Available online at www.sciencedirect.com

ScienceDirect

Journal of the Franklin Institute 359 (2022) 3333–3365

www.elsevier.com/locate/jfranklin

Event-triggered adaptive control of multi-agent systems with saturated input and partial state constraints

Yong Zhao[a,b], Hui Yu[a,b,*], Xiaohua Xia[c]

[a] Three Gorges Mathematical Research Center, China Three Gorges University, Yichang 443002, China

[b] College of Science, China Three Gorges University, Yichang 443002, China

[c] Centre of New Energy Systems, Department of Electrical, Electronics and Computer Engineering, University of Pretoria, Pretoria, South Africa

Received 6 September 2021; received in revised form 7 January 2022; accepted 5 April 2022

Available online 9 April 2022

Abstract

In this paper, we study the consensus tracking control problem of a class of strict-feedback multi-agent systems (MASs) with uncertain nonlinear dynamics, input saturation, output and partial state constraints (PSCs) which are assumed to be time-varying. An adaptive distributed control scheme is proposed for consensus achievement via output feedback and event-triggered strategy in directed networks containing a spanning tree. To handle saturated control inputs, a linear form of the control input is adopted by transforming the saturation function. The radial basis function neural network (RBFNN) is applied to approximate the uncertain nonlinear dynamics. Since the system outputs are the only available data, a high-gain adaptive observer based on RBFNN is constructed to estimate the unmeasurable states. To ensure that the constraints of system outputs and partial states are never violated, a barrier Lyapunov function (BLF) with time-varying boundary function is constructed. Event-triggered control (ETC) strategy is applied to save communication resources. By using backstepping design method, the proposed distributed controller can guarantee the boundedness of all system signals, consensus tracking with a bounded error and avoidance of Zeno behavior. Finally, the correctness of the theoretical results is verified by computer simulation.

https://doi.org/10.1016/j.jfranklin.2022.04.004

1. Introduction

Autonomy, allocation and coordination make MASs to have strong reliability and robustness when solving practical problems. After the development of recent decades, it has been widely applied to unmanned aerial vehicles [1], mobile robots [2] and sensor networks [3], for example. However, it is worth emphasizing that consensus problem is the key problem in MASs. The essence of the consensus problem is to construct a suitable distributed controller for the states of MAS to converge to a same value. Therefore, the consensus problem of MASs has been studied by many scholars from different research directions, for example, the finite time consensus problem [4–6], the adaptive consensus problem [7–9], the second-order consensus problem [9–11], the event-triggered consensus problem [12–14], and so on.

In practical applications, it is always a hard problem to solve due to limitation of the channel bandwidth and computing power. However, the ETC method, which can reduce communication burden and ensure dependable system performance, has been widely paid attention to by researchers. The time-triggered controller executes periodically, while the ETC updates it control input only when a given trigger condition is satisfied. The ETC strategy has been widely used in various applications [15–19]. In [20], the authors addressed the problem of quantized ETC for non-affine pure-feedback stochastic MASs by adaptive fuzzy control method. A sampling data based adaptive distributed ETC scheme for MASs with directed network topology was developed in [21]. In [22], ETC strategy was adopted to study the defense of MASs against false data-injection attack. An adaptive ETC protocol for MASs with undirected graphs was proposed in [23]. In [24], the authors studied the consensus problem of a class of linear MASs with directed graphs via adaptive ETC method.

It is worth mentioning that the system state or output may need to be confined to specific areas duo to physical constraints. In the process of system running, the violation of constraint conditions may lead to the degradation of system performance and even instability. In recent years, the BLF has been adopted to solve the above mentioned problem. In [25], the state constraints problem of strict-feedback switched nonlinear systems was solved by using integral BLF. In [26,27], the output constraints problem of strict feedback nonlinear systems was solved by using BLF. In [28], the adaptive fuzzy control problem of nonlinear systems with constant state constraints was studied in finite time. The control problem of nonlinear systems with time-varying PSCs and input saturation by adaptive ETC strategy was studied in [29]. The problem of tracking control for nonlinear systems with full state constraints (FSCs) was investigated in [30]. An adaptive control strategy was proposed in [31] for nonlinear stochastic systems with FSCs and unknown parameters. In [32,33], the authors addressed the tracking control problem of unknown nonlinear strict-feedback systems and nonlinear pure-feedback systems with time-varying FSCs by a neural network adaptive control method. The BLF is also used in MASs [34–36]. A finite-time distributed tracking control scheme for pure-feedback MASs with FSCs was developed in [34]. In [35], the authors proposed an adaptive consensus control strategy for non-affine MASs, in which partially unknown control directions and FSCs problems were considered simultaneously. In [36], the authors addressed the fast consensus problem of high-order nonlinear non-strict-feedback MASs with asymmetric time-

* Corresponding author at: Three Gorges Mathematical Research Center, China Three Gorges University, Yichang 443002, China.

E-mail address: yuhui@ctgu.edu.cn (H. Yu).

varying FSCs in finite time. In [37], the authors proposed an ETC method for MASs with output constraint.

Moreover, in some actual physical systems, the problem of input saturation [38–40] is often encountered. When system input is limited to a bounded area due to the physical device's limitations, the system performance will become worse, or even causing the system to be unstable. To overcome this drawback, the input saturation problem is also considered in this paper and the designed saturated controller can ensure the achievements of the control objectives.

Through the above literature analysis, to reduce communication burden and overcome some physical limitations, the adaptive tracking control problem of MASs with input saturation and partial state constraints via ETC strategy has not been further studied, which motivates the research of this paper.

This paper studies the consensus tracking control problem for MASs with uncertain nonlinearity and external disturbance in directed networks containing a spanning tree. The output and partial states of the MASs are constrained. A novel adaptive distributed ETC scheme is employed to solve the consensus tracking problem via output feedback. Because the outputs of the system are the only available date, a high-gain observer is then designed. Compared with the existing works, the contributions of this paper are mainly reflected in the following points: i) An adaptive distributed ETC scheme with observer is proposed, which can guarantee consensus tracking of the given leader signal with a bounded error, the boundedness of all system signals and avoidance of the Zeno behavior. Compared with time-triggered algorithms for MASs [34–36], this paper adopts ETC strategy, which can decrease the communication burden. ETC strategy for MASs with output constraint was also considered in [37]. However, not only output constraints but also partial state constraints and saturated control problem are taken into account in this paper. Compared with [29], the dynamic surface control scheme was used in this paper to reduce computational complexity of the virtual controllers. In addition, the event-triggered scheme proposed in [29] is based on state feedback. However, an event-triggered distributed control scheme is proposed in this paper via output feedback and then an observer is designed. ii) In this paper, the model considered is more general. MASs with uncertain nonlinearity and external disturbance via output feedback in directed networks including a spanning tree are considered. The uncertain nonlinear dynamics are approximated by RBFNN, the unknown parameters are estimated by adaptive control method and the unmeasured states are estimated by a high-gain observer via output feedback. Compared with [29], the uncertain nonlinear dynamics are assumed to be linearly parameterizable. Compared with [37], only undirected network topologies are considered. iii) BLFs are designed to guarantee that the output and partial states of the system can be constrained by time-varying boundary functions in this paper. Compared with [34–36], the time-varying boundary functions are assumed to be constant in [34], and only FSCs, which is a special case of partial state constraints, are considered in these paper.

The remaining of the work is arranged as below. Preliminaries of algebraical graph theory and RBFNN are introduced and the problem formulation is stated in Section 2. In Section 3, the state observer is constructed. In section 4, the event-triggered controller is designed. In Section 5, the stability analysis is given. In Section 6, a simulation example is given. In Section 7, conclusions are drawn.

Notations : $R^+ = [0, +\infty)$. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix. $\|\cdot\|_2$ denotes the 2-norm and $\|\cdot\|_\infty$ the ∞-norm. $\min\{a_i\}$ and

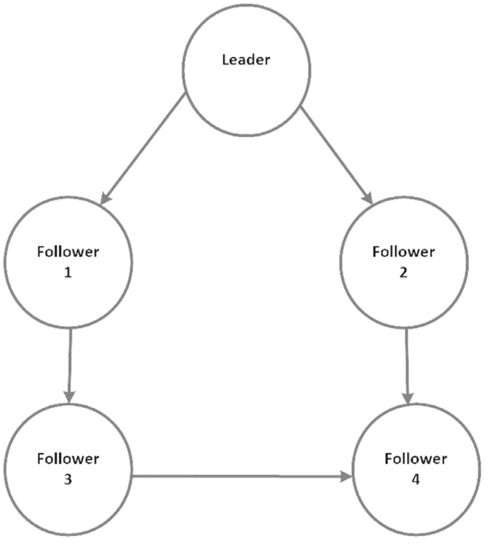

Fig. 1. Connected graph.

$\max\{a_i\}$, $i = 1, 2, \ldots, n$, represent the smallest one and the biggest in a_i, respectively. For a matrix A, $A > 0$ means that A is symmetric positive definite.

2. Preliminaries and problem statement

2.1. Graph theory

In MASs, the directed graph $\mathcal{G}$ is usually employed to describe the relations of information transmission among agents. In graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, $\mathcal{V} = \{1, 2, \ldots, N\}$, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ represent the node set, the directed edge set, and weighted adjacency matrix, respectively. An edge $(j, i) \in \mathcal{E}$ means that node i can receive information from node j and node j is a neighbor of node i. In this case, $a_{ij} > 0$, otherwise $a_{ij} = 0$. We assume that graph $\mathcal{G}$ is simple in this paper, that is, $a_{ii} = 0$. $\mathcal{N}_i = \{j|(j, i) \in \mathcal{E}\}$ stands for node $i's$ neighbors set. The in-degree matrix of graph $\mathcal{G}$ is denoted by $\mathcal{D} = \text{diag}\{d_1, d_2, \ldots, d_N\}$, where $d_i = \Sigma_{j\in\mathcal{N}_i} a_{ij}$. Denoted by $\mathcal{L} = (l_{ij})_{N\times N} = \mathcal{D} - \mathcal{A}$ the Laplacian matrix of $\mathcal{G}$.

The leader node is labeled as 0, similar to a_{ij}, if node i can access the leader, then, $a_{i0} > 0$ and $a_{i0} = 0$ otherwise. Let $\mathcal{B} = \text{diag}(a_{10}, \ldots, a_{N0})$ and $\bar{\mathcal{G}}$ be a directed graph with node set $\{0, 1, \ldots, N\}$. We say that a directed graph containing a directed spanning tree if there exists at least a directed path from the root note to all other nodes.

An example of a digraph including a spanning tree for four agents and a leader is given in Fig. 1. Obviously, $a_{ij} = 0$ except for $a_{31} = a_{42} = a_{43} = 1$, $d_1 = d_2 = d_3 = 1$ and $d_4 = 2$. Then,

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}, \mathcal{D} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \mathcal{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$

Assumption 1. There exists a directed spanning tree in graph $\bar{\mathcal{G}}$ and the leader is its root node.

Lemma 1. [41] *For graph $\bar{\mathcal{G}}$, under Assumption 1, all eigenvalues of the matrix $\mathcal{L}+\mathcal{B}$ have positive real parts.*

2.2. System description

Consider MASs with input saturation:

$$\begin{cases} \dot{x}_{iq} & = & x_{i,q+1} + f_{iq}(\underline{x}_{iq}) + h_{iq}(x_i, t), \\ & & q = 1, 2, \dots, n-1, \\ \dot{x}_{in} & = & \mathrm{sat}_i(u_i(t)) + f_{in}(\underline{x}_{in}) + h_{in}(x_i, t), \\ y_i & = & x_{i1}, i = 1, 2, \dots, N, \end{cases} \tag{1}$$

where the state vector is denoted by $\underline{x}_{iq} = [x_{i1}, x_{i2}, \dots, x_{iq}]^T \in R^q, q = 1, 2, \dots, n$, and let $x_i = \underline{x}_{in}$. $y_i \in R$ is the output of system. The full states $x_i = [x_{i1}, \dots, x_{in}]^T$ are divided into two parts. We assume that the state $[x_{i1}, \dots, x_{i\Lambda}]^T$ is constrained and the state $[x_{i,\Lambda+1}, \dots, x_{in}]^T$ is unconstrained, where $1 \le \Lambda \le n$. In the special case, when $\Lambda = 1$, only the output of the system is constrained; when $\Lambda = n$, the full states of the system are constrained. The constrained states $x_{iq}, q = 1, \dots, \Lambda$, for agent i, are constrained to time-varying regions $|x_{iq}| < k_{c_{iq}}(t)$, where $k_{c_{iq}}(t)$ is a boundary function. $f_{iq}(\cdot) \in R$ is an unknown smooth function. $h_{iq}(x_i, t) \in R$ is unknown but bounded external disturbance. $\mathrm{sat}_i(u_i(t)) \in R$ is the saturated control input, defined as

$$\mathrm{sat}_i(u_i(t)) = \begin{cases} \mathrm{sign}(u_i(t)) u_{iM}, & \text{if} |u_i(t)| > u_{iM}, \\ u_i(t), & \text{if} |u_i(t)| \le u_{iM}, \end{cases} \tag{2}$$

where u_{iM} is a positive constant. It can be rewritten as

$$\mathrm{sat}_i(u_i(t)) = \chi_i(u_i(t)) u_i(t), \tag{3}$$

where

$$\chi_i(u_i(t)) = \begin{cases} \mathrm{sign}(u_i(t)) \frac{u_{iM}}{u_i(t)}, & \text{if} |u_i(t)| > u_{iM}, \\ 1, & \text{if} |u_i(t)| \le u_{iM}. \end{cases} \tag{4}$$

The function $\chi_i(u_i(t)) \in (0, 1]$ indicates the degree of saturation of u_i. In the special case of $\chi_i(u_i(t)) = 1$, that means no saturation occurs. Assume that controller u_i does not go to infinity, which is reasonable for practical applications. Under this assumption, one gets [42]

$$0 < \iota_i \le \min(\chi_i(u_i(t))) \le 1, \tag{5}$$

where ι_i is an unknown constant, which will be estimated later in adaptive law.

In order to reduce the communication burden, an ETC mechanism is designed as:

• *Controller* :

$$u_i(t) = z_i(t_k^i), \forall t \in [t_k^i, t_{k+1}^i). \tag{6}$$

•*ETC mechanism* :

$$t_{k+1}^i = \inf\{t > t_k^i \,|\, |\rho_i(t)| \ge \varrho_i |u_i(t)| + \pi_i\}, \tag{7}$$

where $z_i(t)$ is an intermediate control function that will be designed later, $\rho_i(t) = z_i(t) - u_i(t)$, $0 < \varrho_i < 1$ and $\pi_i > 0$ are design parameters. t_k^i is update time of the controller. When Eq. (7) is triggered, the next update time t_{k+1}^i will be generated. $z_i(t_k^i)$ is invariant in $[t_k^i, t_{k+1}^i)$. According to the above ETC rules, $|z_i(t) - u_i(t)| \le \varrho_i |u_i(t)| + \pi_i$ holds in all time.

Similar to the discussion in [43], the control function $z_i(t)$ satisfies

$$z_i(t) = (1 + \triangle_1(t)\varrho_i)u_i(t) + \triangle_2(t)\pi_i, \tag{8}$$

where $\triangle_1$ and $\triangle_2$ satisfy $|\triangle_1| \le 1$ and $|\triangle_2| \le 1$, respectively.

From Eq. (8), one has

$$u_i(t) = \frac{z_i(t)}{1 + \triangle_1\varrho_i} - \frac{\triangle_2\pi_i}{1 + \triangle_1\varrho_i}. \tag{9}$$

To handle partial state constraints, a BLF candidate is used for control design. Define

$$U(t) = \frac{1}{2}\log\frac{\flat^2(t)}{\flat^2(t) - s^2(t)}, \tag{10}$$

where log is the natural logarithm, $s(t)$ is restricted by $|s(t)| < \flat(t)$ with $\flat(t) > 0$, where $\flat(t)$ is a boundary constant function.

Lemma 2. *[44] For given* $\flat(t) > 0$ *and all* $s(t)$ *satisfying* $|s(t)| < \flat(t)$,

$$\log\frac{\flat^2(t)}{\flat^2(t) - s^2(t)} < \frac{s^2(t)}{\flat^2(t) - s^2(t)}. \tag{11}$$

Control Objectives : The purpose of this paper is to design an ETC scheme for MASs (1) to achieve the following objectives: i) System outputs $y_i, i = 1, \ldots, N$, can follow the leader y_0 and tracking errors remain within a small neighborhood of the origin. ii) System output and partial state constraints are not violated, ie. $|x_{iq}| < k_{c_{iq}}(t), q = 1, 2, \ldots, \Lambda, \forall t > 0$. iii) All the resulting closed-loop systems signals are bounded. iv) Zeno behavior [45] can be avoided.

Assumption 2. For the leader, $y_0(t)$, $\dot{y}_0$ and $\ddot{y}_0$ are continuous and bounded, ie., there exist positive constants a_0, a_1 and a_2, such that $|y_0| \le a_0$, $|\dot{y}_0| \le a_1$ and $|\ddot{y}_0| \le a_2, \forall t \ge 0$.

Assumption 3. For the uncertain and smooth nonlinear function $f_{iq}(\cdot) \in R$, assume that the following inequality

$$|f_{iq}(\underline{y}_{iq}) - f_{iq}(\underline{x}_{iq})| \le L_{iq}(|y_{i1} - x_{i1}| + \ldots + |y_{iq} - x_{iq}|), \tag{12}$$

holds, where L_{iq} are known positive constants and $\underline{y}_{iq} = [y_{i1}, y_{i2}, \ldots, y_{iq}]$, $y_{iq} \in R, q = 1, 2, \ldots, n$.

Assumption 4. For the bounded external disturbance $h_i(x, t)$, there exist an unknown constant $\bar{h}_i > 0$ such that

$$|h_i(x, t)| \le \bar{h}_i. \tag{13}$$

Lemma 3. *[46] For any* $\vartheta > 0$ *and* γ,

$$0 \le |\gamma| - \gamma\tanh(\frac{\gamma}{\vartheta}) \le 0.2785\vartheta. \tag{14}$$

Lemma 4. *[29] For any given $\flat(t) > 0$, let the sets $S := \{s \in R : |s| < \flat(t)\}$ and $M := R^r \times S$. Consider*

$$\dot{y} = g(t, y), \tag{15}$$

where $y = [\omega, s]^T \in M$, and $g : R^+ \times M \to R^{r+1}$ is piecewise continuous in t, uniformly in t and locally Lipschitz in s, on $R^+ \times M$. Assume that exist continuously differentiable and positive definite function $U : R^r \to R^+$ and $V : S \to R^+$, such that

$$V(s) \to \infty \text{ as } \frac{|s|}{\flat(t)} \to 1, \tag{16}$$

and

$$\beta_1(\|\omega\|_2) \le U(\omega) \le \beta_2(\|\omega\|_2), \tag{17}$$

where β_1 and β_2 are $\mathcal{K}_\infty$ function. Let $W(y) := V(s) + U(\omega)$, and $s(0)$ belongs to the interval $(-\flat(t), \flat(t))$. If the inequality

$$\dot{W} = \frac{\partial W}{\partial y} g \le -m_1 W + m_2, \tag{18}$$

holds for $y \in M$ with constants $m_1 > 0$, $m_2 > 0$, then $-\flat(t) < s(t) < \flat(t)$ for $\forall 0 \le t$.

2.3. RBFNN

The uncertain nonlinear functions are approximated by RBFNN [47]

$$\Gamma(x) = \theta^T \psi(x), \tag{19}$$

with l_0 nodes, where $\theta = (\theta_1, \dots, \theta_{l_0})^T \in R^{l_0}$ is the weight vector, $\psi(x) = (\psi_1(x), \dots, \psi_{l_0}(x))^T \in R^{l_0}$ is the basis function vector and $\psi_i(x)$ is selected as

$$\psi_i(x) = \exp[-\frac{(x - r_i)^T (x - r_i)}{\mu_i^2}], \tag{20}$$

with μ_i being the width and r_i the center.

Using RBFNN, nonlinear function $f_{iq}(\underline{\hat{x}}_{iq})$ can be approximated by

$$\hat{f}_{iq}(\underline{\hat{x}}_{iq}|\theta_{iq}) = \theta_{iq}^T \psi_{iq}(\underline{\hat{x}}_{iq}), \tag{21}$$

where $\underline{\hat{x}}_{iq} = [\hat{x}_{i1}, \hat{x}_{i2}, \dots, \hat{x}_{iq}]^T, q = 1, 2, \dots, n,$ is the estimation of $\underline{x}_{iq}$. Defined $\theta_{iq}^* = \arg\min_{\theta_{iq} \in \Omega_{\theta_{iq}}} [\sup_{\underline{\hat{x}}_{iq} \in \Omega_{\underline{\hat{x}}_{iq}}} |\hat{f}_{iq}(\underline{\hat{x}}_{iq}|\theta_{iq}) - f_{iq}(\underline{\hat{x}}_{iq})|]$ as the optimal parameter vector, where $\Omega_{\theta_{iq}}$ and $\Omega_{\underline{\hat{x}}_{iq}}$ are compact sets corresponding to θ_{iq} and $\underline{\hat{x}}_{iq}$, respectively.

Then, the minimum approximation error $\varepsilon_{iq}(\underline{\hat{x}}_{iq})$ is

$$\varepsilon_{iq}(\underline{\hat{x}}_{iq}) = f_{iq}(\underline{\hat{x}}_{iq}) - \hat{f}_{iq}(\underline{\hat{x}}_{iq}|\theta_{iq}^*). \tag{22}$$

Lemma 5. *[48] For any given constant $\varepsilon > 0$, there exists a continuous function $f(x)$ defined on a compact set Ω, such that*

$$\sup_{x \in \Omega} |f(x) - \theta^T \psi(x)| \le \varepsilon. \tag{23}$$

Assumption 5. $\varepsilon_{iq}(\hat{\underline{x}}_{iq})$ is bounded, that is,

$$|\varepsilon_{iq}(\hat{\underline{x}}_{iq})| \leq \bar{\varepsilon}_{iq}, \tag{24}$$

where $\bar{\varepsilon}_{iq} > 0$ is an unknown constant.

Taking Eq. (3) into system Eq. (1), one has

$$\begin{cases} \dot{x}_i = A_i x_i + K_i y_i + f_i + B\chi_i(u_i(t))u_i(t) + h_i, \\ y_i = Cx_i, i = 1, 2, \ldots, N, \end{cases} \tag{25}$$

where

$$A_i = \begin{pmatrix} -k_{i1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -k_{i,n-1} & 0 & \cdots & 1 \\ -k_{in} & 0 & \cdots & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, f_i = \begin{pmatrix} f_{i1}(\underline{x}_{i1}) \\ \vdots \\ \vdots \\ f_{i,n-1}(\underline{x}_{i,n-1}) \\ f_{in}(\underline{x}_{in}) \end{pmatrix},$$

$$h_i = \begin{pmatrix} h_{i1} \\ \vdots \\ h_{i,n-1} \\ h_{in} \end{pmatrix}, x_i = \begin{pmatrix} x_{i1} \\ \vdots \\ x_{i,n-1} \\ x_{in} \end{pmatrix}, K_i = \begin{pmatrix} k_{i1} \\ \vdots \\ k_{i,n-1} \\ k_{in} \end{pmatrix},$$

$C = [1, \ldots, 0, 0]$ and $k_{iq}, q = 1, 2, \ldots, n$, are cofficients of the Hurwitz polynomial

$$p_i(s) = s^n + k_{in}s^{n-1} + \cdots + k_{i2}s + k_{i1}. \tag{26}$$

In system (25), A_i is a Hurwitz matrix by choosing the vector K_i. Then, for given $Q_i > 0$, there exists $P_i > 0$ such that

$$A_i^T P_i + P_i A_i = -Q_i. \tag{27}$$

3. Observer design

In this section, a high-gain state observer is constructed as follows

$$\begin{cases} \dot{\hat{x}}_{iq} = & \hat{x}_{i,q+1} + \theta_{iq}^T \psi_{iq}(\hat{\underline{x}}_{iq}) + k_{iq} g_i^q (y_i - \hat{x}_{i1}), \\ & q = 1, 2, \ldots, n-1, \\ \dot{\hat{x}}_{in} = & \chi_i(u_i(t))u_i(t) + \theta_{in}^T \psi_{in}(\hat{\underline{x}}_i) + k_{in} g_i^n (y_i - \hat{x}_{i1}), \\ \hat{y}_i = & \hat{x}_{i1}, i = 1, 2, \ldots, N, \end{cases} \tag{28}$$

where $g_i > 1$ is the gain, θ_{iq} is the RBFNN weight vector.

Let $\tilde{x}_i = x_i - \hat{x}_i = [\tilde{x}_{i1}, \tilde{x}_{i2}, \ldots, \tilde{x}_{in}]^T$ be the observer errors, $\xi_i = [\tilde{x}_{i1}, \frac{\tilde{x}_{i2}}{g_i}, \ldots, \frac{\tilde{x}_{i2}}{g_i^{n-1}}]^T$ and $\xi = [\xi_1^T, \xi_2^T, \ldots, \xi_N^T]^T$.

Let $\tilde{f}_{iq} = f_{iq}(\underline{x}_{iq}) - f_{iq}(\hat{\underline{x}}_{iq}), q = 1, 2, \ldots, n$. From (25), one gets

$$\dot{\xi}_i = g_i A_i \xi_i + \hbar_i, \tag{29}$$

where $\hbar_i = [\tilde{f}_{i1} + \tilde{\theta}_{i1}^T \psi_{i1}(\hat{\underline{x}}_{i1}) + \varepsilon_{i1} + h_{i1}, \frac{1}{g_i}(\tilde{f}_{i2} + \tilde{\theta}_{i2}^T \psi_{i2}(\hat{\underline{x}}_{i2}) + \varepsilon_{i2} + h_{i2}), \ldots, \frac{1}{g_i^{n-1}}(\tilde{f}_{in} + \tilde{\theta}_{in}^T \psi_{in}(\hat{\underline{x}}_{in}) + \varepsilon_{in} + h_{in})]^T$, $\tilde{\theta}_{iq} = \theta_{iq}^* - \theta_{iq}, q = 1, 2, \ldots, n$.

Constructs a Lyapunov function V_0 as follows

$$V_0 = \Sigma_{i=1}^N V_{0i} = \Sigma_{i=1}^N \frac{1}{\eta_0} \xi_i^T P_i \xi_i, \tag{30}$$

where η_0 is a positive design parameter.

From Eqs. (29) and (30), one has

$$\begin{aligned}\dot{V}_{0i} &= \frac{2}{\eta_0} \xi_i^T P_i \dot{\xi}_i = \frac{1}{\eta_0} (g_i \xi_i^T (A_i^T P_i + P_i A_i) \xi_i + 2\xi_i^T P_i \hbar_i) \\ &\leq \frac{1}{\eta_0} (-g_i \lambda_{\min}(Q_i) \xi_i^T \xi_i + 2\xi_i^T P_i \hbar_i).\end{aligned} \tag{31}$$

Under Assumptions 3–5, using Cauchy-Schwarz inequality and noting that $\psi_i(\underline{\hat{x}}_i)\psi_i^T(\underline{\hat{x}}_i) \leq 1$, one has

$$\begin{aligned}&2\xi_i^T P_i [\tilde{f}_{i1}, \frac{\tilde{f}_{i2}}{g_i}, \ldots, \frac{\tilde{f}_{in}}{g_i^{n-1}}]^T \\ &\leq 2(\xi_i^T P_i \xi_i)^{\frac{1}{2}} ([\tilde{f}_{i1}, \frac{\tilde{f}_{i2}}{g_i}, \ldots, \frac{\tilde{f}_{in}}{g_i^{n-1}}] P_i [\tilde{f}_{i1}, \frac{\tilde{f}_{i2}}{g_i}, \ldots, \frac{\tilde{f}_{in}}{g_i^{n-1}}]^T)^{\frac{1}{2}} \\ &\leq 2\lambda_{\max}(P_i)(\xi_i^T \xi_i)^{\frac{1}{2}} (\tilde{f}_i^T \tilde{f}_i)^{\frac{1}{2}} \\ &\leq 2\lambda_{\max}(P_i) \xi_i^T \xi_i \Sigma_{q=1}^n q L_{iq},\end{aligned} \tag{32}$$

$$\begin{aligned}&2\xi_i^T P_i [\tilde{\theta}_{i1}^T \psi_{i1}(\underline{\hat{x}}_{i1}), \frac{\tilde{\theta}_{i2}^T \psi_{i2}(\underline{\hat{x}}_{i2})}{g_i}, \ldots, \frac{\tilde{\theta}_{in}^T \psi_{in}(\underline{\hat{x}}_{in})}{g_i^{n-1}}]^T \\ &\leq \lambda_{\max}^2(P_i) \xi_i^T \xi_i + \tilde{\theta}_i^T \psi_i(\underline{\hat{x}}_i) \psi_i(\underline{\hat{x}}_i)^T \tilde{\theta}_i \\ &\leq \lambda_{\max}^2(P_i) \xi_i^T \xi_i + \Sigma_{q=1}^n \tilde{\theta}_{iq}^T \tilde{\theta}_{iq},\end{aligned} \tag{33}$$

and

$$\begin{aligned}&2\xi_i^T P_i [\varepsilon_{i1} + h_{i1}, \frac{\varepsilon_{i2} + h_{i2}}{g_i}, \ldots, \frac{\varepsilon_{in} + h_{in}}{g_i^{n-1}}]^T \\ &\leq 2\lambda_{\max}(P_i) \|\xi_i\|_2 (\|[\varepsilon_{i1}, \frac{\varepsilon_{i2}}{g_i}, \ldots, \frac{\varepsilon_{in}}{g_i^{n-1}}]\|_2 + \|[h_{i1}, \frac{h_{i2}}{g_i}, \ldots, \frac{h_{in}}{g_i^{n-1}}]\|_2) \\ &\leq 2\lambda_{\max}(P_i) \|\xi_i\|_2 (\|\varepsilon_i\|_2 + \|h_i\|_2) \\ &\leq \lambda_{\max}^2(P_i) \|\bar{\varepsilon}_i\|_2^2 + 2\xi_i^T \xi_i + \lambda_{\max}^2(P_i) \|\bar{h}_i\|_2^2,\end{aligned} \tag{34}$$

where $\tilde{f}_i = [\tilde{f}_{i1}, \ldots, \tilde{f}_{in}]^T$, $\varepsilon_i = [\varepsilon_{i1}, \ldots, \varepsilon_{in}]^T$, $h_i = [h_{i1}, \ldots, h_{in}]^T$, $\bar{\varepsilon}_i = [\bar{\varepsilon}_{i1}, \ldots, \bar{\varepsilon}_{in}]^T$, $\psi_i(\underline{\hat{x}}_i) = [\psi_{i1}(\underline{\hat{x}}_{i1}), \psi_{i2}(\underline{\hat{x}}_{i2}), \ldots, \psi_{in}(\underline{\hat{x}}_{in})]^T$, $\bar{h}_i = [\bar{h}_{i1}, \ldots, \bar{h}_{i,n-1}, \bar{h}_{in}]^T$ and $\tilde{\theta}_i = diag[\tilde{\theta}_{i1}^T, \tilde{\theta}_{i2}^T, \ldots, \tilde{\theta}_{in}^T]^T$.

Substituting Eqs. (32)–(34) into Eq. (31), one has

$$\begin{aligned}\dot{V}_{0i} \leq \frac{1}{\eta_0} (&-g_i \lambda_{\min}(Q_i) \xi_i^T \xi_i + \lambda_{\max}^2(P_i) \xi_i^T \xi_i + \Sigma_{q=1}^n \tilde{\theta}_{iq}^T \tilde{\theta}_{iq} + \lambda_{\max}^2(P_i) \|\bar{\varepsilon}_i\|_2^2 \\ &+ 2\xi_i^T \xi_i + \lambda_{\max}^2(P_i) \|\bar{h}_i\|_2^2 + 2\lambda_{\max}(P_i) \xi_i^T \xi_i \Sigma_{q=1}^n q L_{iq}).\end{aligned} \tag{35}$$

From Eqs. (30) and (35), one gets

$$\begin{aligned}\dot{V}_0 &= \Sigma_{i=1}^{N}\dot{V}_{0i}\\ &\leq \Sigma_{i=1}^{N}[-(g_i\lambda_{\min}(Q_i)-2-2\lambda_{\max}(P_i)\Sigma_{q=1}^{n}qL_{iq}-\lambda_{\max}^2(P_i))\frac{1}{\eta_0}\xi_i^T\xi_i\\ &\quad+\frac{1}{\eta_0}(\lambda_{\max}^2(P_i)(\|\bar{\varepsilon}_i\|_2^2+\|\bar{h}_i\|_2^2)+\Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq})].\end{aligned} \tag{36}$$

4. Backstepping design

An adaptive ETC scheme is designed in this section to solve the control problem considered in the paper.

Let

$$s_{i1} = \Sigma_{j\in N_i}a_{ij}(y_i-y_j)+a_{i0}(y_i-y_0), \tag{37}$$

$$s_{iq} = \hat{x}_{iq}-\beta_{iq}, \tag{38}$$

and

$$w_{iq} = \beta_{iq}-\alpha_{i,q-1}, q=2,\ldots,n, \tag{39}$$

where s_{i1} is the local consensus error, s_{iq} and $\alpha_{iq}, q=2,\ldots,n$, are the error variable and the virtual control function, respectively, β_{iq} evolves according to the following first-order filter

$$\kappa_{iq}\dot{\beta}_{iq}+\beta_{iq} = \alpha_{i,q-1}, \beta_{iq}(0)=\alpha_{i,q-1}(0), q=2,\ldots,n, \tag{40}$$

where $\kappa_{iq}>0$ is a constant.

Let $s_1=[s_{11},s_{21},\ldots,s_{N1}]^T$, $y=[y_1,y_2,\ldots,y_N]^T$ and $\bar{y}=y-1_Ny_0=[\bar{y}_1,\bar{y}_2,\ldots,\bar{y}_N]^T$. Thus,

$$s_1 = (\mathcal{L}+\mathcal{B})(y-1_Ny_0). \tag{41}$$

Remark 1 : Similar to Wang et al. [37], a first-order filter Eq. (40) is introduced. Compare with the traditional backstepping method, the problem of complexity explosion resulting from repeatedly differentiating virtual control signal $\alpha_{i,q-1}$ can be avoided.

Step 1 : From Eqs. (1), (21)–(22) and (37)–(39), one has

$$\begin{aligned}\dot{s}_{i1} &= (\Sigma_{j\in N_i}a_{ij}+a_{i0})\dot{y}_i-\Sigma_{j\in N_i}a_{ij}\dot{y}_j-a_{i0}\dot{y}_0\\ &= (d_i+a_{i0})(s_{i2}+w_{i2}+\alpha_{i1}+\tilde{x}_{i2}+\tilde{\theta}_{i1}^T\psi_{i1}(\hat{x}_{i1})+\theta_{i1}^T\psi_{i1}(\hat{x}_{i1})+\varepsilon_{i1}\\ &\quad+h_{i1})-\Sigma_{j\in N_i}a_{ij}(\hat{x}_{j2}+\tilde{x}_{j2}+\varepsilon_{j1}+\tilde{\theta}_{j1}^T\psi_{j1}(\hat{x}_{j1})+\theta_{j1}^T\psi_{j1}(\hat{x}_{j1})+h_{j1})\\ &\quad-a_{i0}\dot{y}_0.\end{aligned} \tag{42}$$

where $d_i=\Sigma_{j\in N_i}a_{ij}$.

The following Lyapunov function is given

$$\begin{aligned}V_1 &= V_0+\Sigma_{i=1}^{N}V_{1i}\\ &= V_0+\Sigma_{i=1}^{N}(\frac{1}{2}\log\frac{\flat_{i1}(t)^2}{\flat_{i1}(t)^2-s_{i1}^2}+\frac{1}{2\gamma_{i1}}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1}+\Sigma_{j\in N_i}\frac{a_{ij}}{2\eta_{j1}}\tilde{\theta}_{j1}^T\tilde{\theta}_{j1}),\end{aligned} \tag{43}$$

where $\tilde{\theta}_{i1} = \theta_{i1}^* - \theta_{i1}$, η_{j1} and γ_{i1} are positive design parameters. $\flat_{i1}(t) > 0$ is a boundary function satisfying $|s_{i1}| < \flat_{i1}(t)$ and will be given later.

It follows from Eqs. (42) and (43) that

$$\begin{aligned}
\dot{V}_{1i} &= \frac{s_{i1}\dot{s}_{i1}}{\flat_{i1}^2 - s_{i1}^2} - \frac{s_{i1}^2\dot{\flat}_{i1}}{\flat_{i1}(\flat_{i1}^2 - s_{i1}^2)} + \frac{1}{\gamma_{i1}}\tilde{\theta}_{i1}^T\dot{\tilde{\theta}}_{i1} + \Sigma_{j\in N_i}\frac{a_{ij}}{\eta_{j1}}\tilde{\theta}_{j1}^T\dot{\tilde{\theta}}_{j1} \\
&= \frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2}[(d_i + a_{i0})(s_{i2} + \alpha_{i1} + \tilde{x}_{i2} + \theta_{i1}^T\psi_{i1}(\hat{x}_{i1}) + w_{i2} + \varepsilon_{i1} + h_{i1} \\
&\quad + \tilde{\theta}_{i1}^T\psi_{i1}(\hat{x}_{i1})) - a_{i0}\dot{y}_0 - \Sigma_{j\in N_i}a_{ij}(\hat{x}_{j2} + \tilde{x}_{j2} + \varepsilon_{j1} + \theta_{j1}^T\psi_{j1}(\hat{x}_{j1}) \\
&\quad + \tilde{\theta}_{j1}^T\psi_{j1}(\hat{x}_{j1}) + h_{j1})] - \frac{s_{i1}^2\dot{\flat}_{i1}}{\flat_{i1}(\flat_{i1}^2 - s_{i1}^2)} + \frac{1}{\gamma_{i1}}\tilde{\theta}_{i1}^T\dot{\tilde{\theta}}_{i1} \\
&\quad + \Sigma_{j\in N_i}\frac{a_{ij}}{\eta_{j1}}\tilde{\theta}_{j1}^T\dot{\tilde{\theta}}_{j1}.
\end{aligned} \tag{44}$$

Under Assumptions 4-5 and using Young's inequality, one has

$$\begin{aligned}
&(d_i + a_{i0})\frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2}(\tilde{x}_{i2} + h_{i1} + \varepsilon_{i1}) \\
&\leq \frac{3}{2}(d_i + a_{i0})^2(\frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2})^2 + \frac{g_i^2}{2}\xi_i^T\xi_i + \frac{1}{2}\bar{h}_{i1}^2 + \frac{1}{2}\bar{\varepsilon}_{i1}^2,
\end{aligned} \tag{45}$$

$$\begin{aligned}
&-\frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2}\Sigma_{j\in N_i}a_{ij}(\tilde{x}_{j2} + h_{j1} + \varepsilon_{j1}) \\
&\leq \frac{3}{2}(\frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2})^2 + \frac{\bar{g}^2}{2}\lambda_{\max}(\mathcal{L}^T\mathcal{L})\xi^T\xi + \frac{1}{2}(\Sigma_{j\in N_i}a_{ij}\bar{h}_{j1})^2 \\
&\quad + \frac{1}{2}(\Sigma_{j\in N_i}a_{ij}\bar{\varepsilon}_{j1})^2,
\end{aligned} \tag{46}$$

and

$$\begin{aligned}
&\frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2}(d_i + a_{i0})(s_{i2} + w_{i2}) \\
&\leq (d_i + a_{i0})^2(\frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2})^2 + \frac{1}{2}s_{i2}^2 + \frac{1}{2}w_{i2}^2,
\end{aligned} \tag{47}$$

where $\bar{g} = \max\{g_1, g_2, \ldots, g_N\}$.

Substituting Eqs. (45)–(47) into Eq. (44), one gets

$$\begin{aligned}
\dot{V}_{1i} &\leq \frac{s_{i1}}{\flat_{i1}^2 - s_{i1}^2}[(d_i + a_{i0})(\alpha_{i1} + \theta_{i1}^T\psi_{i1}(\hat{x}_{i1})) - a_{i0}\dot{y}_0 - \Sigma_{j\in N_i}a_{ij}(\hat{x}_{j2} \\
&\quad + \theta_{j1}^T\psi_{j1}(\hat{x}_{j1})) + \frac{5(d_i + a_{i0})s_{i1}}{2(\flat_{i1}^2 - s_{i1}^2)} + \frac{3s_{i1}}{2(d_i + a_{i0})(\flat_{i1}^2 - s_{i1}^2)} \\
&\quad - \frac{s_{i1}\dot{\flat}_{i1}}{(d_i + a_{i0})\flat_{i1}}] + \frac{g_i^2}{2}\xi_i^T\xi_i + \frac{1}{2}\bar{h}_{i1}^2 + \frac{1}{2}\bar{\varepsilon}_{i1}^2 + \frac{1}{2}s_{i2}^2 + \frac{1}{2}w_{i2}^2 \\
&\quad + \frac{\bar{g}^2}{2}\lambda_{\max}(\mathcal{L}^T\mathcal{L})\xi^T\xi + \frac{1}{2}(\Sigma_{j\in N_i}a_{ij}\bar{h}_{j1})^2 + \frac{1}{2}(\Sigma_{j\in N_i}a_{ij}\bar{\varepsilon}_{j1})^2
\end{aligned}$$

$$+\Sigma_{j\in N_i}\frac{a_{ij}}{\eta_{j1}}\tilde{\theta}_{j1}^T(-(\frac{s_{i1}}{\flat_{i1}^2-s_{i1}^2})\eta_{j1}\psi_{j1}(\hat{x}_{j1})-\dot{\theta}_{j1})$$
$$+\frac{1}{\gamma_{i1}}\tilde{\theta}_{i1}^T((d_i+a_{i0})(\frac{s_{i1}}{\flat_{i1}^2-s_{i1}^2})\gamma_{i1}\psi_{i1}(\hat{x}_{i1})-\dot{\theta}_{i1}). \tag{48}$$

Select the virtual controller α_{i1} as

$$\alpha_{i1}=-\frac{c_{i1}s_{i1}}{d_i+a_{i0}}-\theta_{i1}^T\psi_{i1}(\hat{x}_{i1})+\frac{s_{i1}\dot{\flat}_{i1}}{\flat_{i1}(d_i+a_{i0})}$$
$$+\frac{1}{d_i+a_{i0}}\Sigma_{j\in N_i}a_{ij}(\hat{x}_{j2}+\theta_{j1}^T\psi_{j1}(\hat{x}_{j1}))$$
$$+\frac{a_{i0}\dot{y}_0}{d_i+a_{i0}}-\frac{5(d_i+a_{i0})s_{i1}}{2(\flat_{i1}^2-s_{i1}^2)}-\frac{3s_{i1}}{2(d_i+a_{i0})(\flat_{i1}^2-s_{i1}^2)}, \tag{49}$$

and design the adaptive law $\dot{\theta}_{i1}$ and $\dot{\theta}_{j1}$ as

$$\dot{\theta}_{i1}=(d_i+a_{i0})\frac{s_{i1}}{\flat_{i1}^2-s_{i1}^2}\gamma_{i1}\psi_{i1}(\hat{x}_{i1})-\sigma_{i1}\theta_{i1}, \tag{50}$$

$$\dot{\theta}_{j1}=-\frac{s_{i1}}{\flat_{i1}^2-s_{i1}^2}\eta_{j1}\psi_{j1}(\hat{x}_{j1})-\sigma_{j1}\theta_{j1}, \tag{51}$$

where $\sigma_{i1}>0$, $\sigma_{j1}>0$ and $c_{i1}>0$ are design parameters.

Taking Eq. (49)–(51) into Eq. (48), one has

$$\dot{V}_{1i}\le-\frac{c_{i1}s_{i1}^2}{\flat_{i1}^2-s_{i1}^2}+\frac{g_i^2}{2}\xi_i^T\xi_i+\frac{1}{2}\bar{h}_{i1}^2+\frac{1}{2}\bar{\varepsilon}_{i1}^2+\frac{1}{2}s_{i2}^2+\frac{\bar{g}^2}{2}\lambda_{\max}(\mathcal{L}^T\mathcal{L})\xi^T\xi$$
$$+\frac{1}{2}w_{i2}^2+\frac{1}{2}(\Sigma_{j\in N_i}a_{ij}\bar{h}_{j1})^2+\frac{1}{2}(\Sigma_{j\in N_i}a_{ij}\bar{\varepsilon}_{j1})^2+\frac{\sigma_{i1}}{\gamma_{i1}}\tilde{\theta}_{i1}^T\theta_{i1}$$
$$+\Sigma_{j\in N_i}\frac{a_{ij}\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}. \tag{52}$$

From Eqs. (43) and (52), we have

$$\dot{V}_1=\dot{V}_0+\Sigma_{i=1}^N\dot{V}_{1i}$$
$$\le\Sigma_{i=1}^N[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i+M_i^{(2)}-\frac{c_{i1}s_{i1}^2}{\flat_{i1}^2-s_{i1}^2}+\frac{1}{2}(s_{i2}^2+w_{i2}^2)+\frac{\sigma_{i1}}{\gamma_{i1}}\tilde{\theta}_{i1}^T\theta_{i1}$$
$$+\frac{1}{\eta_0}\Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}+\Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}], \tag{53}$$

where $M_i^{(1)}=(g_i\lambda_{\min}(Q_i)-2-\frac{g_i}{2}-2\lambda_{\max}(P_i)\Sigma_{q=1}^n qL_{iq}-\lambda_{\max}^2(P_i)-\frac{\bar{g}^2\eta_0}{2}\lambda_{\max}(\mathcal{L}^T\mathcal{L}))>0$ and $M_i^{(2)}=\frac{1}{\eta_0}\lambda_{\max}^2(P_i)(\|\bar{\varepsilon}_i\|_2^2+\|\bar{h}_i\|_2^2)+\frac{1}{2}(\bar{h}_{i1}^2+\bar{\varepsilon}_{i1}^2+(\Sigma_{j\in N_i}a_{ij}\bar{h}_{j1})^2+(\Sigma_{j\in N_i}a_{ij}\bar{\varepsilon}_{j1})^2)$.

Step 2 : From Eqs. (28), (38) and (39), we have

$$\dot{s}_{i2}=\dot{\hat{x}}_{i2}-\dot{\beta}_{i2}$$
$$=s_{i3}+w_{i3}+\alpha_{i2}+\theta_{i2}^T\psi_{i2}(\underline{\hat{x}}_{i2})+k_{i2}g_i^2(y_i-\hat{x}_{i1})-\dot{\beta}_{i2}. \tag{54}$$

The Lyapunov function V_2 is chosen as

$$V_2=V_1+\Sigma_{i=1}^N V_{2i}$$

$$= V_1 + \Sigma_{i=1}^{N}(\frac{1}{2}\log\frac{\flat_{i2}(t)^2}{\flat_{i2}(t)^2 - s_{i2}^2} + \frac{1}{2\gamma_{i2}}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \frac{1}{2}w_{i2}^2), \tag{55}$$

where $\tilde{\theta}_{i2} = \theta_{i2}^* - \theta_{i2}$, $|s_{i2}| < \flat_{i2}(t)$ and γ_{i2} is a positive design parameter.

From Eqs. (54) and (55), one has

$$\begin{aligned}\dot{V}_{2i} = &\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2}[(s_{i3} + \tilde{\theta}_{i2}^T\psi_{i2}(\hat{\underline{x}}_{i2}) + w_{i3} + \alpha_{i2} - \tilde{\theta}_{i2}^T\psi_{i2}(\hat{\underline{x}}_{i2})\\ &+\theta_{i2}^T\psi_{i2}(\hat{x}_{i2}) + k_{i2}g_i^2(y_i - \hat{x}_{i1}) - \dot{\beta}_{i2}] - \frac{s_{i2}^2\dot{\flat}_{i2}}{\flat_{i2}(\flat_{i2}^2 - s_{i2}^2)}\\ &+\frac{1}{\gamma_{i2}}\tilde{\theta}_{i2}^T\dot{\tilde{\theta}}_{i2} + w_{i2}\dot{w}_{i2}.\end{aligned} \tag{56}$$

According to Young's inequality and $\psi_{i2}(\hat{\underline{x}}_{i2})\psi_{i2}^T(\hat{\underline{x}}_{i2}) \le 1$, one has

$$-\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2}\tilde{\theta}_{i2}^T\psi_{i2}(\hat{\underline{x}}_{i2}) \le \frac{1}{2}(\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2})^2 + \frac{1}{2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2}, \tag{57}$$

and

$$\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2}(s_{i3} + w_{i3}) \le (\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2})^2 + \frac{1}{2}w_{i3}^2 + \frac{1}{2}s_{i3}^2. \tag{58}$$

Based on the above calculation, one has

$$\dot{\alpha}_{iq} = \varsigma_{iq}(s_{i1}, \ldots, s_{iq}, y_0, \dot{y}_0, \ddot{y}_0, w_{i2}, \ldots, w_{iq}), q = 2, 3, \ldots, n, \tag{59}$$

where ς_{iq} is continuous and then there exists a constant $\bar{\delta}_{i,q+1} > 0$ such that $|\varsigma_{iq}| \le \bar{\delta}_{i,q+1}$ in some compact set. Therefore, from Eqs. (39) and (40), one can obtain that

$$\dot{w}_{iq} = -\frac{w_{iq}}{\kappa_{iq}} + \varsigma_{i,q-1} \le -\frac{w_{iq}}{\kappa_{iq}} + \bar{\delta}_{iq}, q = 2, 3, \ldots, n. \tag{60}$$

Substituting Eqs. (57), (58) and (60) into Eq. (56), one gets

$$\begin{aligned}\dot{V}_{2i} \le &\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2}(\alpha_{i2} + \theta_{i2}^T\psi_{i2}(\hat{\underline{x}}_{i2}) + k_{i2}g_i^2(y_i - \hat{x}_{i1}) - \dot{\beta}_{i2}) + \frac{1}{2}s_{i3}^2\\ &+\frac{1}{2}w_{i3}^2 + \frac{3}{2}(\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2})^2 + \frac{1}{2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} - \frac{s_{i2}^2\dot{\flat}_{i2}}{\flat_{i2}(\flat_{i2}^2 - s_{i2}^2)} - \frac{w_{i2}^2}{\kappa_{i2}} + \bar{\delta}_{i2}|w_{i2}|\\ &+\frac{1}{\gamma_{i2}}\tilde{\theta}_{i2}^T(\frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2}\gamma_{i2}\psi_{i2}(\hat{\underline{x}}_{i2}) - \dot{\theta}_{i2}).\end{aligned} \tag{61}$$

Select the virtual controller α_{i2} as

$$\begin{aligned}\alpha_{i2} = &-c_{i2}s_{i2} - k_{i2}g_i^2(y_i - \hat{x}_{i1}) - \theta_{i2}^T\psi_{i2}(\hat{\underline{x}}_{i2}) - \frac{3s_{i2}}{2(\flat_{i2}^2 - s_{i2}^2)}\\ &+\frac{s_{i2}\dot{\flat}_{i2}}{\flat_{i2}} + \dot{\beta}_{i2} - \frac{s_{i2}(\flat_{i2}^2 - s_{i2}^2)}{2},\end{aligned} \tag{62}$$

and design $\dot{\theta}_{i2}$ as

$$\dot{\theta}_{i2} = \frac{s_{i2}}{\flat_{i2}^2 - s_{i2}^2}\gamma_{i2}\psi_{i2}(\hat{\underline{x}}_{i2}) - \sigma_{i2}\theta_{i2}, \tag{63}$$

where $\sigma_{i2} > 0$ and $c_{i2} > 0$ are design parameters.

By Young's inequality, we have

$$\bar{\delta}_{i2}|w_{i2}| \leq \frac{1}{2}\bar{\delta}_{i2}^2 + \frac{1}{2}w_{i2}^2. \tag{64}$$

Taking Eqs. (62), (63) and (64) into Eq. (61), one has

$$\begin{aligned}\dot{V}_{2i} \leq & -\frac{c_{i2}s_{i2}^2}{\flat_{i2}^2 - s_{i2}^2} - \frac{1}{2}s_{i2}^2 + \frac{1}{2}s_{i3}^2 + \frac{1}{2}w_{i3}^2 + \frac{1}{2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} \\ & + \frac{\sigma_{i2}}{\gamma_{i2}}\tilde{\theta}_{i2}^T\theta_{i2} + \frac{1}{2}\bar{\delta}_{i2}^2 - (\frac{1}{\kappa_{i2}} - \frac{1}{2})w_{i2}^2.\end{aligned} \tag{65}$$

From Eqs. (55) and (65), we have

$$\begin{aligned}\dot{V}_2 = & \dot{V}_1 + \Sigma_{i=1}^N\dot{V}_{2i} \\ \leq & \Sigma_{i=1}^N[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i - \Sigma_{q=1}^2\frac{c_{iq}s_{iq}^2}{\flat_{iq}^2 - s_{iq}^2} + \Sigma_{q=1}^2\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq} + \frac{1}{2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} \\ & + M_i^{(2)} + \frac{1}{\eta_0}\Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{2}s_{i3}^2 + \frac{1}{2}\Sigma_{q=2}^3 w_{iq}^2 + \frac{1}{2}\bar{\delta}_{i2}^2 - (\frac{1}{\kappa_{i2}} - \frac{1}{2})w_{i2}^2 \\ & + \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}].\end{aligned} \tag{66}$$

Step $q, q = 3, \ldots, \Lambda$: From Eqs. (28), (38) and (39), we have

$$\dot{s}_{iq} = s_{i,q+1} + w_{i,q+1} + \alpha_{iq} + \theta_{iq}^T\psi_{iq}(\underline{\hat{x}}_{iq}) + k_{iq}g_i^q(y_i - \hat{x}_{i1}) - \dot{\beta}_{iq}. \tag{67}$$

The Lyapunov function V_q is chosen as

$$\begin{aligned}V_q = & V_{q-1} + \Sigma_{i=1}^N V_{qi} \\ = & V_{q-1} + \Sigma_{i=1}^N(\frac{1}{2}\log\frac{\flat_{iq}(t)^2}{\flat_{iq}(t)^2 - s_{iq}^2} + \frac{1}{2\gamma_{iq}}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{2}w_{iq}^2),\end{aligned} \tag{68}$$

where $\tilde{\theta}_{iq} = \theta_{iq}^* - \theta_{iq}$, $|s_{iq}| < \flat_{iq}(t)$ and γ_{iq} is a positive design parameter.

From Eqs. (67) and (68), one has

$$\begin{aligned}\dot{V}_{qi} = & \frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2}[(s_{i,q+1} + \tilde{\theta}_{iq}^T\psi_{iq}(\underline{\hat{x}}_{iq}) + w_{i,q+1} + \alpha_{iq} - \tilde{\theta}_{iq}^T\psi_{iq}(\underline{\hat{x}}_{iq}) + \theta_{iq}^T\psi_{iq}(\hat{x}_{iq}) \\ & + k_{iq}g_i^q(y_i - \hat{x}_{i1}) - \dot{\beta}_{iq}] - \frac{s_{iq}^2\dot{\flat}_{iq}}{\flat_{iq}(\flat_{iq}^2 - s_{iq}^2)} + \frac{1}{\gamma_{iq}}\tilde{\theta}_{iq}^T\dot{\tilde{\theta}}_{iq} + w_{iq}\dot{w}_{iq}.\end{aligned} \tag{69}$$

According to Young's inequality and $\psi_{iq}(\underline{\hat{x}}_{iq})\psi_{iq}^T(\underline{\hat{x}}_{iq}) \leq 1$, one has

$$-\frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2}\tilde{\theta}_{iq}^T\psi_{iq}(\underline{\hat{x}}_{iq}) \leq \frac{1}{2}(\frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2})^2 + \frac{1}{2}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}, \tag{70}$$

and

$$\frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2}(s_{i,q+1} + w_{i,q+1}) \leq (\frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2})^2 + \frac{1}{2}w_{i,q+1}^2 + \frac{1}{2}s_{i,q+1}^2. \tag{71}$$

Substituting Eqs. (60), (70) and (71) into Eq. (69), one gets

$$\begin{aligned}\dot{V}_{qi} \leq{}& \frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2}(\alpha_{iq} + \theta_{iq}^T\psi_{iq}(\underline{\hat{x}}_{iq}) + k_{iq}g_i^q(y_i - \hat{x}_{i1}) - \dot{\beta}_{iq}) + \frac{1}{2}s_{i,q+1}^2 \\ &+\frac{1}{2}w_{i,q+1}^2 + \frac{3}{2}(\frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2})^2 + \frac{1}{2}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} - \frac{s_{iq}^2\dot{\flat}_{iq}}{\flat_{iq}(\flat_{iq}^2 - s_{iq}^2)} \\ &+\frac{1}{\gamma_{iq}}\tilde{\theta}_{iq}^T(\frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2}\gamma_{iq}\psi_{iq}(\underline{\hat{x}}_{iq}) - \dot{\theta}_{iq}) + \frac{1}{2}\bar{\delta}_{iq}^2 - (\frac{1}{\kappa_{iq}} - \frac{1}{2})w_{iq}^2.\end{aligned} \tag{72}$$

Select the virtual controller α_{iq} as

$$\begin{aligned}\alpha_{iq} ={}& -c_{iq}s_{iq} - k_{iq}g_i^q(y_i - \hat{x}_{i1}) - \theta_{iq}^T\psi_{iq}(\underline{\hat{x}}_{iq}) - \frac{3s_{iq}}{2(\flat_{iq}^2 - s_{iq}^2)} \\ &+\frac{s_{iq}\dot{\flat}_{iq}}{\flat_{iq}} + \dot{\beta}_{iq} - \frac{s_{iq}(\flat_{iq}^2 - s_{iq}^2)}{2},\end{aligned} \tag{73}$$

and design $\dot{\theta}_{iq}$ as

$$\dot{\theta}_{iq} = \frac{s_{iq}}{\flat_{iq}^2 - s_{iq}^2}\gamma_{iq}\psi_{iq}(\underline{\hat{x}}_{iq}) - \sigma_{iq}\theta_{iq}, \tag{74}$$

where $\sigma_{iq} > 0$ and $c_{iq} > 0$ are design parameters.

Taking Eqs. (73) and (74) into Eq. (72), one has

$$\begin{aligned}\dot{V}_{qi} \leq{}& -\frac{c_{iq}s_{iq}^2}{\flat_{iq}^2 - s_{iq}^2} - \frac{1}{2}s_{iq}^2 + \frac{1}{2}s_{i,q+1}^2 + \frac{1}{2}w_{i,q+1}^2 + \frac{1}{2}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} \\ &+\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq} + \frac{1}{2}\bar{\delta}_{iq}^2 - (\frac{1}{\kappa_{iq}} - \frac{1}{2})w_{iq}^2.\end{aligned} \tag{75}$$

From Eqs. (68) and (75), we have

$$\begin{aligned}\dot{V}_q ={}& \dot{V}_{q-1} + \Sigma_{i=1}^N\dot{V}_{qi} \\ \leq{}& \Sigma_{i=1}^N[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i - \Sigma_{l=1}^q\frac{c_{il}s_{il}^2}{\flat_{il}^2 - s_{il}^2} + \Sigma_{l=1}^q\frac{\sigma_{il}}{\gamma_{il}}\tilde{\theta}_{il}^T\theta_{il} + \frac{1}{2}\Sigma_{l=2}^q\tilde{\theta}_{il}^T\tilde{\theta}_{il} \\ &+M_i^{(2)} + \frac{1}{\eta_0}\Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{2}s_{i,q+1}^2 + \frac{1}{2}\Sigma_{l=2}^{q+1}w_{il}^2 - \Sigma_{l=2}^q(\frac{1}{\kappa_{il}} - \frac{1}{2})w_{il}^2 \\ &+\frac{1}{2}\Sigma_{l=2}^q\bar{\delta}_{il} + \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}].\end{aligned} \tag{76}$$

Step $\Lambda + 1$: From Eqs. (28), (38) and (39), we have

$$\begin{aligned}\dot{s}_{i,\Lambda+1} ={}& s_{i,\Lambda+2} + w_{i,\Lambda+2} + \alpha_{i,\Lambda+1} + \theta_{i,\Lambda+1}^T\psi_{i,\Lambda+1}(\underline{\hat{x}}_{i,\Lambda+1}) \\ &+k_{i,\Lambda+1}g_i^{\Lambda+1}(y_i - \hat{x}_{i1}) - \dot{\beta}_{i,\Lambda+1}.\end{aligned} \tag{77}$$

The Lyapunov function $V_{\Lambda+1}$ is chosen as

$$\begin{aligned}V_{\Lambda+1} ={}& V_\Lambda + \Sigma_{i=1}^N V_{\Lambda+1,i} \\ ={}& V_\Lambda + \Sigma_{i=1}^N(\frac{1}{2}s_{i,\Lambda+1}^2 + \frac{1}{2\gamma_{i,\Lambda+1}}\tilde{\theta}_{i,\Lambda+1}^T\tilde{\theta}_{i,\Lambda+1} + \frac{1}{2}w_{i,\Lambda+1}^2),\end{aligned} \tag{78}$$

where $\tilde{\theta}_{i,\Lambda+1} = \theta^*_{i,\Lambda+1} - \theta_{i,\Lambda+1}$, and $\gamma_{i,\Lambda+1}$ is a positive design parameter.

From Eqs. (77) and (78), one has

$$\begin{aligned}\dot{V}_{\Lambda+1,i} = {} & s_{i,\Lambda+1}(s_{i,\Lambda+2} + w_{i,\Lambda+2} + \alpha_{i,\Lambda+1} + \tilde{\theta}^T_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) \\ & -\tilde{\theta}^T_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) + \theta^T_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) - \dot{\beta}_{i,\Lambda+1} \\ & +k_{i,\Lambda+1}g_i^{\Lambda+1}(y_i - \hat{x}_{i1})) + \frac{1}{\gamma_{i,\Lambda+1}}\tilde{\theta}^T_{i,\Lambda+1}\dot{\tilde{\theta}}_{i,\Lambda+1} + w_{i,\Lambda+1}\dot{w}_{i,\Lambda+1}.\end{aligned} \tag{79}$$

Since $\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1})\psi^T_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) \leq 1$, thus

$$-s_{i,\Lambda+1}\tilde{\theta}^T_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) \leq \frac{1}{2}s^2_{i,\Lambda+1} + \frac{1}{2}\tilde{\theta}^T_{i,\Lambda+1}\tilde{\theta}_{i,\Lambda+1}, \tag{80}$$

and

$$s_{i,\Lambda+1}(s_{i,\Lambda+2} + w_{i,\Lambda+2}) \leq s^2_{i,\Lambda+1} + \frac{1}{2}s^2_{i,\Lambda+2} + \frac{1}{2}w^2_{i,\Lambda+2}. \tag{81}$$

Substituting Eqs. (60), (80) and (81) into Eq. (79), one gets

$$\begin{aligned}\dot{V}_{\Lambda+1,i} \leq {} & s_{i,\Lambda+1}(\frac{3}{2}s_{i,\Lambda+1} + \alpha_{i,\Lambda+1} + k_{i,\Lambda+1}g_i^{\Lambda+1}(y_i - \hat{x}_{i1}) \\ & +\theta^T_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) - \dot{\beta}_{i,\Lambda+1}) + \frac{1}{2}s^2_{i,\Lambda+2} + \frac{1}{2}w^2_{i,\Lambda+2} \\ & +\frac{1}{2}\tilde{\theta}^T_{i,\Lambda+1}\tilde{\theta}_{i,\Lambda+1} + \frac{1}{2}\bar{\delta}^2_{i,\Lambda+1} - (\frac{1}{\kappa_{i,\Lambda+1}} - \frac{1}{2})w^2_{i,\Lambda+1} \\ & +\frac{1}{\gamma_{i,\Lambda+1}}\tilde{\theta}^T_{i,\Lambda+1}(s_{i,\Lambda+1}\gamma_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) - \dot{\theta}_{i,\Lambda+1}).\end{aligned} \tag{82}$$

Select the virtual controller $\alpha_{i,\Lambda+1}$ as

$$\begin{aligned}\alpha_{i,\Lambda+1} = {} & -(c_{i,\Lambda+1} + 2)s_{i,\Lambda+1} - k_{i,\Lambda+1}g_i^{\Lambda+1}(y_i - \hat{x}_{i1}) + \dot{\beta}_{i,\Lambda+1} \\ & -\theta^T_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}),\end{aligned} \tag{83}$$

and design the adaptive law $\dot{\theta}_{i,\Lambda+1}$ as

$$\dot{\theta}_{i,\Lambda+1} = s_{i,\Lambda+1}\gamma_{i,\Lambda+1}\psi_{i,\Lambda+1}(\hat{\underline{x}}_{i,\Lambda+1}) - \sigma_{i,\Lambda+1}\theta_{i,\Lambda+1}, \tag{84}$$

where $\sigma_{i,\Lambda+1} > 0$ and $c_{i,\Lambda+1} > 0$ are design parameters.

Taking Eqs. (83) and (84) into Eq. (82), one has

$$\begin{aligned}\dot{V}_{\Lambda+1,i} \leq {} & -c_{i,\Lambda+1}s^2_{i,\Lambda+1} - \frac{1}{2}s^2_{i,\Lambda+1} + \frac{1}{2}w^2_{i,\Lambda+2} + \frac{1}{2}\tilde{\theta}^T_{i,\Lambda+1}\tilde{\theta}_{i,\Lambda+1} \\ & +\frac{1}{2}s^2_{i,\Lambda+2} + \frac{\sigma_{i,\Lambda+1}}{\gamma_{i,\Lambda+1}}\tilde{\theta}^T_{i,\Lambda+1}\theta_{i,\Lambda+1} + \frac{1}{2}\bar{\delta}^2_{i,\Lambda+1} - (\frac{1}{\kappa_{i,\Lambda+1}} - \frac{1}{2})w^2_{i,\Lambda+1}.\end{aligned} \tag{85}$$

From Eqs. (78) and (85), we have

$$\begin{aligned}\dot{V}_{\Lambda+1} = {} & \dot{V}_\Lambda + \Sigma_{i=1}^N \dot{V}_{\Lambda+1,i} \\ \leq {} & \Sigma_{i=1}^N[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i - \Sigma_{q=1}^{\Lambda}\frac{c_{iq}s^2_{iq}}{\flat^2_{iq} - s^2_{iq}} - c_{i,\Lambda+1}s^2_{i,\Lambda+1} + \Sigma_{q=1}^{\Lambda+1}\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}^T_{iq}\theta_{iq}\end{aligned}$$

$$+M_i^{(2)} + \frac{1}{2}\Sigma_{q=2}^{\Lambda+1}\bar{\delta}_{iq}^2 + \frac{1}{2}\Sigma_{q=2}^{\Lambda+1}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{\eta_0}\Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{2}s_{i,\Lambda+2}^2$$
$$+\frac{1}{2}\Sigma_{q=2}^{\Lambda+2}w_{iq}^2 - \Sigma_{q=2}^{\Lambda+1}(\frac{1}{\kappa_{iq}} - \frac{1}{2})w_{iq}^2 + \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}]. \tag{86}$$

Step $q, q = \Lambda+2, \ldots, n-1$: From Eqs. (28), (38) and (39), we have

$$\dot{s}_{iq} = s_{i,q+1} + w_{i,q+1} + \alpha_{iq} + \theta_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq}) + k_{iq}g_i^q(y_i - \hat{x}_{i1}) - \dot{\beta}_{iq}. \tag{87}$$

The Lyapunov function V_q is chosen as

$$V_q = V_{q-1} + \Sigma_{i=1}^N V_{qi} = V_{q-1} + \Sigma_{i=1}^N(\frac{1}{2}s_{iq}^2 + \frac{1}{2\gamma_{iq}}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{2}w_{iq}^2), \tag{88}$$

where $\tilde{\theta}_{iq} = \theta_{iq}^* - \theta_{iq}$, and γ_{iq} is a positive design parameter.
From Eqs. (87) and (88), one has

$$\dot{V}_{qi} = s_{iq}(s_{i,q+1} + \alpha_{iq} + \tilde{\theta}_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq}) - \tilde{\theta}_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq}) + \theta_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq})$$
$$-\dot{\beta}_{iq} + w_{i,q+1} + k_{iq}g_i^q(y_i - \hat{x}_{i1})) + \frac{1}{\gamma_{iq}}\tilde{\theta}_{iq}^T\dot{\tilde{\theta}}_{iq} + w_{iq}\dot{w}_{iq}. \tag{89}$$

According to Young's inequality and $\psi_{iq}(\hat{\underline{x}}_{iq})\psi_{iq}^T(\hat{\underline{x}}_{iq}) \le 1$, one has

$$-s_{iq}\tilde{\theta}_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq}) \le \frac{1}{2}s_{iq}^2 + \frac{1}{2}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}, \tag{90}$$

and

$$s_{iq}(s_{i,q+1} + w_{i,q+1}) \le s_{iq}^2 + \frac{1}{2}s_{i,q+1}^2 + \frac{1}{2}w_{i,q+1}^2. \tag{91}$$

Substituting Eqs. (60), (90) and (91) into Eq. (89), one gets

$$\dot{V}_{qi} \le s_{iq}(\frac{3}{2}s_{iq} + \alpha_{iq} + \theta_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq}) + k_{iq}g_i^q(y_i - \hat{x}_{i1}) - \dot{\beta}_{iq})$$
$$+\frac{1}{2}s_{i,q+1}^2 + \frac{1}{2}w_{i,q+1}^2 + \frac{1}{2}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{2}\bar{\delta}_{iq}^2 - (\frac{1}{\kappa_{iq}} - \frac{1}{2})w_{iq}^2$$
$$+\frac{1}{\gamma_{iq}}\tilde{\theta}_{iq}^T(s_{iq}\gamma_{iq}\psi_{iq}(\hat{\underline{x}}_{iq}) - \dot{\theta}_{iq}). \tag{92}$$

Select the virtual controller α_{iq} as

$$\alpha_{iq} = -(c_{iq}+2)s_{iq} - k_{iq}g_i^q(y_i - \hat{x}_{i1}) + \dot{\beta}_{iq} - \theta_{iq}^T\psi_{iq}(\hat{\underline{x}}_{iq}), \tag{93}$$

and design the adaptive law $\dot{\theta}_{iq}$ as

$$\dot{\theta}_{iq} = s_{iq}\gamma_{iq}\psi_{iq}(\hat{\underline{x}}_{iq}) - \sigma_{iq}\theta_{iq}, \tag{94}$$

where $\sigma_{iq} > 0$ and $c_{iq} > 0$ are design parameters.
Taking Eqs. (93) and (94) into Eq. (92), one has

$$\dot{V}_{qi} \le -c_{iq}s_{iq}^2 - \frac{1}{2}s_{iq}^2 + \frac{1}{2}w_{i,q+1}^2 + \frac{1}{2}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}$$

$$+\frac{1}{2}s_{i,q+1}^2+\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq}+\frac{1}{2}\bar{\delta}_{iq}^2-(\frac{1}{\kappa_{iq}}-\frac{1}{2})w_{iq}^2. \tag{95}$$

From Eqs. (88) and (95), one has

$$\begin{aligned}\dot{V}_q \le{}& \Sigma_{i=1}^N[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i-\Sigma_{l=1}^{\Lambda}\frac{c_{il}s_{il}^2}{\flat_{il}^2-s_{il}^2}-\Sigma_{l=\Lambda+1}^{q}c_{il}s_{il}^2+\Sigma_{l=1}^{q}\frac{\sigma_{il}}{\gamma_{il}}\tilde{\theta}_{il}^T\theta_{il}\\&+M_i^{(2)}+\frac{1}{2}\Sigma_{l=2}^{q}\tilde{\theta}_{il}^T\tilde{\theta}_{il}+\frac{1}{\eta_0}\Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}+\frac{1}{2}s_{i,q+1}^2+\frac{1}{2}\Sigma_{l=2}^{q+1}w_{il}^2\\&+\frac{1}{2}\Sigma_{l=2}^{q}\bar{\delta}_{il}^2-\Sigma_{l=2}^{q}(\frac{1}{\kappa_{il}}-\frac{1}{2})w_{il}^2+\Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}].\end{aligned} \tag{96}$$

Step n : We will structure a real controller in this step. From Eqs. (28), (38) and (39), we have

$$\dot{s}_{in}=\chi_i(u_i(t))u_i(t)+\theta_{in}^T\psi_{in}(\hat{\underline{x}}_{in})+k_{in}g_i^n(y_i-\hat{x}_{i1})-\dot{\beta}_{in}. \tag{97}$$

Chose Lyapunov function V_n as

$$\begin{aligned}V_n&=V_{n-1}+\Sigma_{i=1}^N V_{ni}\\&=V_{n-1}+\Sigma_{i=1}^N(\frac{1}{2}s_{in}^2+\frac{1}{2\gamma_{in}}\tilde{\theta}_{in}^T\tilde{\theta}_{in}+\frac{\iota_i}{2\bar{\gamma}_i}\tilde{\epsilon}_i^2+\frac{1}{2}w_{in}^2),\end{aligned} \tag{98}$$

where $\tilde{\theta}_{iq}=\theta_{iq}^*-\theta_{iq}$ and $\epsilon_i=\frac{1}{\iota_i}$, ι_i is a constant satisfying Eq. (5). Let $\hat{\epsilon}_i$ be the estimation of ϵ_i and $\tilde{\epsilon}_i=\epsilon_i-\hat{\epsilon}_i$. $\bar{\gamma}_i$ and γ_{in} are positive design parameters.

It follows from Eqs. (97) and (98) that

$$\begin{aligned}\dot{V}_{ni}={}&s_{in}(\chi_i(u_i(t))u_i(t)+\tilde{\theta}_{in}^T\psi_{in}(\hat{\underline{x}}_{in})-\tilde{\theta}_{in}^T\psi_{in}(\hat{\underline{x}}_{in})+\theta_{in}^T\psi_{in}(\hat{\underline{x}}_{in})\\&-\dot{\beta}_{in}+k_{in}g_i^n(y_i-\hat{x}_{i1}))+\frac{1}{\gamma_{in}}\tilde{\theta}_{in}^T\dot{\tilde{\theta}}_{in}+\frac{\iota_i}{\bar{\gamma}_i}\tilde{\epsilon}_i\dot{\tilde{\epsilon}}_i+w_{in}\dot{w}_{in}.\end{aligned} \tag{99}$$

According to Young's inequality and $\psi_{in}(\hat{\underline{x}}_{in})\psi_{in}^T(\hat{\underline{x}}_{in})\le 1$, one has

$$-s_{in}\tilde{\theta}_{in}^T\psi_{in}(\hat{\underline{x}}_{in})\le\frac{1}{2}s_{in}^2+\frac{1}{2}\tilde{\theta}_{in}^T\tilde{\theta}_{in}. \tag{100}$$

Substituting Eqs. (60) and (100) into Eq. (99), one gets

$$\begin{aligned}\dot{V}_{ni}\le{}&s_{in}(\chi_i(u_i(t))u_i(t)+\theta_{in}^T\psi_{in}(\hat{\underline{x}}_{in})+k_{in}g_i^n(y_i-\hat{x}_{i1})-\dot{\beta}_{in})+\frac{1}{2}s_{in}^2\\&+\frac{1}{2}\tilde{\theta}_{in}^T\tilde{\theta}_{in}+\frac{1}{2}\bar{\delta}_{in}^2-(\frac{1}{\kappa_{in}}-\frac{1}{2})w_{in}^2+\frac{1}{\gamma_{in}}\tilde{\theta}_{in}^T(s_{in}^3\gamma_{in}\psi_{in}(\hat{\underline{x}}_{in})-\dot{\tilde{\theta}}_{in})\\&+\frac{\iota_i}{\bar{\gamma}_i}\tilde{\epsilon}_i\dot{\tilde{\epsilon}}_i.\end{aligned} \tag{101}$$

Form Eqs. (98) and (101), one gets

$$\begin{aligned}\dot{V}_n&=\dot{V}_{n-1}+\Sigma_{i=1}^N\dot{V}_{ni}\\&\le\Sigma_{i=1}^N[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i-\Sigma_{q=1}^{\Lambda}\frac{c_{iq}s_{iq}^2}{\flat_{iq}^2-s_{iq}^2}-\Sigma_{q=\Lambda+1}^{n-1}c_{iq}s_{iq}^2+\Sigma_{q=1}^{n-1}\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq}\end{aligned}$$

$$+M_i^{(2)} + \frac{1}{2}\Sigma_{q=2}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} - \Sigma_{q=2}^{n}(\frac{1}{\kappa_{iq}} - 1)w_{iq}^2 + \frac{1}{2}\Sigma_{q=2}^{n}\bar{\delta}_{iq}^2 + \frac{1}{\eta_0}\Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}$$
$$+s_{in}(\chi_i(u_i(t))u_i(t) + \theta_{in}^T\psi_{in}(\hat{\underline{x}}_{in}) - \dot{\beta}_{in} + k_{in}g_i^n(y_i - \hat{x}_{i1})) + s_{in}^2$$
$$+\frac{1}{\gamma_{in}}\tilde{\theta}_{in}^T(s_{in}^3\gamma_{in}\psi_{in}(\hat{\underline{x}}_{in}) - \dot{\theta}_{in}) + \frac{\iota_i}{\bar{\gamma}_i}\tilde{\epsilon}_i\dot{\hat{\epsilon}}_i + \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}]. \tag{102}$$

Design the intermediate continuous control function $z_i(t)$ in Eq. (6) as

$$z_i(t) = -(1+\varrho_i)(\alpha_{in}\hat{\epsilon}_i\tanh(\frac{s_{in}\alpha_{in}\hat{\epsilon}_i}{\vartheta_i}) + \frac{\pi_i}{1-\varrho_i}\tanh(\frac{s_{in}\pi_i}{(1-\varrho_i)\vartheta_i})), \tag{103}$$

where $\vartheta_i > 0, i = 1\ldots, N,$ are constants.

By Lemma 3 and Eq. (9), one has

$$\frac{s_{in}\chi_i(u_i(t))z_i(t)}{1+\triangle_1(t)\varrho_i} \le -s_{in}\alpha_{in} + \iota_i\tilde{\epsilon}_i|s_{in}\alpha_{in}| + 0.2785\vartheta_i$$
$$-\frac{s_{in}\chi_i(u_i(t))\pi_i}{1-\varrho_i}\tanh(\frac{s_{in}\pi_i}{(1-\varrho_i)\vartheta_i}), \tag{104}$$

and

$$-\frac{s_{in}\chi_i(u_i(t))\triangle_2(t)\pi_i}{1+\triangle_1(t)\varrho_i} \le \frac{|s_{in}|\chi_i(u_i(t))|\triangle_2(t)|\pi_i}{1+\triangle_1(t)\varrho_i} \le \frac{|s_{in}|\chi_i(u_i(t))\pi_i}{1-\varrho_i}. \tag{105}$$

From Eqs. (9), (104), (105) and Lemma 3, we have

$$s_{in}\chi_i(u_i(t))u_i(t) \le -s_{in}\alpha_{in} + \iota_i\tilde{\epsilon}_i|s_{in}\alpha_{in}| + 0.557\vartheta_i. \tag{106}$$

Substituting Eq. (106) into Eq. (102), then, one has

$$\dot{V}_n \le \Sigma_{i=1}^{N}[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i - \Sigma_{q=1}^{\Lambda}\frac{c_{iq}s_{iq}^2}{\flat_{iq}^2 - s_{iq}^2} - \Sigma_{q=\Lambda+1}^{n-1}c_{iq}s_{iq}^2 + \Sigma_{q=1}^{n-1}\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq}$$
$$+\frac{1}{2}\Sigma_{q=2}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} - \Sigma_{q=2}^{n}(\frac{1}{\kappa_{iq}} - 1)w_{iq}^2 + \frac{1}{2}\Sigma_{q=2}^{n}\bar{\delta}_{iq}^2 + \frac{1}{\eta_0}\Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}$$
$$+0.557\vartheta_i + M_i^{(2)} + s_{in}[-\alpha_{in} + \theta_{in}^T\psi_{in}(\hat{\underline{x}}_{in}) - \dot{\beta}_{in} + k_{in}g_i^n(y_i - \hat{x}_{i1})]$$
$$+s_{in}^2 + \frac{\iota_i}{\bar{\gamma}_i}\tilde{\epsilon}_i(\bar{\gamma}_i|s_{in}\alpha_{in}| + \dot{\hat{\epsilon}}_i) + \frac{1}{\gamma_{in}}\tilde{\theta}_{in}^T(s_{in}\gamma_{in}\psi_{in}(\hat{\underline{x}}_{in}) - \dot{\theta}_{in})$$
$$+\Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}]. \tag{107}$$

Select the virtual controller α_{in} as

$$\alpha_{in} = (c_{in}+1)s_{in} + k_{in}g_i^n(y_i - \hat{x}_{i1}) + \theta_{in}^T\psi_{in}(\hat{\underline{x}}_{in}) - \dot{\beta}_{in}, \tag{108}$$

and design the adaptive law $\dot{\theta}_{in}$ and $\dot{\hat{\epsilon}}_i$ as

$$\dot{\theta}_{in} = s_{in}\gamma_{in}\psi_{in}(\hat{\underline{x}}_{in}) - \sigma_{in}\theta_{in}, \tag{109}$$

and

$$\dot{\hat{\epsilon}}_i = \bar{\gamma}_i|s_{in}\alpha_{in}| - \bar{\sigma}_i\hat{\epsilon}_i. \tag{110}$$

Substituting Eqs. (108)–(110) into Eq. (107), one gets

$$\dot{V}_n \leq \Sigma_{i=1}^{N}[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i - \Sigma_{q=1}^{\Lambda}\frac{c_{iq}s_{iq}^2}{\flat_{iq}^2 - s_{iq}^2} - \Sigma_{q=\Lambda+1}^{n}c_{iq}s_{iq}^2 + \Sigma_{q=1}^{n}\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq}$$
$$+M_i^{(2)} + 0.557\vartheta_i + \frac{1}{2}\Sigma_{q=2}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{1}{\eta_0}\Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{\iota_i\bar{\sigma}_i}{\bar{\gamma}_i}\tilde{\epsilon}_i\hat{\epsilon}_i + \frac{1}{2}\Sigma_{q=2}^{n}\bar{\delta}_{iq}^2$$
$$-\Sigma_{q=2}^{n}(\frac{1}{\kappa_{iq}} - 1)w_{iq}^2 + \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{2\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1}]. \tag{111}$$

By completing square, one has

$$\frac{\sigma_{iq}}{\gamma_{iq}}\tilde{\theta}_{iq}^T\theta_{iq} \leq -\frac{\sigma_{iq}}{2\gamma_{iq}}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \frac{\sigma_{iq}}{2\gamma_{iq}}\theta_{iq}^{*T}\theta_{iq}^*, \tag{112}$$

$$\frac{\sigma_{j1}}{\eta_{j1}}\tilde{\theta}_{j1}^T\theta_{j1} \leq -\frac{\sigma_{j1}}{2\eta_{j1}}\tilde{\theta}_{j1}^T\tilde{\theta}_{j1} + \frac{\sigma_{j1}}{2\eta_{j1}}\theta_{j1}^{*T}\theta_{j1}^*, \tag{113}$$

and

$$\frac{\iota_i\bar{\sigma}_i}{\bar{\gamma}_i}\tilde{\epsilon}_i\hat{\epsilon}_i \leq -\frac{\iota_i\bar{\sigma}_i}{2\bar{\gamma}_i}\tilde{\epsilon}_i^2 + \frac{\iota_i\bar{\sigma}_i}{2\bar{\gamma}_i}\epsilon_i^2 = -\frac{\iota_i\bar{\sigma}_i}{2\bar{\gamma}_i}\tilde{\epsilon}_i^2 + \frac{\bar{\sigma}_i}{2\bar{\gamma}_i\iota_i}. \tag{114}$$

Taking Eqs. (112)–(114) into Eq. (111) and from Lemma 2, one gets

$$\dot{V}_n \leq \Sigma_{i=1}^{N}[-M_i^{(1)}\frac{1}{\eta_0}\xi_i^T\xi_i - \Sigma_{q=1}^{\Lambda}c_{iq}\log\frac{\flat_{iq}^2}{\flat_{iq}^2 - s_{iq}^2} - \Sigma_{q=\Lambda+1}^{n}c_{iq}s_{iq}^2$$
$$-(\frac{\sigma_{i1}}{2\gamma_{i1}} - \frac{1}{\eta_0})\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} - \Sigma_{q=2}^{n}(\frac{\sigma_{iq}}{2\gamma_{iq}} - \frac{1}{2} - \frac{1}{\eta_0})\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} - \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{2\eta_{j1}}\tilde{\theta}_{j1}^T\tilde{\theta}_{j1}$$
$$-\Sigma_{q=2}^{n}(\frac{1}{\kappa_{iq}} - 1)w_{iq}^2 - \frac{\iota_i\bar{\sigma}_i}{2\bar{\gamma}_i}\tilde{\epsilon}_i^2 + \frac{\bar{\sigma}_i}{2\bar{\gamma}_i\iota_i} + \frac{1}{2}\Sigma_{q=2}^{n}\bar{\delta}_{iq}^2 + 0.557\vartheta_i$$
$$+M_i^{(2)} + \Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{2\eta_{j1}}\theta_{j1}^{*T}\theta_{j1}^* + \Sigma_{q=1}^{n}\frac{\sigma_{iq}}{2\gamma_{iq}}\theta_{iq}^{*T}\theta_{iq}^*]. \tag{115}$$

Select parameters $\eta_0, \sigma_{i1}, \gamma_{i1}, \sigma_{iq}, \gamma_{iq}, \kappa_{iq}, q = 2, \ldots, n$, such that $\sigma_{i1}\eta_0 - 2\gamma_{i1} > 0$, $\sigma_{iq}\eta_0 - \gamma_{iq}(\eta_0 + 2) > 0$ and $\frac{1}{\kappa_{iq}} > 0$. Let

$$\nu = \min\{\frac{M_i^{(1)}}{\lambda_{max}(P_i)}, c_{i1}, c_{iq}, \bar{\sigma}_i, \sigma_{i1}\eta_0 - 2\gamma_{i1}, \sigma_{iq}\eta_0 - \gamma_{iq}(\eta_0 + 2), \frac{2}{\kappa_{iq}} - 2\}, \tag{116}$$

for $i = 1, \ldots, N, q = 2, \ldots, n$.

$$M_i^{(3)} = M_i^{(2)} + \Sigma_{q=1}^{n}\frac{\sigma_{iq}}{2\gamma_{iq}}\theta_{iq}^{*T}\theta_{iq}^* + \frac{\bar{\sigma}_i}{2\bar{\gamma}_i\iota_i} + 0.557\vartheta_i + \frac{1}{2}\Sigma_{q=2}^{n}\bar{\delta}_{iq}^2$$
$$+\Sigma_{j\in N_i}a_{ij}\frac{\sigma_{j1}}{2\eta_{j1}}\theta_{j1}^{*T}\theta_{j1}^*, \tag{117}$$

and

$$M^{(3)} = \Sigma_{i=1}^{N}M_i^{(3)}. \tag{118}$$

The following inequality holds

$$\dot{V}_n \leq -\nu V_n + M^{(3)}. \tag{119}$$

5. Stability analysis and parameter design

5.1. Stability analysis

Theorem 1. *For the MASs Eq. (1) with input saturation, under Assumptions 1–5, state observer Eq. (28), and the ETC scheme Eqs. (6), (7), (103), associated with adaptive laws Eqs. (50), (51), (63), (74), (84), (94), (109) and (110), the intermediate control functions Eqs. (49), (62), (73), (83), (93) and (108), consensus tracking can be achieved with consensus errors remaining within small neighborhoods of the origin. Moreover, the following goals can be ensured.*

i) Error signals s_{iq}, observer errors $\tilde{x}_{iq}$, adaptive parameter errors $\tilde{\theta}_{iq}$ and $\tilde{\epsilon}_i$ are bounded and satisfy

$$|s_{iq}| \leq \flat_{iq}(t)(1 - e^{-2V_n(0) - \frac{2M^{(3)}}{\nu}})^{\frac{1}{2}}, q = 1, 2, \ldots, \Lambda,$$

$$|s_{iq}| \leq (2V_n(0) + \frac{2M^{(3)}}{\nu})^{\frac{1}{2}}, q = \Lambda + 1, \ldots, n,$$

$$\tilde{x}_{iq} \leq g_i^{q-1}\sqrt{\eta_0(V_n(0) + \frac{M^{(3)}}{\nu})/\lambda_{min}(P_i)}, q = 1, \ldots, n,$$

$$|\tilde{\epsilon}_i| \leq (\frac{2\bar{\gamma}_i}{\iota_i}(V_n(0) + \frac{M^{(3)}}{\nu}))^{\frac{1}{2}},$$

and

$$|\tilde{\theta}_{iq}| \leq (2\gamma_{iq}V_n(0) + \frac{2\gamma_{iq}M^{(3)}}{\nu})^{\frac{1}{2}}, q = 1, \ldots, n,$$

for $i = 1, \ldots, N$.

ii) Output and PSCs are ensured, ie., $|x_{iq}| < k_{c_{iq}}(t), q = 1, 2, \ldots, \Lambda, \forall t > 0$.

iii) All resulting systems signals are bounded.

iv) Zeno behavior can be avoided.

Proof. It follows from Eq. (119) and applying Comparison principle, we have

$$0 \leq V_n \leq (V_n(0) - \frac{M^{(3)}}{\nu})e^{-\nu t} + \frac{M^{(3)}}{\nu}, \tag{120}$$

and then

$$V_n(t) \leq \frac{M^{(3)}}{\nu}, t \to \infty. \tag{121}$$

□

From Eqs. (98) and (120), one has

$$\frac{1}{2}\log\frac{\flat_{iq}^2}{\flat_{iq}^2 - s_{iq}^2} \leq V_n \leq V_n(0) + \frac{M^{(3)}}{\nu}, q = 1, \ldots, \Lambda. \tag{122}$$

Then, we have

$$|s_{iq}| \leq \flat_{iq}(t)(1 - e^{-2V_n(0) - \frac{2M^{(3)}}{\nu}})^{\frac{1}{2}} < \flat_{iq}(t), q = 1, \ldots, \Lambda. \tag{123}$$

Let $\bar{\flat}_1(t) = \max_{i=1,\ldots,N}\{\flat_{i1}(t)\}$. From Lemma 1, $\mathcal{L} + \mathcal{B}$ is invertible. It follows from Eq. (41) that $|y_i - y_0| \leq \|\bar{y}\|_\infty \leq \|(\mathcal{L} + \mathcal{B})^{-1}\|_\infty \|s_1\|_\infty \leq \|(\mathcal{L} + \mathcal{B})^{-1}\|_\infty \bar{\flat}_1(t)(1 - e^{-2V_n(0) - \frac{2M^{(3)}}{\nu}})^{\frac{1}{2}}$. Therefore, consensus tracking can be achieved with consensus errors remaining within small neighborhoods of the origin. i) From Eqs. (98) and (120), one has

$$\frac{1}{2}s_{iq}^2 \leq V_n \leq V_n(0) + \frac{M^{(3)}}{\nu}, q = \Lambda + 1, \ldots, n. \tag{124}$$

Then, we have

$$|s_{iq}| \leq (2V_n(0) + \frac{2M^{(3)}}{\nu})^{\frac{1}{2}}, q = \Lambda + 1, \ldots, n. \tag{125}$$

Similarly, we have

$$\tilde{x}_{iq} \leq g_i^{q-1}\sqrt{\eta_0(V_n(0) + \frac{M^{(3)}}{\nu})/\lambda_{min}(P_i)}, q = 1, \ldots, n, \tag{126}$$

$$|\tilde{\epsilon}_i| \leq (\frac{2\bar{\gamma}_i}{\iota_i}(V_n(0) + \frac{M^{(3)}}{\nu}))^{\frac{1}{2}}, \tag{127}$$

and

$$|\tilde{\theta}_{iq}| \leq (2\gamma_{iq}V_n(0) + \frac{2\gamma_{iq}M^{(3)}}{\nu})^{\frac{1}{2}}, q = 1, \ldots, n. \tag{128}$$

From Eqs. (123), (125), (126), (127) and (128), $s_{iq}, \tilde{x}_{iq}, \tilde{\epsilon}_i$ and $\tilde{\theta}_{iq}$ are bounded. ii) From Assumption 2, one has $|y_0| \leq a_0$. Thus $|x_{i1}| \leq \|y\|_\infty \leq \|(\mathcal{L} + \mathcal{B})^{-1}\|_\infty \|s_1\|_\infty + a_0 \leq \|(\mathcal{L} + \mathcal{B})^{-1}\|_\infty \bar{\flat}_1(t) + a_0$. Choose $\flat_{i1}(t) \leq \frac{k_{c_{i1}}(t) - a_0}{\|(\mathcal{L}+\mathcal{B})^{-1}\|_\infty}$. Then, we can obtain that $|x_{i1}| < k_{c_{i1}}(t)$.

Since $s_{i2} = \hat{x}_{i2} - \beta_{i2} = x_{i2} - \tilde{x}_{i2} - w_{i2} - \alpha_{i1}$, thus $|x_{i2}| \leq |\tilde{x}_{i2}| + |\alpha_{i1}| + |s_{i2}| + |w_{i2}|$. Because α_{i1} is continuous in some compact set, thus $|\alpha_{i1}| \leq \bar{a}_{i1}$ with $\bar{a}_{i1} > 0$ being a constant, $i = 1, \ldots, N$. From Eq. (120), one has $|w_{i2}| \leq \|(w_{i1}, w_{i2}, \ldots, w_{in})\|_2 \leq \sqrt{2(V_n(0) + \frac{M^{(3)}}{\nu})}$ and Eq. (126). Let

$$\bar{c}_i = g_i^{\Lambda-1}\sqrt{\eta_0(V_n(0) + \frac{M^{(3)}}{\nu})/\lambda_{min}(P_i)}, \tag{129}$$

and

$$\bar{r}_i = \sqrt{2(V_n(0) + \frac{M^{(3)}}{\nu})}, \tag{130}$$

choose $\flat_{i2}(t) = k_{c_{i2}}(t) - \bar{c}_i - \bar{a}_{i1} - \bar{r}_i$, we can obtain $|x_{i2}| < k_{c_{i2}}(t)$. Similar to x_{i2} and choosing $\flat_{iq}(t) = k_{c_{iq}}(t) - \bar{c}_i - \bar{a}_{i,q-1} - \bar{r}_i$, we can obtain $|x_{iq}| < k_{c_{iq}}(t)$ for $q = 3, \ldots, \Lambda$. Therefore, output and partial states constraints are never violated. iii) Since $s_{i,\Lambda+1} = \hat{x}_{i,\Lambda+1} - \beta_{i,\Lambda+1} = x_{i,\Lambda+1} - \tilde{x}_{i,\Lambda+1} - w_{i,\Lambda+1} - \alpha_{i\Lambda}$, thus $|x_{i,\Lambda+1}| \leq |\tilde{x}_{i,\Lambda+1}| + |\alpha_{i\Lambda}| + |s_{i,\Lambda+1}| + |w_{i,\Lambda+1}|$. From the boundedness of $\tilde{x}_{i,\Lambda+1}, \alpha_{i\Lambda}, s_{i,\Lambda+1}$ and $w_{i,\Lambda+1}$, we can obtain that $x_{i,\Lambda+1}$ are bounded. Similar to $x_{i,\Lambda+1}$, $x_{iq}, q = \Lambda + 2, \dots, n$, are bounded. From Eq. (126) and $\hat{x}_{iq} = x_{iq} - \tilde{x}_{iq}, q = 1, \dots, n$, we can obtain that $\hat{x}_{iq}$ are bounded. Therefore, the unconstrained states are also bounded. In addition, from Eqs. (127) and(128), one has $|\hat{\epsilon}_i| \le \epsilon_i + |\tilde{\epsilon}_i| \le \epsilon_i + (\frac{2\bar{\gamma}_i}{\iota_i}(V_n(0) + \frac{M^{(3)}}{\nu}))^{\frac{1}{2}}$ and $|\hat{\theta}_{iq}| \le |\theta_{iq}| + |\tilde{\theta}_{iq}| \le |\theta_{iq}| + (2\gamma_{iq}V_n(0) + \frac{2\gamma_{iq}M^{(3)}}{\nu})^{\frac{1}{2}}, q = 1, \dots, n$, respectively. Then, all systems signals are bounded. iv) Now, let's prove that the Zeno behavior does not occur. It needs to prove that $t^i_{k+1} - t^i_k \ge t^*$ for all i with $t^* > 0$ and $\forall k \in z^+$. Since $\rho_i(t) = z_i(t) - u_i(t)$, $\forall t \in [t^i_k, t^i_{k+1})$, thus

$$\frac{d}{dt}|\rho_i| = \frac{d}{dt}(\rho_i^2)^{\frac{1}{2}} = \text{sgn}(\rho_i)\dot{\rho}_i \le |\dot{z}_i|. \tag{131}$$

From Eqs. (103) and (108), $\dot{z}_i$ is a function of bonded signals. Thus, $|\dot{z}_i| \le c$ with $c > 0$ being a constant. Note that $\rho_i(t^i_k) = 0$ and $\lim_{t \to t^i_{k+1}} \rho_i = \varrho_i|u_i(t)| + \pi_i$. Integrating both sides of Eq. (131) from t^i_k to t^i_{k+1} and letting $t^* = \frac{\varrho_i|u_i(t)|+\pi_i}{c} > 0$, the lower bound t^* of inter-execution interval is obtained such that $t^i_{k+1} - t^i_k \ge t^* > 0$. Therefore, the Zeno behavior does not occur. □

5.2. Parameter design

In this subsection, we discuss the parameter design method to ensure correct operation of the proposed algorithm.

For given interconnected graph $\mathcal{G}$ with adjacency matrix $\mathcal{A}$, $\mathcal{B}$ and Laplacian matrix $\mathcal{L}$, the bound a_0 of the reference signal $y_0(t)$, the bound a_1 of $\dot{y}_0(t)$, the saturated bound u_{iM} and the time-varying boundary function $k_{c_{iq}}(t), i = 1, \dots, N, q = 1, \dots, \Lambda$, the parameter design process is given as follows: i) Set the initial value of $x_{iq}(0)$, $\hat{x}_{iq}(0)$, $\theta_{iq}(0)$, and $\hat{\epsilon}_i(0)$, $i = 1, 2, \dots, N$, $q = 1, \dots, n$, where the constrained states x_{iq} and its estimate $\hat{x}_{iq}$ should satisfy $x_{iq}(0) < k_{c_{iq}}(0)$and $\hat{x}_{iq}(0) < k_{c_{iq}}(0), q = 1, \dots, \Lambda$. The initial value of the filter output $\beta_{iq}(0) = \alpha_{i,q-1}(0), i - 1, \dots, N, q = 2, \dots, n$, in Eq. (40) is calculated according to Eqs. (49), (62), (73), (83) and (93). ii) Select appropriate center r^k_{iq} and width $\mu^k_{iq}, k = 1, \dots, l$, to construct RBFNN such that $f_{iq}(\underline{x}_{iq}) = \theta^T_{iq}\psi_{iq}(\hat{\underline{x}}_{iq}), i = 1, \dots, N, q = 1, \dots, n$. iii) Select the Hurwitz polynomial coefficients $k_{iq}, q = 1, \dots, n$, in Eq. (26) such that the matrix A_i is Hurwitz. For given matrix $Q_i > 0$, solve Lyapunov Eq. (27) to obtain positive definite matrix P_i and then calculate the maximum and minimum eigenvalues of matrices Q_i and P_i. iv) For $i, j = 1, \dots, N$, select high gain parameters $g_i > 1$, positive design parameters η_0, $\bar{\gamma}_i$, $\bar{\sigma}_i$, η_{j1}, c_{iq}, γ_{iq}, $\sigma_{iq}, q = 1, \dots, n$, and $\kappa_{iq}, q = 2, \dots, n$, such that $M^{(1)}_i > 0, \sigma_{i1}\eta_0 - 2\gamma_{i1} > 0$, $\sigma_{iq}\eta_0 - \gamma_{iq}(\eta_0 + 2) > 0$, and $\frac{2}{\kappa_{iq}} - 2 > 0$. v) Select appropriate boundary function $\flat_{iq}(t), q = 1, \dots, \Lambda$, satisfying $\flat_{i1}(t) \le \frac{k_{c_{i1}}(t) - a_0}{\|(\mathcal{L}+\mathcal{B})^{-1}\|_\infty}$ and $\flat_{iq}(t) = k_{c_{iq}}(t) - \bar{c}_i - \bar{a}_{i,q-1} - \bar{r}_i, q = 2, \dots, \Lambda$, where $\bar{c}_i$ and $\bar{r}_i$ are defined in Eqs. (129) and (130), respectively. vi) Solve differential equations subject to system Eq. (1), observer Eq. (28), adaptive system Eqs. (50), (51), (63), (74), (84), (94), (109) and (110), and the first-order filter Eq. (40), in which the virtual controller α_{iq} is calculated according Eqs. (49), (62), (73), (83) and (93), the intermediate control function $z_i(t)$ according to Eq. (103), and the saturated controller $\text{sat}_i(u_i(t))$ according to Eq. (2) associated with the ETC mechanism Eqs. (6) and (7).

Remark 2 : By selecting appropriate parameters, stability of closed-loop system is ensured by Eq. (119). From Eq. (119), we know that larger ν and smaller $M^{(3)}$ lead to faster convergence and smaller error bounds. From Eq. (116), increasing $c_{i1}, c_{iq}, \sigma_{i1}, \bar{\sigma}_i$ and σ_{iq}, and decreasing η_0, γ_{i1}, γ_{iq} and $\kappa_{iq}, q = 2, \dots, n$, lead to faster convergence but may lead to lager error bound duo to Eqs. (117) and (118). Therefore, we need to find a balance between a faster convergence speed and an appropriate error bound. Moreover, smaller parameters ϱ_i and π_i in the ETC mechanism Eq. (7) lead to better system performance but may increase unnecessary communication burden. Obviously, when ϱ_i and π_i are equal to zero, it becomes time-triggered scheme as the special case of ETC scheme. As a result, several attempts are needed to obtain better system performance.

6. Numerical example

In this section, an example is given to verify the correctness of the theoretical results.

The consensus control problem of four one-link manipulators tracking a reference signal $y_0 = \sin(t)$ is considered. The communication relationship among four manipulators and the reference signal can be modeled by a digraph including a spanning tree shown in Fig. 1. The four manipulators are given as follows

$$\varpi_i^{(1)}\ddot{p}_i + \varpi_i^{(2)}\dot{p}_i + \varpi_i^{(3)}\sin(p_i) = \mathrm{sat}_i(u_i) + \tau_i, i = 1, 2, 3, 4, \tag{132}$$

where p_i is the angle of the rigid link, $\dot{p}_i$ the angular velocity and $\ddot{p}_i$ the link acceleration; $\mathrm{sat}_i(u_i)$ is the saturated control input given in Eq. (3); τ_i is the torque disturbance; $\varpi_i^{(1)}$, $\varpi_i^{(2)}$ and $\varpi_i^{(3)}$ are unknown constants.

Letting $x_{i1} = p_i$ and $x_{i2} = \dot{p}_i$. Equation (132) can be rewritten as

$$\begin{cases} \dot{x}_{i1} &= x_{i2}, \\ \dot{x}_{i2} &= \frac{1}{\varpi_i^{(1)}}\mathrm{sat}_i(u_i(t)) + \frac{1}{\varpi_i^{(1)}}\tau_i - \frac{\varpi_i^{(2)}}{\varpi_i^{(1)}}x_{i2} - \frac{\varpi_i^{(3)}}{\varpi_i^{(1)}}\sin(x_{i1}), \\ y_i &= x_{i1}, \end{cases} \tag{133}$$

where $\varpi_i^{(1)} = \varpi_i^{(2)} = 1$, $\varpi_i^{(3)} = 2$, $\tau_i = 0.1\cos(10t)$. The states $x_{i1}, i = 1, 2, 3, 4$, are constrained by $k_{c_{i1}}$. The main parameters are chosen as $k_{i1} = 10, k_{i2} = 15, g_i = 10$, $c_{11} = c_{21} = 15, c_{31} = 10, c_{41} = 5, c_{12} = c_{22} = c_{32} = c_{42} = 30, k_{c_{11}} = k_{c_{21}} = 2e^{-t} + 1.2, k_{c_{31}} = 4e^{-t} + 2.4$, $k_{c_{41}} = 4e^{-t} + 2.6$, $\kappa_{i2} = 0.05$, $\varrho_i = 0.1$, $u_{iM} = 8$, $Q_i = \mathrm{diag}[1, 1]$, $\pi_i = 1.5$, $\vartheta_1 = \vartheta_2 = 1$, $\vartheta_3 = \vartheta_4 = 0.5$, $\gamma_{11} = \gamma_{21} = \gamma_{12} = \gamma_{22} = 0.005$, $\gamma_{31} = \gamma_{41} = \gamma_{32} = \gamma_{42} = 0.002$, $\sigma_{11} = \sigma_{21} = \sigma_{12} = \sigma_{22} = 25$, $\bar{\gamma}_i = 0.5$, $\sigma_{31} = \sigma_{41} = \sigma_{32} = \sigma_{42} = 8$, $\bar{\sigma}_i = 1$, for $1 < i < 4$. Set $x_{i1}(0) = 0.1$ and other initial states are set to 0. The basis functions of the RBFNN with 5 nodes are chosen as follows:

$$\psi_i^1(x_{iq}) = \exp[-\frac{(x_{iq}-2)^2}{2^2}], \psi_i^2(x_{iq}) = \exp[-\frac{(x_{iq}-1)^2}{2^2}],$$

$$\psi_i^3(x_{iq}) = \exp[-\frac{x_{iq}^2}{2^2}], \psi_i^4(x_{iq}) = \exp[-\frac{(x_{iq}+1)^2}{2^2}],$$

$$\psi_i^5(x_{iq}) = \exp[-\frac{(x_{iq}+2)^2}{2^2}], i = 1, 2, 3, 4, q = 1, 2.$$

The simulation is conducted in 20 s time, and the simulation results are shown in Figs. 2–12. The trajectories of the states $x_{i1}, i = 1, 2, 3, 4$, are shown in Fig. 2 and we can see that the trajectories do not violate the restricted bounds. Figure 3 depicts the trajectories of reference signal y_0 and outputs $y_i, i = 1, 2, 3, 4$. The trajectories of reference signal y_0 and outputs $\hat{y}_i$, are shown in Fig. 4. From Figs. 3 and 4, we can see that system output and observer output

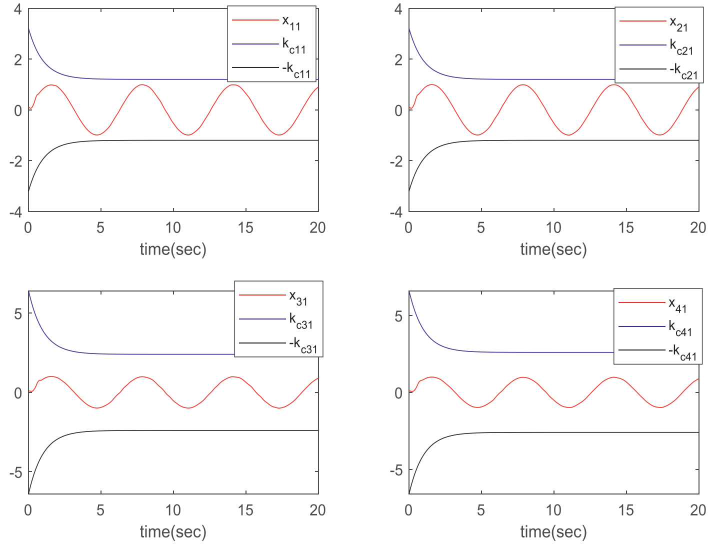

Fig. 2. The constrains on state x_{11}, x_{21}, x_{31} and x_{41}.

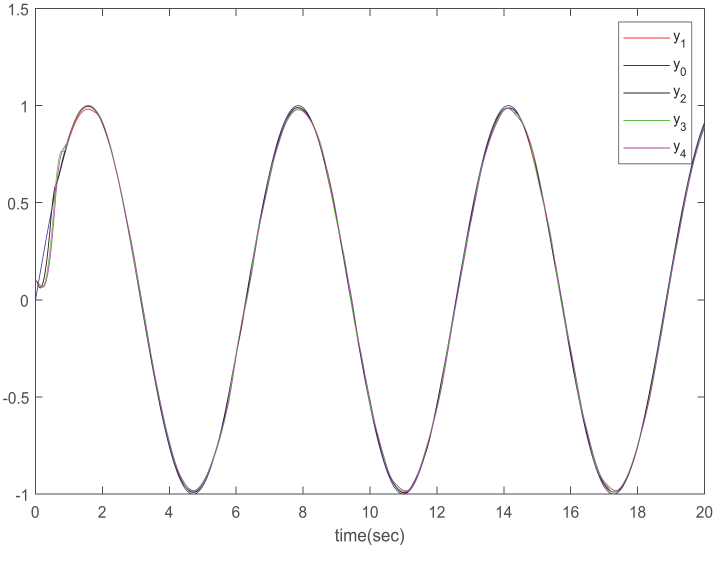

Fig. 3. The trajectories of y_0, y_1, y_2, y_3, and y_4 with ETC scheme.

can track the reference signal well and tracking errors remain within a small neighborhood of the origin. Figure 5 shows the trajectories of observer errors $\tilde{x}_{i1}$, and we can see that the trajectories of $\tilde{x}_{i1}, i = 1, 2, 3, 4,$ fluctuate very little in a certain range. Figure 6 shows the trajectories of the local consensus error $s_{i1}, i = 1, 2, 3, 4$, which does not violate the restricted bounds. The trajectories of the states x_{i2} and the observer states $\hat{x}_{i2}, i = 1, 2, 3, 4$, are shown in Fig. 7 and we can see that the observer states $\hat{x}_{i2}, i = 1, 2, 3, 4,$ are reasonable estimates of the states x_{i2} with small errors. Figures 8–11 show the trajectories of control inputs $u_i(t)$

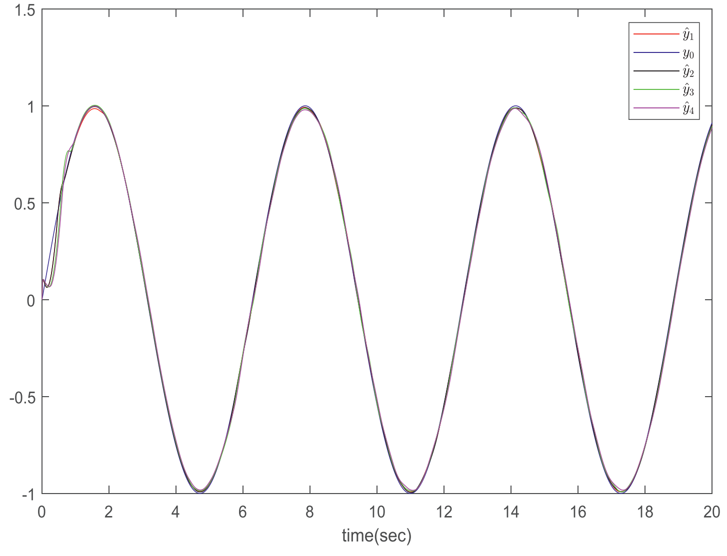

Fig. 4. The trajectories of $y_0, \hat{y}_1, \hat{y}_2, \hat{y}_3$, and $\hat{y}_4$ with ETC scheme.

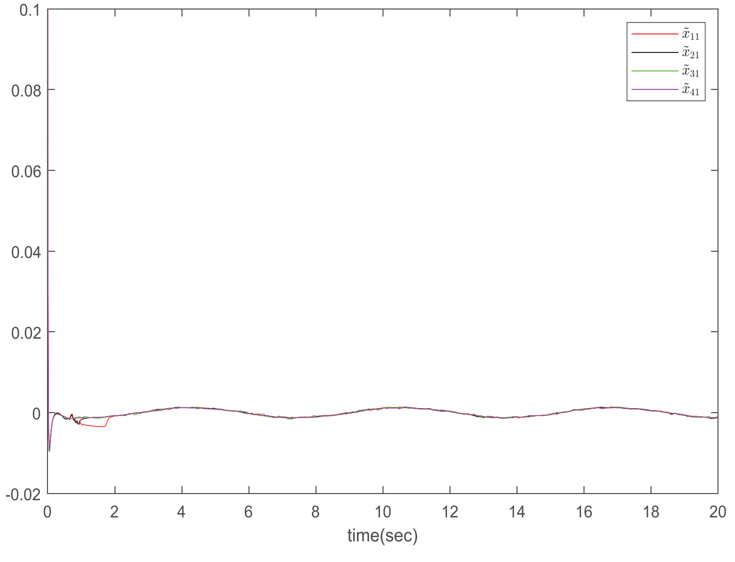

Fig. 5. The trajectories of $\tilde{x}_{i1}$ with ETC scheme.

and saturated input $\text{sat}_i(u_i(t))$. As can be seen from the detail magnification in Figs. 8–11, when the system needs a larger control input (blue line), the actual saturated control input (red line) can still achieve better control effect. Figure 12 shows the trigger time instants t_k^i and inter-event time $t_{k+1}^i - t_k^i$ for agent $i, i = 1, 2, 3, 4$. The event-triggered controller updates only at time instant t_k^i and remains constant within time interval $[t_k^i, t_{k+1}^i)$, thus reducing the communication burden.

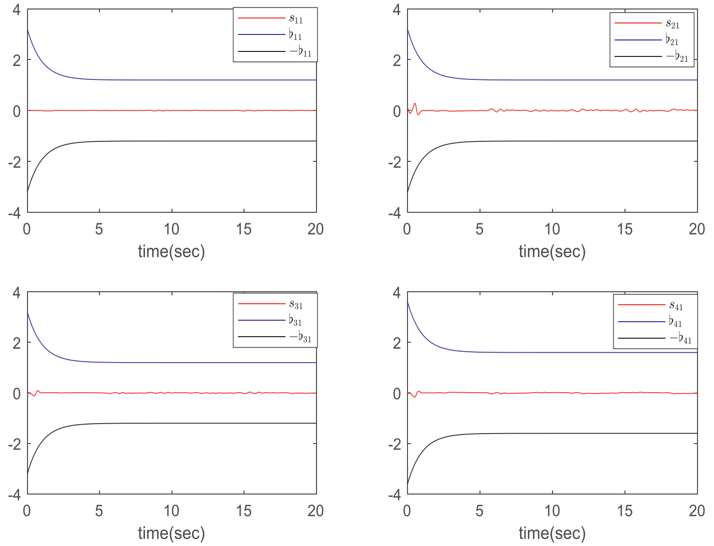

Fig. 6. The constrains on error variable s_{11}, s_{21}, s_{31} and s_{41}.

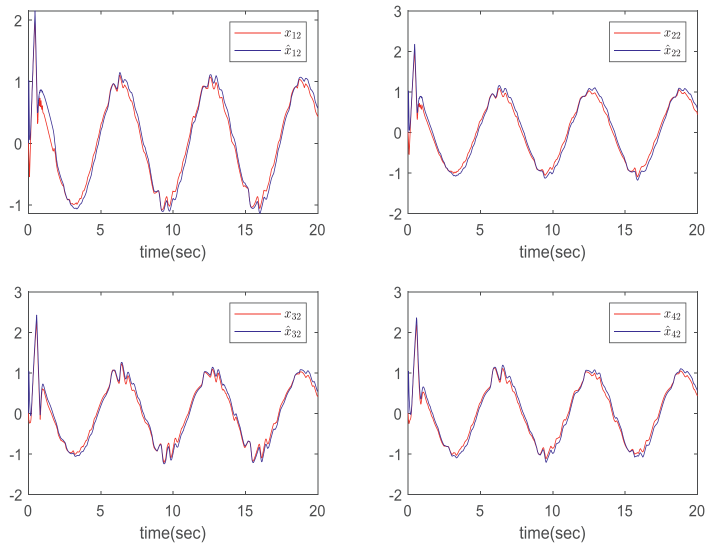

Fig. 7. The trajectories of x_{i2}, and $\hat{x}_{i2}$ with ETC scheme.

Compared with related works for MASs, the cases of FSCs [34–36], constant boundary [34] with time-triggered scheme and output constraint with ETC scheme in undirected networks [37] are special cases of this paper. In the related works such as [29,37] and this paper, almost the same ETC mechanism is used. To highlight the advantages of the proposed event-based control strategy, a comparison with time-based control strategy is given. Figures 13 and 14 show the tracking performance and tracking error trajectories with time-triggered control method, respectively. For time-triggered control scheme, the sampling period is selected as

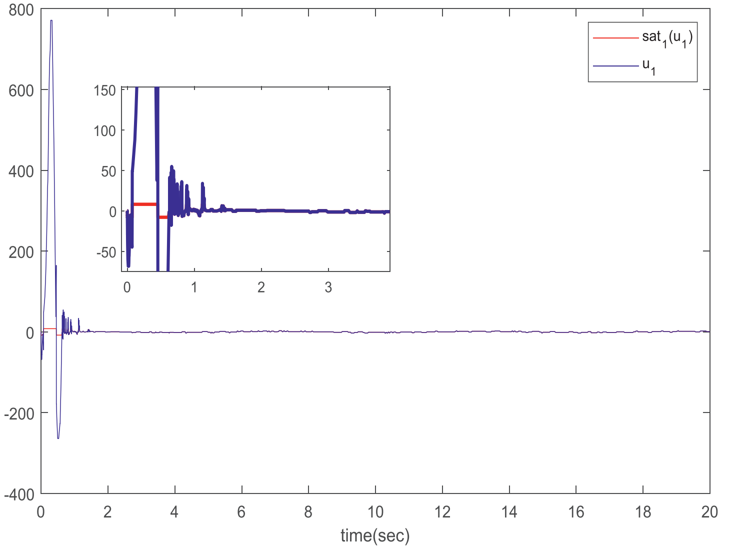

Fig. 8. Input $u_1(t)$ and saturated input $\text{sat}_1(u_1(t))$.

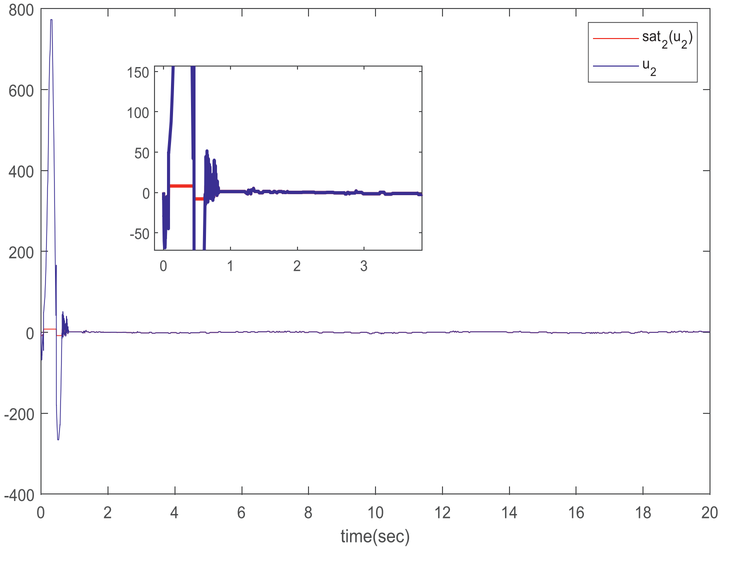

Fig. 9. Input $u_2(t)$ and saturated input $\text{sat}_2(u_2(t))$.

$T = 0.01s$. The other design parameters are chosen to be the same as event-triggered control method. A comparison considering the number of transmissions generated by the triggering mechanism is given in Table 1. Obviously, the triggering number for event-based method is much smaller than time-based one.

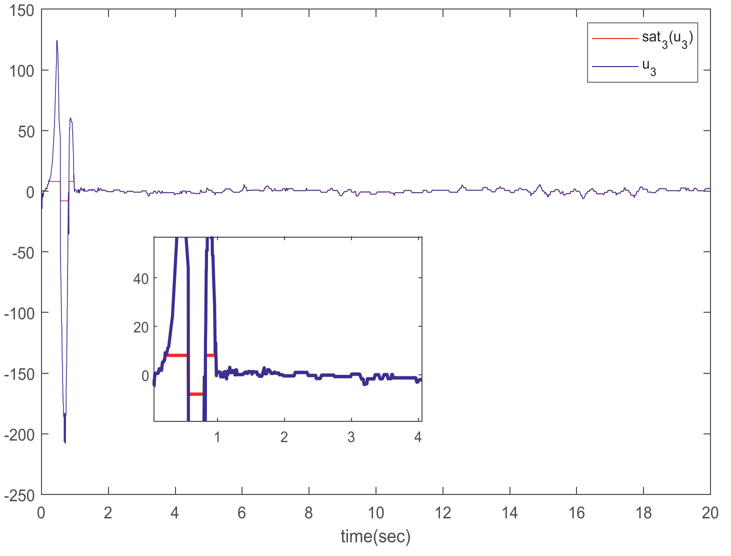

Fig. 10. Input $u_3(t)$ and saturated input $\mathrm{sat}_3(u_3(t))$.

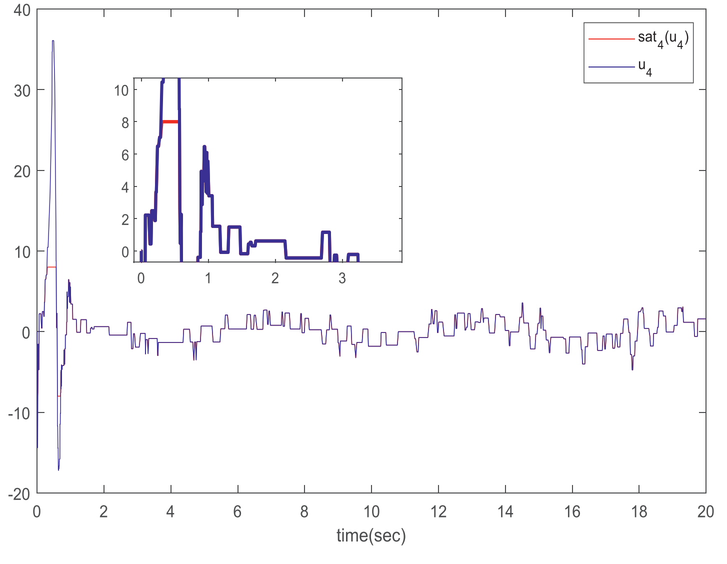

Fig. 11. Input $u_4(t)$ and saturated input $\mathrm{sat}_4(u_4(t))$.

Table 1
Triggering numbers for agents.

control method	agent 1	agent 2	agent 3	agent 4
Time-triggered	2000	2000	2000	2000
Event-triggered	590	611	324	200

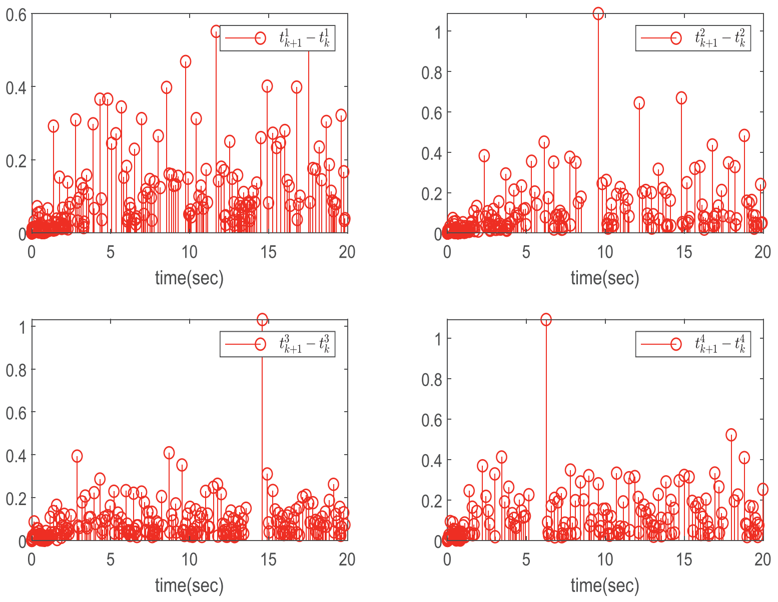

Fig. 12. The trigger time instants t^i_k and inter-event time $t^i_{k+1} - t^i_k$ for agent i.

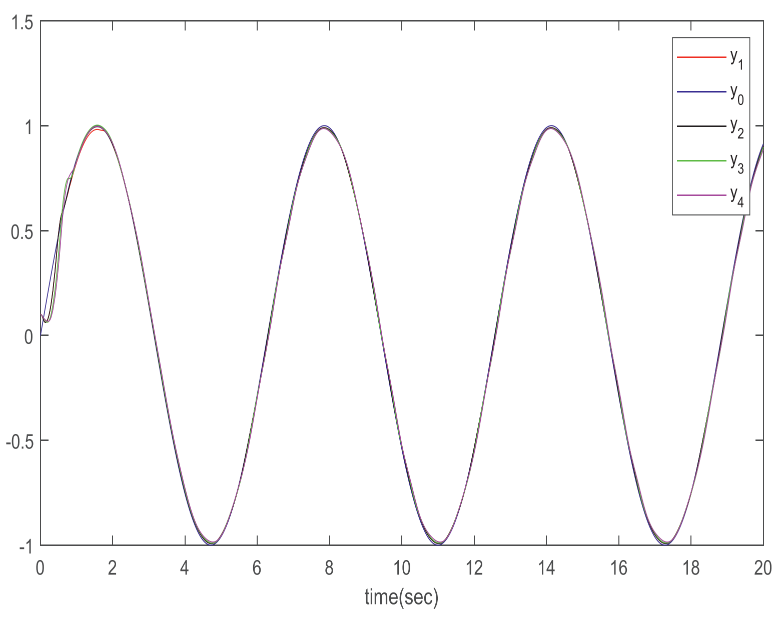

Fig. 13. The trajectories of y_0, y_1, y_2, y_3, and y_4 with time-triggered control scheme.

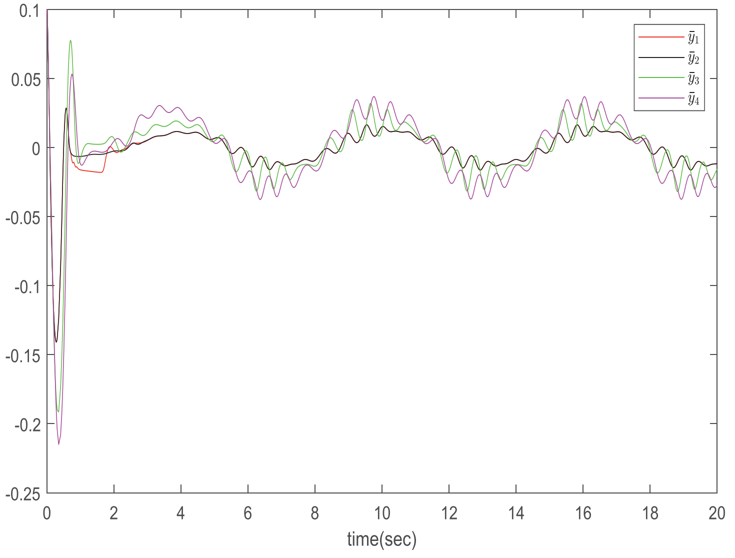

Fig. 14. The trajectories of $\bar{y}_i, i = 1, 2, 3, 4,$ with time-triggered control scheme.

7. Conclusion

In this paper, an adaptive distributed ETC strategy with observer has been presented. RBFNN is used to handle the nonlinearity of the system, and the unmeasured states are handled by designing a high gain state observer. A new saturation controller is proposed for MAS, which is more suitable for practical applications. Besides, time-varying barrier Lyapunov functions are introduced, which can ensure that the partial states do not violate the constraint conditions. Considering the benefit of communication resource saving, an adaptive ETC strategy has been proposed to guarantee consensus tracking of MASs. The proposed ETC strategy can ensure the boundedness of all system signals, each agent being able to track the given leader signal within a bounded error and avoiding Zeno behavior successfully. Finally, the correctness of the theoretical results is verified by computer simulation.

Declaration of Competing Interest

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, “Event-triggered adaptive control of multi-agent systems with saturated input and partial state constraints”.

References

[1] X. Dong, B. Yu, Z. Shi, Y. Zhong, Time-varying formation control for unmanned aerial vehicles: theories and applications, IEEE Trans. Control Syst. Technol. 23 (1) (2015) 340–348.
[2] L. Consolini, F. Morbidi, D. Prattichizzo, M. Tosques, Leader-follower formation control of nonholonomic mobile robots with input constraints, Automatica 44 (2008) 1343–1349.

[3] C. Huang, D.W.C. Ho, J. Lu, Partial-information-based distributed filtering in two-targets tracking sensor networks, IEEE Trans. Circuits Syst. I 59 (4) (2012) 820–832.
[4] Y. Zhao, Z. Duan, G. Wen, Finite-time consensus for second-order multi-agent systems with saturated control protocols, IET Control Theory Appl. 9 (3) (2014) 312–319.
[5] C. Li, Z. Qu, Distributed finite-time consensus of nonlinear systems under switching topologies, Automatica 50 (6) (2014) 1626–1631.
[6] H. Du, Y. He, Y. Cheng, Finite-time synchronization of a class of second-order nonlinear multi-agent systems using output feedback control, IEEE Trans. Circuits Syst. I 61 (6) (2014) 1778–1788.
[7] Y.Y. Qian, L. Liu, G. Feng, Distributed event-triggered adaptive control for consensus of linear multi-agent systems with external disturbances, IEEE Trans. Cybern. 50 (5) (2020) 2197–2208.
[8] J. Li, D.W. Ho, J. Li, Adaptive consensus of multi-agent systems under quantized measurements via the edge Laplacian, Automatica 92 (2018) 217– 224.
[9] L. Zhao, J. Yu, C. Lin, Distributed adaptive output consensus tracking of nonlinear multi-agent systems via state observer and command filtered backstepping, Inf. Sci. 478 (2019) 355–374.
[10] H. Su, Y. Liu, Z. Zeng, Second-order consensus for multiagent systems via intermittent sampled position data control, IEEE Trans. Cybern. 50 (5) (2020) 2063–2072.
[11] J. Mei, W. Ren, J. Chen, Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph, IEEE Trans. Automat. Control 61 (8) (2016) 2019–2034.
[12] Z. Wu, Y. Xu, R. Lu, Y. Wu, T. Huang, Event-triggered control for consensus of multiagent systems with fixed/switching topologies, IEEE Trans. Syst. Man Cybern. (99) (2017) 1–11.
[13] S.M. Noorbakhsh, J. Ghaisari, Distributed event-triggered consensus strategy for multi-agent systems under limited resources, Int. J. Control 89 (1) (2016) 156–168.
[14] K. Liu, Z. Ji, X. Zhang, Periodic event-triggered consensus of multi-agent systems under directed topology, Neurocomputing 385 (2020) 33–41.
[15] J. Cai, R. Yu, B. Wang, C. Mei, L. Shen, Decentralized event-triggered control for interconnected systems with unknown disturbances, J. Franklin Inst. 357 (3) (2020) 1494–1515.
[16] J. Sun, J. Yang, S. Li, X. Wang, G. Li, Event-triggered output consensus disturbance rejection algorithm for multi-agent systems with time-varying disturbances, J. Franklin Inst. 357 (17) (2020) 12870–12885.
[17] X. Li, X. Yang, J. Cao, Event-triggered impulsive control for nonlinear delay systems, Automatica 117 (2020) 108981.
[18] C. Gao, X. Liu, Y. Yang, X. Liu, P. Li, Event-triggered finite-time adaptive neural control for nonlinear non-strict-feedback time-delay systems with disturbances, Inf. Sci. 536 (2020) 1–24.
[19] L. Liu, X. Li, Y.J. Liu, S. Tong, Neural network based adaptive event trigger control for a class of electromagnetic suspension systems, Control Eng. Pract. 106 (2021) 104675.
[20] G. Liu, Y. Pan, H.K. Lam, H. Liang, Event-triggered fuzzy adaptive quantized control for nonlinear multi-agent systems in nonaffine pure-feedback form, Fuzzy Sets Syst. (2020).
[21] G. Guo, L. Ding, Q.L. Han, A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems, Automatica 50 (5) (2014) 1489–1496.
[22] Q. Zhou, W. Wang, H. Liang, M.V. Basin, B. Wang, Observer-based event-triggered fuzzy adaptive bipartite containment control of multi-agent systems with input quantization,, IEEE Trans. Fuzzy Syst. 29 (2) (2020) 372–384.
[23] X. Li, Y. Tang, H.R. Karimi, Consensus of multi-agent systems via fully distributed event-triggered control, Automatica 116 (2020) 108898.
[24] X. Li, Z. Sun, Y. Tang, H.R. Karimi, Adaptive event-triggered consensus of multiagent systems on directed graphs, IEEE Trans. Automat. Control 66 (4) (2021) 1670–1685.
[25] L. Liu, A. Chen, S. Tong, C.L. Philip Chen, Integral barrier Lyapunov function-based adaptive control for switched nonlinear systems, Sci. China Inf. Sci. 63 (2020) 132203.
[26] K.P. Tee, S.S. Ge, E.H. Tay, Barrier Lyapunov functions for the control of output-constrained nonlinear systems, Automatica 45 (4) (2009) 918–927.
[27] K.P. Tee, B. Ren, S.S. Ge, Control of nonlinear systems with time-varying output constraints, Automatica 47 (11) (2011) 2511–2516.
[28] H. Zhang, Y. Liu, Y. Wang, Observer-based finite-time adaptive fuzzy control for nontriangular nonlinear systems with full-state constraints, IEEE Trans. Cybern. 51 (3) (2021) 1110–1120.
[29] L. Wang, C.L.P. Chen, H. Li, Event-triggered adaptive control of saturated nonlinear systems with time-varying partial state constraints, IEEE Trans. Cybern. 50 (4) (2020) 1485–1497.

[30] Y.J. Liu, W. Zhao, L. Liu, D. Li, S. Tong, C.L.P. Chen, Adaptive neural network control for a class of nonlinear systems with function constraints on states, IEEE Trans. Neural Netw. Learn. Syst. (2021) 1–10.
[31] Y.J. Liu, S. Lu, S. Tong, X. Chen, C.P. Chen, D.J. Li, Adaptive control-based barrier Lyapunov functions for a class of stochastic nonlinear systems with full state constraints, Automatica 87 (2018) 83–93.
[32] Y. Liu, L. Ma, L. Liu, S. Tong, C.L.P. Chen, Adaptive neural network learning controller design for a class of nonlinear systems with time-varying state constraints, IEEE Trans. Neural Netw. Learn. Syst. 31 (1) (2020) 66–75.
[33] T. Gao, Y. Liu, L. Liu, D. Li, Adaptive neural network-based control for a class of nonlinear pure-feedback systems with time-varying full state constraints, IEEE/CAA J. Autom. Sin. 5 (5) (2018) 923–933.
[34] Q. Ji, G. Chen, Q. He, Neural network-based distributed finite-time tracking control of uncertain multi-agent systems with full state constraints, IEEE Access 8 (2020) 174365–174374.
[35] F. Yuan, J. Lan, Y. Liu, L. Liu, Adaptive NN control for nonlinear multi-agent systems with unknown control direction and full state constraints, IEEE Access 9 (2021) 24425–24432.
[36] L. Shang, M. Cai, Adaptive practical fast finite-time consensus protocols for high-order nonlinear multi-agent systems with full state constraints, IEEE Access 9 (2021) 81554–81563.
[37] L. Wang, J. Dong, C. Xi, Event-triggered adaptive consensus for fuzzy output-constrained multi-agent systems with observers, J. Franklin Inst. 357 (1) (2020) 82–105.
[38] C. Wen, J. Zhou, Z. Liu, H. Su, Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance, IEEE Trans. Automat. Control 56 (7) (2011) 1672–1678.
[39] Y. Li, S. Tong, T. Li, Adaptive fuzzy output-feedback control for output constrained nonlinear systems in the presence of input saturation, Fuzzy Sets Syst. 248 (2014) 138–155.
[40] Q. Zhou, L. Wang, C. Wu, H. Li, H. Du, Adaptive fuzzy control for nonstrict-feedback systems with input saturation and output constraint, IEEE Trans. Syst. Man Cybern. 47 (1) (2017) 1–12.
[41] W. Ren, R. Beard, Consensus seeking in multiagent systems under dynamically changing interaction topologies, IEEE Trans. Automat. Control 50 (5) (2005) 655–661.
[42] Z. Zhu, Y. Xia, M. Fu, Adaptive sliding mode control for attitude stabilization with actuator saturation, IEEE Trans. Ind. Electron. 58 (10) (2011) 4898–4907.
[43] Z. Zhu, Y. Pan, Q. Zhou, C. Lu, Event-triggered adaptive fuzzy control for stochastic nonlinear systems with unmeasured states and unknown backlash-like hysteresis, IEEE Trans. Fuzzy Syst. 29 (5) (2021) 1273–1283.
[44] B. Ren, S.S. Ge, K.P. Tee, T.H. Lee, Adaptive neural control for output feedback nonlinear systems using a barrier Lyapunov function, IEEE Trans. Neural Netw. 21 (5499019) (2010) 1339–1345.
[45] K.H. Johansson, M. Egerstedt, J. Lygeros, S. Sastry, On the regularization of Zeno hybrid automata, Syst. Control Lett. 38 (3) (1999) 141–150.
[46] M.M. Polycarpou, P.A. Ioannou, A robust adaptive nonlinear control design, Automatica 32 (1996) 423–427.
[47] T. Zhang, S. Ge, C. Hang, Adaptive neural network control for strict-feedback nonlinear systems using backstepping design, Automatica 36 (12) (2000) 1835–1846.
[48] Y.L. S. Tong Y. Li, Y. Liu, Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems, IEEE Trans. Syst. Man Cybern.Part B 41 (2011) 1693–1704.