the "maintenance instant." The number of actions are integers, thus there is an integer nature with the maintenance intensity. Thereafter, the maintenance plan optimization is implemented via selecting proper maintenance intensities and time scheduling, such that energy savings and other performances can be maximized against failures. The cash flows during operation can also be obtained from the item group populations and maintenance intensities. The economy performances, for example, the financial payback, can be identified consequently. In this way, the aggregated performances, which are under the influence of the population decay and maintenance, can be evaluated.

Such a population dynamics can be characterized and formulated as a control system, where the MPO is cast into an optimal control problem and control system approaches can be introduced. In our previous studies,[4,5] a set of grouping based energy efficiency models have been developed, subject to population decay models following statistical laws. However, the possible interacting energy efficiency effects between different categories of devices have been ignored in these previous studies. Actually, such interplay in buildings have been a widely investigated topic. For example, the ASHRAE* 2009 fundamental handbook[14] introduced the complicated composition of the heating/cooling load for an air conditioning system in buildings. For example, the ASHRAE† fundamental handbook of 2009[14] gives a detailed introduction to the heating/cooling load for a building air conditioning system, which involves a series of heat transmissions, including the heat transmission through the building envelope and ventilation, the heat gain from the solar radiation and appliances. The air conditioner heating/cooling load computation, which is essential to the air conditioning system energy consumption estimation, takes into account such heat dynamics.

According to existing studies, the air conditioner heating/cooling load receive significant impacts from the nearby electrical appliances, such as the computers, monitors, lights, etc, which contribute significant heat gains to the heating/cooling load. Sezgen[15] and Zmeureanu[16] evaluated the lights and air conditioners' interacting energy effects. Ahn et al[17] incorporates such an interaction into the energy efficiency of air conditioning system by directing the convective heat from LED lighting. When taking into account the malfunctions of the retrofitted devices, such an interacting energy efficiency effects can further result in interplays in the population decay of different groups. Breuker and Braun[18] identified that the rooftop air conditioners suffer from a series of failures that are resulted from the unmatched heating/cooling loads. For example, when the working load gets very high, the compressor motor can overload and damage itself. When the load is too small than the designed value, liquid flood back can take place, which can cause the failure of the compressor. Generally, the building air conditioners are sized on a basis of normal working loads. The rated power of an air conditioner is selected to match the peak demand, which is estimated with an assumption that the appliances are all working properly. In practice, the appliance degradation, especially the item population decay, can lead to abnormal working load of the air conditioning system, resulting in a higher failure rate. For example, the population decay of lighting can significantly reduce the working load, and the failure of fan coil units implies higher working load for each working air conditioner in the area. Such interacting energy efficiency and reliability effects suggest that population dynamics actually have coupled state variables. Furthermore, there can be hidden interplays within the energy efficiency

*American Society of Heating, Refrigerating and Air-conditioning Engineers.

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models. It is reasonable to assume that significant interacting effects must be taken into account, and the previous models in References 4 and 5 can be incomplete.

This study aims at improving the existing control system framework by incorporating the aforementioned interacting energy efficiency and reliability effects. The possible impacts of such interplays are evaluated, based on which the energy efficiency and population decay models are modified from ones with independent parameters to ones with coupled parameters and decision variables. Results from the aforementioned studies are employed to evaluate the interplays between lighting and air conditioning units. Due to the lack of the progress in these relevant studies, the current stage model also focuses on the lighting and air conditioning unit groups, where several assumptions are employed to allow the control system framework modeling. First, only lights and air conditioners are investigated. Second, a common space is involved, where all items are installed and contribute to heating/cooling loads. Third, heat sources other than lights and air conditioners are considered to have been identified a priori. The current investigation adopts the control framework from previous studies. There are two coupled state variables in the model: the lights group and air conditioner group populations. The decision variables are the maintenance intensities upon predecided maintenance instants. The control objectives are to maximize the aggregate energy savings and the financial payback. The constraints are the targeted energy savings, the budget limit, and the human comfort satisfactions. The uncertainties are inevitable in such an MPO problem. The uncertainty factors can come from various resources, for example, modeling uncertainties, measurement uncertainties, and sampling uncertainties.[19] Detailed explanations can be found from previous studies.[20-22] Given the main focus of the proposed approach is improving the existing control system framework, the uncertainties are simplified to be a random disturbance on the state variables, as such an interpretation is incomprehensive but easier to implement. A model predictive control (MPC) approach is adopted for the optimal control based maintenance plan optimization. Taking advantage of the data from a practical retrofitting project, a case study is investigated, where simulation results exhibit the effectiveness of the proposed approach. For the sake of the comparative study, a maintenance plan without considering the interaction is generated and tested.

The remainder of the paper consists of four sections: Section 2 models the coupled system dynamics given the interacting energy and reliability effects. Section 3 introduces the MPC based approach to solve the formulated MPO problem. Section 4 introduces the simulation exercise, illustrates the results, analysis, and comparisons. Section 5 is the conclusion.

2 | CONTROL PROBLEM FORMULATION

2.1 | Variable definition

Given that two retrofitted item groups are investigated at the current stage. Let $x_{\mathrm{L}}(t_k)$ denote the lighting group population, and $x_{\mathrm{AC}}(t_k)$ the air conditioner group population, where $t_k = kS, k = 0, 1, 2, \dots, T$ are the sampling instants. A finite time horizon namely the sustainability period is defined as $[0, TS)$, over which the item group populations and performances are estimated. S denotes the sampling interval. The system state is given as follows:

$$\mathbf{x}(t_k) = (x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k))^T, \tag{1}$$

where $\mathbf{x}(t_0) = (x_{\mathrm{L}}^0, x_{\mathrm{AC}}^0)^T$. The control inputs upon time t_k are represented as follows:

$$\mathbf{u}(t_k) = (u_{\mathrm{L}}(t_k), u_{\mathrm{AC}}(t_k))^T. \tag{2}$$

For simplicity of the derivation, let $\mathbf{x}$ denote the system states and $\mathbf{u}$ the control inputs. At the current stage, $\mathbf{x}$ and $\mathbf{u}$ are integers given the integer nature of the population and maintenance intensities. In order to characterize the maintenance time scheduling, $Q = \{m_1, m_2, \dots, m_N\}$ is adopted to describe the maintenance instants. For simplicity, the maintenance instants are assumed to be commensurate with t_k, that is, the elements of Q are selected from $k = 0, 1, 2, \dots, T$, such that m_i with $i = 1, 2, \dots, N$ are actually indices of the sampling instants. In this way, $u_{\mathrm{AC}}(t_k)$ are forced to be 0 at t_k with $k \notin Q$. As aforementioned, the maintenance instants are decided a priori, therefore Q is a collection of constants.

2.2 | Interacting energy efficiency modeling

The energy consumption per item is assumed to have been identified from the site visit. Let $a_{\mathrm{L}}(t_k)$ denote the lighting consumption per unit time per item. The overall lighting energy consumption is the summation of all light consumptions, that is, $E_{\mathrm{L}}(t_k) = a_{\mathrm{L}}(t_k)x_{\mathrm{L}}(t_k)$. According to previous studies, a lighting unit manifest nearly constant power consumption during operation.[3] Such an electric light contribute to the sensible heat gain of the building. The heat gain is computed from its instantaneous rate q_{el} (Btu/h):

$$q_{\mathrm{el}} = 3.41WF_{\mathrm{ul}}F_{\mathrm{sa}}, \tag{3}$$

where W is the rated power of a light. F_{ul} denotes the lighting use factor, which is the ratio of the light's actual power consumption against its overall installed capacity. F_{sa} denotes the lighting special allowance factor, which is the ratio of the lighting fixtures' power consumption, including lamps and ballast, to the nominal power consumption of the lamps.[14] 3.41 is the conversion factor. Thereafter, the lighting heat gain $Q_{\mathrm{L}}(t_k)$ is estimated according to (3):

$$Q_{\mathrm{L}}(t_k) = f_{\mathrm{r}}E_{\mathrm{L}}(t_k), \tag{4}$$

where f_{r} denotes the fraction of lighting heat gain goes into the room, namely the space fraction. Reference values to estimate the space fraction can be found from References 14 and 23.

Similarly, the air conditioner working load can be estimated. It consists of two kinds, the heating load and cooling load. An air conditioner instantaneous energy consumption is decided by its working load as well as the heating/cooling efficiency. Let $Q_{\mathrm{HD}}(t_k)$ denote the total heating load in the working zone and $Q_{\mathrm{CD}}(t_k)$ the total cooling load. The estimations of $Q_{\mathrm{HD}}(t_k)$ and $Q_{\mathrm{CD}}(t_k)$ are given as follows:

$$\begin{aligned} Q_{\mathrm{HD}}(t_k) &= Q'_{\mathrm{HD}}(t_k) - Q_{\mathrm{L}}(t_k), \\ Q_{\mathrm{CD}}(t_k) &= Q'_{\mathrm{CD}}(t_k) + Q_{\mathrm{L}}(t_k), \end{aligned} \tag{5}$$

where the heating/cooling load is separated into two parts. $Q_{\mathrm{L}}(t_k)$ indicates the heat emitted by the working lights.[14] $Q'_{\mathrm{HD}}(t_k)$ indicates the total heat loss from the other resources, and $Q'_{\mathrm{CD}}(t_k)$ the total heat gain from the other resources. As aforementioned, $Q'_{\mathrm{HD}}(t_k)$ and $Q'_{\mathrm{CD}}(t_k)$ are identified a priori. The air conditioner energy consumption during $[t_{k-1}, t_k)$ is estimated by:

$$a_{\mathrm{AC}}(t_k) = \rho_{\mathrm{h}}(t_k)\varepsilon_{\mathrm{h}}(t_k)\frac{Q_{\mathrm{HD}}(t_k, x_{\mathrm{L}}(t_k))}{x_{\mathrm{AC}}(t_k)} + \rho_{\mathrm{c}}(t_k)\varepsilon_{\mathrm{c}}(t_k)\frac{Q_{\mathrm{CD}}(t_k, x_{\mathrm{L}}(t_k))}{x_{\mathrm{AC}}(t_k)}, \tag{6}$$

where $\rho_{\mathrm{h}}(t_k)$ and $\rho_{\mathrm{c}}(t_k)$ are heating/cooling indicators, subject to

$$\begin{cases} \rho_{\mathrm{h}}(t_k) + \rho_{\mathrm{c}}(t_k) = 1, \\ \rho_{\mathrm{h}}(t_k), \rho_{\mathrm{c}}(t_k) = 0 \text{ or } 1. \end{cases} \tag{7}$$

$(t_k, x_{\mathrm{L}}(t_k))$ is employed in (6) to emphasize the interplays between lights and air conditioners, following an assumption that working air conditioners undertake evenly distributed heating/cooling load in a common working zone. Furthermore, the heating efficiency is denoted by $\varepsilon_{\mathrm{h}}(t_k)$ and cooling efficiency $\varepsilon_{\mathrm{c}}(t_k)$. The estimation of the hearing/cooling efficiency is difficult. The heating seasonal performance factor (HSPF) and seasonal energy efficiency ratio (SEER), defined in AHRI Standard 210/240,[‡] are the measurements of the air conditioner heating/cooling efficiency. From the documents, the heating/cooling efficiency varies under the impact of a series of factors, for example, the ratio of the actual heating/cooling load against the system's rated capacity. Taking into account the aforementioned interplays, $\varepsilon_{\mathrm{h}}(t_k)$ and $\varepsilon_{\mathrm{h}}(t_k)$ are rewritten into $\varepsilon_{\mathrm{h}}(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k))$ and $\varepsilon_{\mathrm{h}}(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k))$. The aggregated air conditioner energy consumption is thus estimated by $E_{\mathrm{AC}}(t_k) = a_{\mathrm{AC}}(t_k)x_{\mathrm{AC}}(t_k)$.

Given the interplays, the energy savings from retrofitted lights, denoted by $S_{\mathrm{L}}(t_k)$, and from retrofitted air conditioners, denoted by $S_{\mathrm{AC}}(t_k)$, are estimated as the following:[6]

$$\begin{aligned} S_{\mathrm{L}}(t_k) &= x_{\mathrm{L}}(t_k)(\bar{a}_{\mathrm{L}}(t_k) - a_{\mathrm{L}}(t_k)), \\ S_{\mathrm{AC}}(t_k) &= \bar{E}_{\mathrm{AC}}(t_k) - a_{\mathrm{AC}}(t_k)x_{\mathrm{AC}}(t_k), \end{aligned} \tag{8}$$

where $\bar{a}_{\mathrm{L}}(t_k)$ represents the preimplementation wattage per light, and $\bar{E}_{\mathrm{AC}}(t_k)$ the aggregate energy consumption from the previous air conditioners. The pose-retrofit consumptions $a_{\mathrm{L}}(t_k)$ and $a_{\mathrm{AC}}(t_k)$ are computed under the same working

[‡]Air Conditioning, Heating, and Refrigeration Institute in its 2008 standard AHRI 210/240, Performance Rating of Unitary Air-Conditioning and Air-Source Heat Pump Equipment.

conditions as $\bar{a}_{\mathrm{L}}(t_k)$ and $\bar{E}_{\mathrm{AC}}(t_k)$. The corresponding cost savings are then estimated:

$$\begin{aligned} C_{\mathrm{L}}(t_k) &= p(t_k)S_{\mathrm{L}}(t_k), \\ C_{\mathrm{AC}}(t_k) &= p(t_k)S_{\mathrm{AC}}(t_k), \end{aligned} \tag{9}$$

where $p(t_k)$ indicates the electricity price upon time. As (5)–(9) describe the interplay between lights and air conditioners, the coupled system states are introduced.

2.3 | The population dynamics modeling with interactions

First, the population dynamics from previous studies are adopted:[3,4]

$$\begin{bmatrix} x_{\mathrm{L}}(t_{k+1}) \\ x_{\mathrm{AC}}(t_{k+1}) \end{bmatrix} = \begin{bmatrix} G_{\mathrm{L}}(x_{\mathrm{L}}(t_k)) \\ G_{\mathrm{AC}}(x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) \end{bmatrix} + \begin{bmatrix} u_{\mathrm{L}}(t_k) \\ u_{\mathrm{AC}}(t_k) \end{bmatrix}, \tag{10}$$

where $x_{\mathrm{L}}(t_0) = x^0_{\mathrm{L}}$, $x_{\mathrm{AC}}(t_0) = x^0_{\mathrm{AC}}$ indicate the initial state. $G_{\mathrm{L}}(x_{\mathrm{L}}(t_k))$ and $G_{\mathrm{AC}}(x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k))$ denote the population decay of the lighting group and air conditioner group, respectively. $u_{\mathrm{L}}(t_k)$ and $u_{\mathrm{AC}}(t_k)$ denote the control inputs, that is, the respective maintenance intensities for the light group and air conditioner group upon time t_k.

The interacting reliability effects between the light group and the air conditioner group are estimated in the following way. First, the lighting group population decay is characterized by a model from Carstens et al.[8] The air conditioner group population decay is subject to Weibull distribution according to Kwak et al,[24] where random failures of air conditioner systems are taken into account. The population decay formulations are therefore given:

$$G_{\mathrm{L}}(x_{\mathrm{L}}(t_k)) = \frac{b_{\mathrm{L}} c_{\mathrm{L}} x_{\mathrm{L}}(t_k)^2}{x^0_{\mathrm{L}}} - b_{\mathrm{L}} x_{\mathrm{L}}(t_k) + x_{\mathrm{L}}(t_k), \tag{11}$$

$$G_{\mathrm{AC}}(x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) = \left(1 - \frac{\beta t_k^{\beta-1}}{\eta^{\beta}}\right) x_{\mathrm{AC}}(t_k). \tag{12}$$

The identifications of decay parameters b_{L} and c_{L} can be found from Reference 1. For the Weibull distribution based population decay, the shape parameter β and scale parameter η must be figured out. According to Reference 9, the mean time between failure (MTBF) can be adopted to estimate η. For an air conditioner, too heavy or too small loads can incur the compressor overloads, which imply a higher chance of failures.[18] As a result, $\eta(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k))$ is employed, which is considered to be a variable related to item group populations. Due to the lack of accurate quantification of η for an air conditioner upon damages and deterioration, an η estimator is employed, based on a piecewise exponential formulation.

Given the qualitative result from the existing study:[18] when the working load q_{AC} increases or decreases far enough from the rated capacity, failures can occur more frequently, implying the reduced MTBF, that is, η. Such an estimation of η can result in additional uncertainties, as soft faults are also possible during operation and there lacks prior knowledge on its impacts to the population decay. In our investigated model, such uncertainties result in the parameter uncertainties in the linear system dynamics (12). The MPC approach has been verified in existing studies to be able to overcome such parameter uncertainties, for example.[25,26] Therefore, such uncertainties are believed to be nondominant for the performance of MPC.

A heating load ratio $p_{\mathrm{h}}(t_k)$ is introduced to indicate the ratio of the actual heating load $Q_{\mathrm{HD}}(t_k)$ against the rated heating capacity $\overline{q}_{\mathrm{h}}$. Similarly, a cooling load ratio $p_{\mathrm{c}}(t_k)$ indicates the ratio of the actual cooling load $Q_{\mathrm{CD}}(t_k)$ against the rated heating capacity $\overline{q}_{\mathrm{c}}$. According to (5) and (6)), $p_{\mathrm{h}}(t_k)$ and $p_{\mathrm{c}}(t_k)$ can be formulated as follows:

$$\begin{cases} p_{\mathrm{h}}(t_k) = \frac{Q_{\mathrm{HD}}(t_k)}{\overline{q}_{\mathrm{h}} x_{\mathrm{AC}}(t_k)} * 100\%, \\ \qquad\quad = \frac{Q'_{\mathrm{HD}}(t_k) - q_{\mathrm{L}}(t_k) x_{\mathrm{L}}(t_k)}{\overline{q}_{\mathrm{h}} x_{\mathrm{AC}}(t_k)} * 100\%, \text{ if } \rho_{\mathrm{h}} = 1, \\ p_{\mathrm{c}}(t_k) = \frac{Q_{\mathrm{CD}}(t_k)}{\overline{q}_{\mathrm{c}} x_{\mathrm{AC}}(t_k)} * 100\%, \\ \qquad\quad = \frac{Q'_{\mathrm{CD}}(t_k) + q_{\mathrm{L}}(t_k) x_{\mathrm{L}}(t_k)}{\overline{q}_{\mathrm{c}} x_{\mathrm{AC}}(t_k)} * 100\%, \text{ if } \rho_{\mathrm{c}} = 1. \end{cases} \tag{13}$$

η_0 is adopted to represent the nominal value of the scale parameter η. The context based $\eta(t_k, x_\mathrm{L}(t_k), x_\mathrm{AC}(t_k))$ is estimated upon the varying load ratios $p_\mathrm{h}(t_k)$ and $p_\mathrm{c}(t_k)$:

$$\eta(t_k, x_\mathrm{L}(t_k), x_\mathrm{AC}(t_k)) = \begin{cases} \eta_0 \frac{1}{a_{h,l}+e^{-k_{h,l}p_\mathrm{h}(t_k)+b_{h,l}}}, & p_\mathrm{h}(t_k) < Th_{\mathrm{h},l}\%, \\ \eta_0, & Th_{\mathrm{h},l}\% \le p_\mathrm{h}(t_k) \le Th_{\mathrm{h},r}\%, \\ \eta_0 \frac{1}{a_{h,r}+e^{k_{h,r}p_\mathrm{h}(t_k)-b_{h,r}}}, & p_\mathrm{h}(t_k) > Th_{\mathrm{h},r}\%, \end{cases} \tag{14}$$

$$\eta(t_k, x_\mathrm{L}(t_k), x_\mathrm{AC}(t_k)) = \begin{cases} \eta_0 \frac{1}{a_{c,l}+e^{-k_{c,l}p_\mathrm{c}(t_k)+b_{c,l}}}, & p_\mathrm{c}(t_k) < Th_{\mathrm{c},l}\%, \\ \eta_0, & Th_{\mathrm{c},l}\% \le p_\mathrm{c}(t_k) \le Th_{\mathrm{c},r}\%, \\ \eta_0 \frac{1}{a_{c,r}+e^{k_{c,r}p_\mathrm{c}(t_k)-b_{c,r}}}, & p_\mathrm{c}(t_k) > Th_{\mathrm{c},r}\%, \end{cases} \tag{15}$$

where the parameters are positive constants. A pair of threshold points are involved, denoted by $Th_{\mathrm{h},l}$ and $Th_{\mathrm{h},r}$, respectively. The threshold points are heating load ratios. η stays constant, when the actual heating load ratio $p_\mathrm{h}(t_k)$ is between $Th_{\mathrm{h},l}\%$ and $Th_{\mathrm{h},r}\%$. It becomes decreasing when $p_\mathrm{h}(t_k) < Th_{\mathrm{h},l}\%$ or $p_\mathrm{h}(t_k) > Th_{\mathrm{h},r}\%$. Similarly, $Th_{\mathrm{c},l}$ and $Th_{\mathrm{c},r}$ represent the pair of threshold points for the cooling load ratio $p_\mathrm{c}(t_k)$. Equation (15) applies when $\rho_\mathrm{h} = 1$ and (15) applies when $\rho_\mathrm{c} = 1$.

In this way, a control system formulation with coupled state variables is derived by (10)-(15). In practice, the model estimation is realized by model parameter identification, and such an estimation can be done from several perspectives. For an existing building, data collection and data fitting over a significant time interval is the best choice. If such a data collection is unavailable, the expert knowledge from the relevant field and the experience on similar problems can be taken advantage of to estimate the parameters.

2.4 | Performance indicators formulation

As aforementioned, taking advantage of the preimplementation audits and evaluation of some parameters are considered known a priori, that is, the lights and air conditioners working load, the lights and air conditioners maintenance cost per item, and the maintenance time scheduling. The unit maintenance costs are represented by $mc_\mathrm{L}(t_k)$ and $mc_\mathrm{AC}(t_k)$, respectively. Furthermore, with (8), the energy savings are formulated as the following:

$$\begin{cases} ES(t_k) = S_\mathrm{L}(t_k) + S_\mathrm{AC}(t_k), \\ ES|_\mathrm{all} = \sum_{k=1}^{T} ES(t_k), \end{cases} \tag{16}$$

where $ES(t_k)$ denotes the aggregate energy savings over a sampling interval and $ES|_\mathrm{all}$ the total amount of the sustainability period energy savings. The cost savings are formulated accordingly:

$$\begin{cases} C(t_k) = C_\mathrm{L}(t_k) + C_\mathrm{AC}(t_k), \\ C|_\mathrm{all} = \sum_{k=1}^{T} C(t_k). \end{cases} \tag{17}$$

The aggregate maintenance cost is:

$$h(t_k) = mc_\mathrm{L}(t_k)u_\mathrm{L}(t_k) + mc_\mathrm{AC}(t_k)u_\mathrm{AC}(t_k), \tag{18}$$

as well as the overall investment:

$$h|_\mathrm{all} = h_0 + \sum_{k=1}^{T} h(t_k). \tag{19}$$

The initial investment, namely the retrofitting cost, is denoted by h_0. Let $P = C|_\mathrm{all} - h|_\mathrm{all}$ denote the project profit. However, the net present value (NPV) is a more common way to evaluate the project's financial payback over $[0, TS)$. The formulation of NPV is given as follows:

$$\mathrm{NPV} = \sum_{k=1}^{T} \frac{C(t_k) - h(t_k)}{(1+d)^{\phi(t_k)-1}} - h_0, \tag{20}$$

where d denotes the discount rate. The NPV is computed on an annual basis, therefore $\phi(t_k) = 1, 2, 3 \ldots$ is employed, such that t_k within a same year can refer to the same $\phi(t_k)$. Thereafter, the financial payback is quantified by the internal rate of return (IRR), which is represented by $d_R|_T$. The $d_R|_T$ is computed via the following:

$$\sum_{k=1}^{T} \frac{B(t_k) - h(t_k)}{(1+d_R|_T)^{\phi(t_k)-1}} - h_0 = 0. \tag{21}$$

Equation (21) finds a discount rate, such that the NPV equals to 0 over the sustainability period. It is difficult to represent IRR in an analytical manner. However, given a nonzero amount of h_0, the IRR is guaranteed to be bounded.[27]

2.5 | Optimal control problem formulation

The maintenance plan optimization is formulated in an optimal control manner as follows:

Optimization Problem $\mathcal{MPO}$:

Find the optimal maintenance plan $\mathbf{u} = \{u_{\mathrm{L}}(t_{m_1}), u_{\mathrm{AC}}(t_{m_1}), \ldots, u_{\mathrm{L}}(t_{m_N}), u_{\mathrm{AC}}(t_{m_N})\}$, which minimizes the following performance index:

$$J(\mathbf{x}_0, Q, \mathbf{u}(\cdot)) = -\lambda_1 \frac{ES|_{\mathrm{all}}}{\alpha} - \lambda_2 d_R|_T, \tag{22}$$

subject to (10)-(15), the predecided maintenance time schedule $Q = \{m_1, \ldots, m_N\}$, and the following constraints:

$$\begin{cases} ES|_{\mathrm{all}} \geq \alpha, \\ \sum_{k=1}^{T} h(t_k) \leq \beta, \\ NPV|_0^{T_p} > 0, \\ x_{\mathrm{L}}(t_k) \leq x_{\mathrm{L}}^0, \; x_{\mathrm{AC}}(t_k) \leq x_{\mathrm{AC}}^0, \\ x_{\mathrm{L}}(t_k) \geq x_{\mathrm{L}}^0/3, \; x_{\mathrm{AC}}(t_k) \geq x_{\mathrm{AC}}^0/2. \end{cases} \tag{23}$$

where the initial condition is $x_{\mathrm{L}}^0, x_{\mathrm{AC}}^0$. The objective function is a weighted sum of the energy efficiency objective $ES|_{\mathrm{all}}$ and economy objective $d_R|_T$. λ_1 and λ_2 are the weights. α denotes the targeted energy saving amount, and β the budget limit. T_p denotes the payback limit, such that the NPV over time horizon $[0, T_pS)$ must be greater than zero. The state variables $x_{\mathrm{L}}(t_k)$ and $x_{\mathrm{AC}}(t_k)$ are physically limited by the initial condition x_{L}^0 and x_{AC}^0, as the maintenance actions only apply to existing items. Lastly, due to the comfort satisfaction limit, it constrains the $x_{\mathrm{L}}(t_k)$ to be greater than $x_{\mathrm{L}}^0/3$, and $x_{\mathrm{AC}}(t_k)$ to be greater than $x_{\mathrm{AC}}^0/2$. Given the finite number of maintenance instants N, $\mathbf{u}(\cdot) \in \mathcal{R}^{2\times N}$, that is, the minimization problem (22)–(23) is a finite dimensional one. The MPC controller is thus designed as follows.

3 | MPC CONTROLLER DESIGN AND NUMERICAL SOLVER

3.1 | MPC controller

A modified MPC based approach from Reference 5 is employed to address the long-term performance indicators involved in constraints (23) of $\mathcal{MPO}$. The modified MPC approach adopts a decreasing horizon manner, such that a varying predictive horizon $N = T - m$ is employed to predict the aggregate performances over the whole remaining sustainability period.[5] The open loop problem is accordingly transformed to be an minimization problem with the following objective:

$$J'(\mathbf{x}(t_m), Q, \mathbf{u}'|_m(\cdot)) = -\lambda_1 \frac{ES'|_m}{\alpha} - \lambda_2 R'_T, \tag{24}$$

where R'_T denotes the discount rate solved from $\text{NPV}'|_m = 0$. $ES'|_m$ and $\text{NPV}'|_m$ are formulated as follows:

$$\begin{cases} ES'|_m = \sum_{k=1}^{m} \overline{E}S(t_k) + \sum_{k=m+1}^{T} ES(t_k), \\ \text{NPV}'|_m = \sum_{k=1}^{m} \frac{\overline{C}(t_k) - \overline{h}(t_k)}{(1+R)^{n-1}} + \sum_{k=m+1}^{T} \frac{C(t_k) - h(t_k)}{(1+R)^{n-1}} - h_0, \end{cases} \tag{25}$$

subject to (10)-(15), and

$$\begin{cases} ES'|_m \geq \alpha, \\ h'|_m \leq \beta', \\ \text{NPV}'|_m^{T_p} \geq 0, \\ x_{\text{L}}(t_k) \leq x_{\text{L}}^0,\ x_{\text{AC}}(t_k) \leq x_{\text{AC}}^0, \\ x_{\text{L}}(t_k) \geq x_{\text{L}}^0/3,\ x_{\text{AC}}(t_k) \geq x_{\text{AC}}^0/2, \end{cases} \tag{26}$$

where

$$\begin{cases} h'|_m = \sum_{k=m+1}^{T} h(t_k), \\ \beta' = \beta - \sum_{k=1}^{m} \overline{h}(t_k), \\ \text{NPV}'|_m^{T_p} = \sum_{k=1}^{m} \frac{\overline{C}(t_k) - \overline{h}(t_k)}{(1+R)^{n-1}} + \sum_{k=m+1}^{T_p} \frac{C(t_k) - h(t_k)}{(1+R)^{n-1}} - h_0. \end{cases} \tag{27}$$

$\overline{E}S(t_k)$ and $\overline{C}(t_k)$ denote the existing energy savings and cost savings, respectively. The maintenance costs from the existing maintenance actions are represented by $\overline{h}(t_k)$. They are observed as a consequence of the control inputs prior to t_m. Given an $m \in Q$, the open loop problem (24)–(27) is solved over the remaining interval $[t_m, TS)$. Let $\mathbf{u}'|_m = \{\mathbf{u}'(t_k) : k = m, m+1, \dots, T-1\}$ denote the obtained sequence of optimal maintenance intensities. Let $\overline{\mathbf{u}}|_m = \{\mathbf{u}'|_m(t_m)\} = \{\overline{\mathbf{u}}|_m(\mathbf{x}(t_m))\}$ denote the obtained maintenance intensity over $[t_m, t_{m+1})$. Only $\overline{\mathbf{u}}|_m$ is applied, where the rest from the sequence are abandoned. $\overline{\mathbf{u}}|_m$ applies with a consequence of the new state variable $\mathbf{x}(t_{m+1})$, which is adopted as the input condition of the next step MPC computation, that is, the prediction over horizon $[t_{m+1}, TS)$. The optimal control inputs $\overline{\mathbf{u}}$ are thus obtained upon the consecutive implementation of the process.

In practice where uncertainties are often significant enough to influence the prediction of state variables, the input state variables to each open loop problem must be adjusted. A simplification of the uncertainties is adopted, where the impact is interpreted as a disturbance on the state variables. Let $\mathbf{d}(t_k) = (d_{\text{L}}(t_k), d_{\text{AC}}(t_k))^T$ denote such a disturbance. The actual state is thus estimated by $\hat{\mathbf{x}}(t_{m+1}) = \mathbf{x}(t_{m+1}) + \mathbf{d}(t_k)$, and adopted as the input condition of the prediction over $[t_{m+1}, TS)$. In $\mathcal{MPO}$, $\hat{\mathbf{x}}(t_{m+1})$ is assumed measurable in practice. Given the physical boundaries of the system states, the stability of such a closed-loop system can be identified. The relevant discussion can be referred to in Reference 28.

MPC algorithm for $\mathcal{MPO}$

Initialization: Given the initial state $\mathbf{x}(t_0) = \mathbf{x}_0$ and $m = 0$.

1. Solve (24)–(27) to obtain the open loop optimal solution $\{\mathbf{u}'|_m(t_k)\}$, with $k = m, m+1, \dots, T-1$.
2. The MPC controller $\overline{\mathbf{u}}|_m = \{\mathbf{u}'|_m(t_m)\}$ is applied at the maintenance instant t_m. The remains of the open loop optimal solution $\{\mathbf{u}'|_m(t_k) : k = m+1, \dots, T-1\}$ are discarded. Apply $\overline{\mathbf{u}}|_m$ to (10)-(15) to obtain the predicted $\mathbf{x}(t_{m+1})$.

Assuming the uncertainties $\mathbf{d}(t_m)$, the actual system state over the next sampling period is updated by:

$$\begin{bmatrix} \hat{x}_{\mathrm{L}}(t_{m+1}) \\ \hat{x}_{\mathrm{AC}}(t_{m+1}) \end{bmatrix} = \begin{bmatrix} x_{\mathrm{L}}(t_{m+1}) \\ x_{\mathrm{AC}}(t_{m+1}) \end{bmatrix} + \begin{bmatrix} d_{\mathrm{L}}(t_m) \\ d_{\mathrm{AC}}(t_m) \end{bmatrix},$$

which can be measured at t_{m+1} and executed over $[t_m, t_{m+1})$.

3. Let $\hat{\mathbf{x}}(t_{m+1}) = (\hat{x}_{\mathrm{L}}(t_{m+1}), \hat{x}_{\mathrm{AC}}(t_{m+1}))^T$ be the initial state for the next predictive horizon, $m := m + 1$ and go back to step 1.

When $m \notin Q$, $\mathbf{u}(t_m) = 0$, step 1 is skipped and $\hat{\mathbf{x}}(t_{m+1})$ is obtained by:

$$\begin{bmatrix} \hat{x}_{\mathrm{L}}(t_{m+1}) \\ \hat{x}_{\mathrm{AC}}(t_{m+1}) \end{bmatrix} = \begin{bmatrix} G_{\mathrm{L}}(x_{\mathrm{L}}(t_m)) \\ G_{\mathrm{AC}}(x_{\mathrm{L}}(t_m), x_{\mathrm{AC}}(t_m)) \end{bmatrix} + \begin{bmatrix} d_{\mathrm{L}}(t_m) \\ d_{\mathrm{AC}}(t_m) \end{bmatrix}.$$

The above MPC algorithm will go over the sustainability period to obtain the optimal control strategy. The open loop problem of step 1 is solved by the a differential evolution (DE) algorithm based numerical solver. The details of the numerical solver can be found from some of our previous studies.[4,5] To be noticed, given the integer nature that has been discussed above, the adopted numerical solver employs a binary coding as introduced in Reference 5. The integer nature increases the difficulty to obtain the optimal solution. However, given the nonlinear and nonanalytic items in the MPO formulation, the DE algorithm is still an efficient tool as the numerical solver.

4 | A CASE STUDY RESULTS AND ANALYSIS

A simulation is conducted with the data input from an actual retrofitting project in Pretoria, South Africa. The preimplementation audit data is employed from the project. As a result of the retrofitting plan, the old appliances are replaced with 480 compact fluorescent lamp (CFL) lights and 16 latest air conditioners. A sustainability period of 10 years is considered, over which the maintenance is scheduled to take place over a 6-month interval. The sampling interval is 1 month. Given that the MPO involves long-time data collection, the simulation verification is adopted at the current stage. For simplicity, the sustainability period is considered to start in January. According to the Pretoria typical year weather profile, May, June, July, and August are categorized into the heating season, and rest months are categorized into the cooling season. It hereby assumes that the set point for the heating system is 24°C, and for the cooling it is 26°C. Table 1 exhibits the exploited data for the simulation.

The $Q^{'}_{\mathrm{HD}}$ and $Q^{'}_{\mathrm{CD}}$ are given in Table 1 in order to apply (5). Such estimations are obtained based on the history occupancy profiles. The $\bar{a}_{\mathrm{L}}$ and $\bar{E}_{\mathrm{AC}}$ are also given according to (8). $\bar{a}_{\mathrm{L}}$ represents the energy consumption per light per month. $\bar{E}_{\mathrm{AC}}$ indicates the previous air conditioners' overall consumptions with respect to each month. The typical year weather profile is also given. The temperature profile is developed based on the historical weather profile of the district, it is assumed to be deterministic based on the statistical average. Given that the main focus of the proposed method is extending the control system framework for the maintenance planning investment decisions, improving the precision of the energy consumption estimation is excluded at the current stage. It is however a topic worthy further exploring and will be one major focus in our future studies.

Table 2 gives the necessary specifications of the retrofitted appliances. First, the initial populations of the retrofitted items groups are given by x^0_{L} and x^0_{AC}. The monthly consumption per retrofitted light can be considered constant, which is given in Table 2. The consumption per air conditioner varies upon time, as analyzed in the previous sections, it is therefore omitted in Table 2. As aforementioned, the working load formulations (5)–(9) and system dynamics (10)-(15) are exploited to compute the air condition consumptions. The retrofitted costs are also given, which imply that the initial investment h_0 is \$14 171.2. The expenditures per maintenance action per item are indicated by the maintenance cost row. The unit of the MTBF values is month. The population decay models are given as follows:

$$G_{\mathrm{L}}(x_{\mathrm{L}}(t_k)) = 0.0692x_{\mathrm{L}}(t_k)^2/68 - 0.1094x_{\mathrm{L}}(t_k) + x_{\mathrm{L}}(t_k), \tag{28}$$

$$G_{\mathrm{AC}}(x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) = \left(1 - \frac{1}{\eta(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k))}\right) x_{\mathrm{AC}}(t_k). \tag{29}$$

The estimated average lifespan of one CFL is 11.9 months. According to Reference 1, it is assumed that $G_{\mathrm{L}}(x_{\mathrm{L}}(t_0)) = 1$ and $G_{\mathrm{L}}(x_{\mathrm{L}}(t_{12})) = 0.5$. By solving the two equations, $b_{\mathrm{L}} = 0.0692$ and $c_{\mathrm{L}} = 0.1094$ can be obtained, as suggested in (12). Equation (29) is developed from (12), where the shape parameter $\beta = 1$. The MTBF of the retrofitted air conditioner unit

Month	Jan	Feb	Mar	Apr
Q'_{HD} (kWh)	n/a	n/a	n/a	n/a
Q'_{CD} (kWh)	4651.77	4560.96	4042.32	3971.5
$\bar{a}_{\mathrm{L}}$ (kWh)	17.28	17.28	17.28	17.28
$\bar{E}_{\mathrm{AC}}$ (kWh)	6340.34	6268.04	6309.68	6150.91
High (°C)	32	31	30	29
Low (°C)	17	17	16	12
Month	May	Jun	Jul	Aug
Q'_{HD} (kWh)	7856.72	9224.1	9473.4	7928.16
Q'_{CD} (kWh)	n/a	n/a	n/a	n/a
$\bar{a}_{\mathrm{L}}$ (kWh)	17.28	17.28	17.28	17.28
$\bar{E}_{\mathrm{AC}}$ (kWh)	0	0	0	0
High (°C)	20	17	15	16
Low (°C)	7	3	3	7
Month	Sep	Oct	Nov	Dec
Q'_{HD} (kWh)	n/a	n/a	n/a	n/a
Q'_{CD} (kWh)	4228.48	4342.72	4492.7	4542.7
$\bar{a}_{\mathrm{L}}$ (kWh)	17.28	17.28	17.28	17.28
$\bar{E}_{\mathrm{AC}}$ (kWh)	6111.19	6339.49	6421.05	6600.86
High (°C)	29	30	31	32
Low (°C)	11	14	15	16

TABLE 1 The estimations of air conditioner working loads from the preretrofit lights and other resources, the previous appliance energy consumptions, and the typical year temperature profile

is 18 months, therefore η_0 should be 25.97 according to Reference 1. During the heating season, the estimation of η is as follows:

$$\eta(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) = \begin{cases} \frac{25.97}{0.835+e^{-7.105p_{\mathrm{c}}(t_k)+1.397}}, & p_{\mathrm{h}}(t_k) < 45\%, \\ 25.97, & 45\% \le p_{\mathrm{h}}(t_k) \le 100\%, \\ \frac{25.97}{0.719+e^{1.195p_{\mathrm{c}}(t_k)+2.465}}, & p_{\mathrm{h}}(t_k) > 100\%, \end{cases} \tag{30}$$

and during cooling season:

$$\eta(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) = \begin{cases} \frac{25.97}{0.835+e^{-7.105p_{\mathrm{c}}(t_k)+1.397}}, & p_{\mathrm{c}}(t_k) < 45\%, \\ 25.97, & 45\% \le p_{\mathrm{c}}(t_k) \le 100\%, \\ \frac{25.97}{0.683+e^{1.239p_{\mathrm{c}}(t_k)+2.389}}, & p_{\mathrm{c}}(t_k) > 100\%. \end{cases} \tag{31}$$

The heating/cooling efficiency ε is estimated as follows:

$$\varepsilon_{\mathrm{h}}(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) = \begin{cases} \frac{2.8}{0.9485+1.1364e^{-7.105p_{\mathrm{c}}(t_k)+1.397}}, & p_{\mathrm{h}}(t_k) < 45\%, \\ -2.7768p_{\mathrm{h}}(t_k)^2 + 5.0672p_{\mathrm{h}}(t_k) + 1.0312, & 45\% \le p_{\mathrm{h}}(t_k) \le 100\%, \\ \frac{2.8}{0.7492+1.0417e^{1.1949p_{\mathrm{c}}(t_k)+2.4649}}, & p_{\mathrm{h}}(t_k) > 100\%, \end{cases} \tag{32}$$

$$\varepsilon_{\mathrm{c}}(t_k, x_{\mathrm{L}}(t_k), x_{\mathrm{AC}}(t_k)) = \begin{cases} \frac{3.62}{0.9274+1.1e^{-7.105p_{\mathrm{c}}(t_k)+1.397}}, & p_{\mathrm{c}}(t_k) < 45\%, \\ -4.3386p_{\mathrm{c}}(t_k)^2 + 6.4881p_{\mathrm{c}}(t_k) + 1.2167, & 45\% \le p_{\mathrm{c}}(t_k) \le 100\%, \\ \frac{3.62}{0.7348+1.0753e^{1.239p_{\mathrm{c}}(t_k)+2.389}}, & p_{\mathrm{c}}(t_k) > 100\%. \end{cases} \tag{33}$$

TABLE 2 Retrofitted items specifications

	26W CFL Lighting	8500 Btu/h Air Conditioner
Maximum possible quantities	480	16
Rated power heating (W)	26	926.8
Rated power cooling (W)	26	740
Rated heating capacity (Btu/h)	n/a	8500
Rated cooling capacity (Btu/h)	n/a	8500
Average monthly consumption (kWh)	6.24	n/a
Electricity price ($/kWh)	0.1661	0.1437
Installation price ($)	14.19	460
Maintenance cost ($)	14.19	180
MTBF (months)	11.9	18

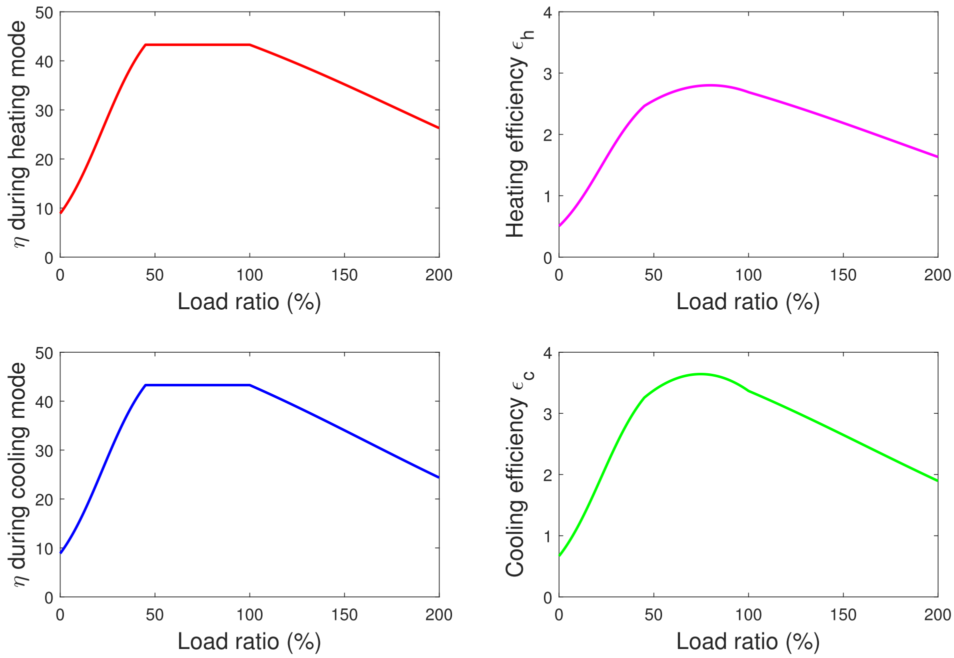

FIGURE 1 The shapes of η, ε_{h}, and ε_{c} with the load ratio against rated capacity [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 1 depicts the shapes of η, ε_{h} and ε_{c}.

There is a targeted overall energy saving amount in this project, which is 671 380.1 kWh over 10 years, against an estimated baseline consumption 1 500 743.6 kWh. A saving below the targeted value is considered unacceptable. In order to achieve this target, several maintenance budget limits are taken into account, including $31 500, $37 500 and $42 500, from very tight to sufficient. The maintenance time schedule for the energy efficiency purpose, as aforementioned, is $Q = \{6, 12, 18, 24, \ldots, 114\}$. For the financial payback calculation, a discount rate at 11% per year is adopted, with a maximum acceptable payback period 48, that is, 4 years. According to (22), a weighted sum approach is employed, with weighting factors $\lambda_1 = 0.5$ and $\lambda_2 = 0.5$.

Two simulation tests are conducted in the case study. First, the uncertainties are ignored, where an open loop dynamic programming problem is solved as the MPO. A comparative case is also simulated, where a maintenance plan is optimized on the basis of the model without considering interplays. The control system (10)-(15) applies both open loop optimal maintenance plans with and without considering interacting effects. Table 3 illustrates the results of the first test. Second, the uncertainties are taken into account as a random noise describe in the MPC algorithm. The random noise varies between ±10% of the state variables $\mathbf{x}(t_k)$. Thereafter, the maintenance plan is derived from the MPC controller. For the second test, an open loop maintenance plan is obtained to be the comparative baseline. Similarly, both the maintenance plans with and without considering uncertainties are applied to the control system (10)-(15). Table 4 gives the results of the second test.

TABLE 3 Comparison of with and without interaction solutions under different budget limits

Cases	Budget Limit ($)	Energy Savings (kWh)	Saving Ratio Exceeding Target	IRR	Payback Period (years)	NPV ($)	Maintenance Cost ($)	Comfort Violation
With interactions	31 500	682 461.3	1.65%	69.89%	1.52	37 041.77	31 414.17	Yes
Without interactions	31 500	660 406.1	−1.64%	67.84%	1.55	34 737.33	31 460.43	Yes
With interactions	37 500	725 261.8	8.02%	68.77%	1.52	35 917.37	37 379.22	No
Without interactions	37 500	720 228.5	7.27%	66.85%	1.54	36 239.4	37 473.3	No
With interactions	42 500	822 015.9	22.43%	70.76%	1.51	41 791.32	42 422.7	No
Without interactions	42 500	818 781.6	21.95%	70.59%	1.53	41 584	42 427.95	No

Abbreviations: IRR, internal rate of return; NPV, net present value.

TABLE 4 Comparison of closed-loop and open loop solutions under different budget limits

Cases	Budget Limit ($)	Energy Savings (kWh)	Percentage Saved Over Baseline	IRR	Payback Period (years)	NPV ($)	Maintenance Cost ($)	Comfort Violation
Closed-loop solution	31 500	669 008.9	−0.35%	68.18%	1.53	36 462.9	31 526.3	Yes
Open loop solution	31 500	607 177.6	−9.56.%	62.34%	1.56	32 020.5	31 560.27	Yes
Closed-loop solution	37 500	709 476.3	5.67%	66.29%	1.54	35 258.5	37 487.1	No
Open loop solution	37 500	666 778.8	−0.685%	63.15%	1.54	33 389.8	37 479.3	Yes
Closed-loop solution	42 500	795 327.1	18.46%	67.95%	1.54	40 791.4	42 499.7	No
Open loop solution	42 500	707 188.9	5.33%	64.2%	1.53	36 062.4	42 417.6	No

Abbreviations: IRR, internal rate of return; NPV, net present value.

The parameters of the DE-based numerical solver has been tuned to obtain the most effective convergence. The tuned parameters include the population size of individual solutions, the learning rate, and the crossover rate. Detailed explanations of the parameters can be found from Reference 4. The problem dimension in the case study is 38, therefore the DE population size is selected to be 120. The learning rate adopts a nonlinear decreasing function as introduced by Reference 29. The crossover rate is 0.75. The results in Tables 3 and 4 are the average of 20-run results. In order to manifest the effectiveness, several performances are illustrated in the tables. The "saving ratio exceeding target" demonstrates the amount of extra savings obtained from the maintenance plans against the targeted saving value. In this way, it emphasizes the improvements resulted from the MPO. The IRR, NPV, and payback period in years are also given to manifest the economy performances. Furthermore, for the sake of comfort requirements, an upper and lower bounded constraint is introduced in (23) to regulate the feasible range of the state variables. Tables 3 and 4 also demonstrated that whether the comfort constraint is violated.

According to Table 3, a negative impact range between 2% and 3% can be delivered in cases where interacting energy and reliability effects are incorrectly addressed. It is believed that the negative impact implies an approximate 2% uncertainty resulting from the interactions between lights and air conditioners. In Table 4, comparing with the MPC controller performances, the open loop solutions are decreased by 5%-10%, which verifies that in practice, where uncertainties are inevitable, the control system framework as well as the closed-loop optimization strategy can effectively improve the performances.

5 | CONCLUSIONS

In this paper, the interacting energy and reliability effects between the light group and air conditioner group in a building energy maintenance plan optimization context is investigated. An interpretation of the interplays between the two item groups are proposed, based on which the maintenance plan optimization problem is cast into an optimal control problem with coupled state variables and system dynamics. The interactions are mainly identified from the energy consumption estimations and the population decay model parameters. A simulation exercise that exploits the data from a practical retrofitting project is conducted to test the effectiveness of the proposed approach. From the simulation results, the effects of the interactions are identified, such that the interactions can deliver an approximate 2% impact to the final energy efficiency.

The suggested approach provides a management level evaluation of the cost-effectiveness of the budget and the manpower over a long future period. As aforementioned, the population decay models are extracted from the statistical sources, which restricts the model accuracy. The optimized maintenance plan can be the reference for actual operation and maintenance; however, due to the very large timescale, implementing the actual maintenance actions must take into account the practical situations, including the load profiles and available manpower at the maintenance instant. Such a risk is inevitable for the investment decision at management level. Addressing such a risk and combining the long period planning and actual operation is one very important topic of our future research.

ORCID

Zhongtao Cheng https://orcid.org/0000-0002-5622-5298
Lei Liu https://orcid.org/0000-0003-3606-122X

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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