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Optimal operation scheduling of a pumping station with multiple pumps

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HIGHLIGHTS

- We formulate the optimal operation scheduling of a pumping station with multiple pumps as a dynamic programming problem.
- The extended reduced dynamic programming algorithm (RDPA) is proposed to solve the optimization problem.
- The extended RDPA reduces the admissible domain of the possible state values at each stage and that of the possible state transfer routes.
- The optimal scheduling strategy reduces the operational cost.
- The extended RDPA performs much more time efficient than the conventional DP algorithms.

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ABSTRACT

The optimal operation scheduling of a pumping station with multiple pumps is formulated as a dynamic programming problem. Based on the characteristics of the problem, an extended reduced dynamic programming algorithm (RDPA) is proposed to solve the problem. Both the energy cost and the maintenance cost are considered in the performance function of the optimization problem. The extended RDPA can significantly reduce the computational time when it is compared to conventional DP algorithms. Simulation shows the feasibility of the reduction of the operation cost.

1. Introduction

The operation of a pumping station is very important in achieving the tasks of the station. The main task is to maintain a suitable water volume in the reservoir and supply the demands. Another important task is to reduce the operation cost. With the operation scheduling optimized, remarkable reduction of the operation cost could be achieved while no change is needed with the physical elements, such as pumps and civil infrastructures [1].

The operation scheduling problem of a pumping station can be formulated as a cost optimization problem, of which the objective is to minimized the operation cost while the state constraints are satisfied. For a reservoir, the water volume/water level should be kept within a range to satisfy the security and operation requirements.

There are mainly two classes of operation costs for a pumping station. One is the energy cost, and the other is the maintenance cost. The maintenance cost, related to the wear of the rotating equipments, is difficult to be quantified. However it is true that the maintenance cost increases when the number of pump switches increases. A simple assumption is that the maintenance cost is proportional to the number of pump switches. Here a pump switch refers to "turning on a pump that was not operating in the previous period" [1].

Energy cost is the main part of the operation cost. When physical elements are not changed, the energy cost is related to the energy consumption and the energy pricing structure. The energy consumption is proportional to the pump power and the operational time length. With the time of use (ToU) electricity pricing structure implemented, the operation scheduling has a heavy influence on the energy cost if there is room for the scheduling of the pump operation. Such kind of load shifting has been studied for various systems, such as in [2] for steel plants and in [3] for a deep-mine water reticulation system.

The problem on the optimal operation scheduling of a pumping station has been studied in many papers in recent years. The problem for a pumping station with fixed-speed motored pumps is intrinsically an integer programming problem (linear or nonlinear), depending on the mathematical models of the hydraulic structures, networks, etc. For such a kind of integer programming problems, various techniques have been employed in load shifting

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for different processes. The linear programming is employed in [4] for a wind/hydro hybrid water supply system while the dynamic programming (DP) in [5–7] for a water supply system and a pump station. The stochastic DP is considered in [8] for a water supply system with the water demand modeled as a Markov process. The binary integer programming (BIP) is used in [9] for the operation scheduling of a colliery. The above methods can theoretically solve the optimal operation scheduling problems, but they are limited in practice when the underlying model is large or complex because of *curse of dimensionality* of the DP or *interminable branch and bound* of the integer programming.

To search the global optimal solution to a programming problem, some modern optimization methods, such as genetic algorithm [10–13], simulated annealing [14,15], particle swarm optimization [16], ant colony optimization [17] and fuzzy optimization [18], are adopted. Those approaches improve the possibility of obtaining the global optimization solution while the computational time is sometimes very long and the algorithms are sometimes too complex, which again limits their application.

In [19], a reduced dynamic programming algorithm is developed to address the optimal operation scheduling problem with the capability of fast computation. The scheduling problem is reformulated as a control sequence optimal scheduling problem. This algorithm is a cost-efficient scheduling approach for the pump operation.

The optimal operation scheduling in [19], is studied for a pumping station with only one pump. The admitted domain of the control variable is $\{0,1\}$. When more pumps' operation is required to be optimized, the method in [19] could not be directly employed. The optimal operation scheduling of a pumping station with multiple pumps are considered in this paper. The approach of RDPA is re-investigated and extended to implement in a pumping station with multiple pumps. The problem studied here for a pumping station with multiple pumps is different from the one in [19] in the following aspects.

(1) The domain of the control variable is larger (the domain is $\{0,1,\ldots,N_p\}$, where N_p is the number of pumps).
(2) The number of possible values for the water volume at the sth stage is larger. For one pump, the number is $s+1$ while for multiple pumps, it is $s\times N_p+1$.
(3) The number of possible routes from the $(s-1)$th stage to the sth stage is larger, too. For one pump, the number is at most two while for multiple pumps, it can be as large as N_p+1.
(4) The maintenance cost is considered in the operation cost.

Compared with the conventional DP algorithm, the nature of fast computation of the extended RDPA is owing to two aspects. One is that the number of possible values at stage s is reduced from $s\times N_p+1$ with RDPA to $k_s^u-k_s^l+1$, which is much less than $s\times N_p+1$ when s is large. The other is that the number of state transfer routes and the comparison of the cost function at stage $(s+1)$ are significantly reduced to less than $(sN_p+1)(N_p+1)$ from $(sN_p+1)((s-1)N_p+1)$ with a conventional DP algorithm.

Simulation shows the feasibility of extended RDPA in the reductions of the energy cost and the number of pump switches. Both the penalty on a pump switch and the time step of the scheduling in extended RDPA have influences on the number of pump switches.

The main contributions of this paper are: (1) The optimal operation scheduling of a pumping station with multiple pumps is formulated as a dynamic programming problem; (2) The maintenance cost can be explicitly considered in the cost function; (3) The RDPA in [19] is extended to solve the problem of a pumping station with multiple pumps.

The structure of this paper is: in Section 2, the optimal operation scheduling problem of a pumping station with multiple pumps is formulated under the ToU electricity tariff structure, followed by the extension of the RDPA in Section 3. Simulation of the extended RDPA for a pumping station with three pumps is given in Section 4. Some conclusions are give in Section 5.

2. Problem formulation

A typical reservoir with a pumping station is shown in Fig. 1. There are several pumps in the station. Generally, the pumps are identical, including the outflow capacity and the corresponding power. The pump's outflow capacity and the power are denoted as b and P_m, respectively.

The water volume in the reservoir is $\mathcal{v}(t)$, which satisfies the following equation,

$$\dot{\mathcal{v}}(t)=a-b\sum_{i=1}^{N_p}u_i(t), \tag{1}$$

where a is the inflow rate and $u_i(t)$ is the ith pump state in the station, which can be either one for the pump being on or zero for the pump being off. Then the sum $\sum_{i=1}^{N_p}u_i(t)$ is an integer denoting the number of pumps being 'on' in the station at time t.

The electricity price structure is as follows,

$$P_e(t)=C(p), t\in[T_e^p,T_e^{p+1}],\quad p=1,\ldots,P$$

where P is the number of the time intervals within the time period $[t_0,t_f]$ and in each of the intervals the electricity price is constant.

The ToU electricity rate can differ by time of day, day of week and season. A typical ToU electricity pricing structure is shown in Fig. 2, which shows the rate by time of a week with off-peak, mid-peak and on-peak intervals given in Table 1.

Generally, an optimal operation scheduling problem of the pumping station is: to find, a control sequence $\{U(k)\}$ ($U(k)=[u_1(k),u_2(k),\ldots,u_{N_p}(k)]^T$) and the corresponding switching time sequence $\{t(k)\}$ such that the energy cost within the time period $[t_0,t_f]$ is minimized with the water volume constraints $\mathcal{v}(t)\in[\mathcal{v}^l,\mathcal{v}^u]$ satisfied. Here $u_i(k)$ is either zero or one within the time interval $[t(k),t(k+1)]$.

Such a problem is reformulated in [20] as a control sequence optimization problem when the time sequence $\{t(k)\}$ is given. Especially, when $t(k)=t_0+k\times T_{\text{sampling}}$, $k=0,\ldots,K$ where T_{sampling} is the sampling time period and constant, the problem is intrinsically a BIP problem.

Within the framework of the dynamic programming problem studied here, a given time period $[t_0,t_f]$, for example, the period of the ToU pricing, could be scheduled into S subintervals according to the ToU electricity pricing structure and the sampling time period T_{sampling}. Within each time subinterval, the time length of the subinterval is T_{sampling} and the electricity price is constant.

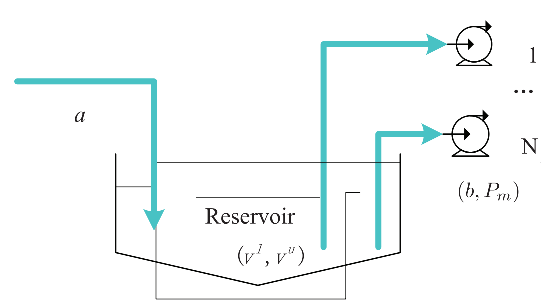

Fig. 1. A typical reservoir with a pumping station.

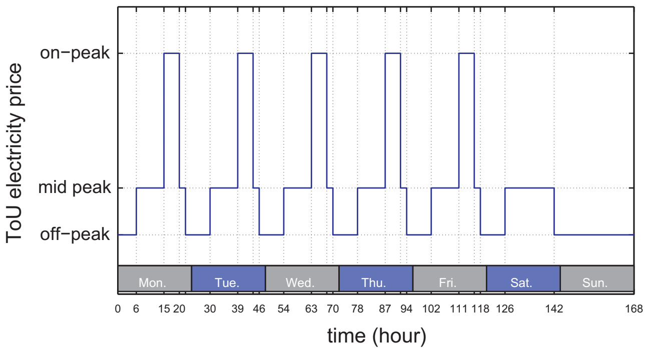

Fig. 2. General ToU electricity price structure.

Table 1
Time of use pricing by time a week.

Time period	Hours of a day	Days of a week
On-peak	3–8 p.m.	Monday–Friday
Mid-peak	6 a.m.–3 p.m.	Monday–Friday
	8–10 p.m.	Monday–Friday
	6 a.m.–10 p.m.	Saturday
Off-peak	10 p.m.–6 a.m.	Everyday
	6 a.m.–10 p.m.	Sunday

With the above time sequence, the optimization problem of the pumps in the pumping station is formulated as follows: to find a control sequence $\{U(k)\}$, such that the energy cost within the scheduling time period $[t_0, t_f]$ is minimized and meanwhile the water volume of the reservoir is bounded in the range $[\nu^l, \nu^u]$.

In the following, the notations are given as follows: $P_e(s)$ is the energy price in the sth subinterval, $u_i(s)$ is the ith pump's status in the sth subinterval, $U(s) = [u_1(s), u_2(s), \ldots, u_{N_{pump}(s)}]^T$ is a vector denoting the pumps' states within the sth subinterval, and $V(s+1, j)$ is the jth possible value of the water volume at the end of sth subinterval.

The mathematical model of the above optimization problem is,

$$\begin{aligned} \min \quad & J = P_m \times \sum_{s=1}^{S}\left(P_e(s) \times \sum_{i=1}^{N_p} u_i(s)\right) + f(\nu(S+1)) \\ \text{s.t.} \quad & \nu(s+1) = \nu(s) + \bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s), \quad s = 2, \ldots, S, \\ & \nu(s) \in [\nu^l, \nu^u], \; u_i(s) \in \{0, \; 1, \} \end{aligned} \tag{2}$$

where $V(1,1) = \nu(1), \bar{a} = a \times T_{\text{sampling}}, \bar{b} = b \times T_{\text{sampling}}$ and $f(\nu(S+1))$ is a penalty for the final water volume. In the following, it is assumed that

$$f(\nu(S+1)) = P_m \times \frac{\nu(S+1)}{b} \times P_e(S).$$

With the domain of the $u_i(s) \in \{0, 1\}$ considered, the above problem is intrinsically a BIP problem. Because the direct solving algorithm to a BIP problem may lead to the problem of *curse of dimensionality*, the reformulation of the above problem into a dynamic programming problem will facilitate solving the problem.

3. Extended reduced dynamic programming algorithm

The above problem can be depicted in Fig. 3.

It can be seen that if $J(s,j)$ denotes the minimum cost from the initial water volume $V(1,1)$ to jth possible value at the sampling point s, then $J(1,1) = 0$,

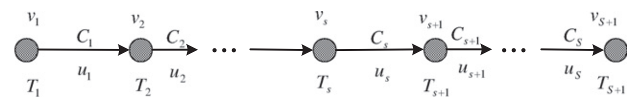

Fig. 3. A process scheme of the BIP problem.

$$J(s+1, j) = \min\{J(s, i) + \delta J(s+1, j, i)\}, \quad s = 1, \ldots, S-1,$$

and

$$J(S+1, j) = \min\{J(S, i) + \delta J(S+1, j, i) + f(V(S+1, j))\},$$

where $\delta J(s+1, j, i)$ represents the cost for $V(s+1, j)$ transferred from $V(s,i)$ with $u_i(s) \in \{0, 1\}$. The corresponding state transfer equation is

$$V(s+1, j) = V(s, i) + \bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s). \tag{3}$$

The cost $\delta J(s+1, j, i)$ will be determined by $u_i(s)$, $i = 1, 2, \ldots, N_p$. If there is no suitable route from $V(s,i)$ to $V(s+1, j)$, the cost will be infinity.

The above reformulated problem is actually in a DP problem structure and could be solved with the conventional DP algorithms.

In a DP algorithm, one task is to obtain the domain of possible values of $\nu(s+1)$, $s = 1, 2, \ldots, S$, i.e., the range of j in (3).

With recursively calculation with (3), it holds that

$$V(s+1, j) = V(1,1) + s\bar{a} - \bar{b}\sum_{m=1}^{s}\sum_{i=1}^{N_p} u_i(m). \tag{4}$$

Because $u_i(m) \in \{0, 1\}$, it is true that

$$\sum_{m=1}^{s}\sum_{i=1}^{N_p} u_i(m) \in \{0, 1, \ldots, sN_p\},$$

which implies that at stage $s + 1$, the number of possible values of state is $sN_p + 1$.

Intuitively, for a specific value $V(s+1, j)$, all the $(s-1)N_p + 1$ possible values of $\nu(s)$ can lead to $V(s+1, j)$, i.e., the number of the possible state transfer routes at stage $s + 1$ is $(sN_p + 1)((s-1)N_p + 1)$. The number of the comparison of the objective function is $(sN_p + 1)((s-1)N_p + 1)$.

With the conventional DP algorithms intuitively employed, the dimension of the problem will be very large and the computational tasks become very heavy when S and N_p are large [19].

The reduced dynamic programming algorithm in [19] could be extended to such an optimization problem with multiple pumps in the pumping station.

Proposition 1. *The domain of possible values of $v(s+1)$ can be written in the following form*

$$V(s+1, j) = V(1,1) + s\bar{a} - (j-1)\bar{b}, \tag{5}$$

where j is a natural number between k^l and k^u with k^l and k^u determined by

$$\begin{aligned} k^l &= \max\left\{1, \text{ceil}\left(\frac{V(1,1) - \nu^u}{\bar{b}} + \frac{s\bar{a}}{\bar{b}} + 1\right)\right\}, \\ k^u &= \min\left\{sN_p, \text{floor}\left(\frac{V(1,1) - \nu^l}{\bar{b}} + \frac{s\bar{a}}{\bar{b}}\right)\right\} + 1. \end{aligned} \tag{6}$$

Proof. The Eq. (4) can be rewritten in the following form,

$$\begin{aligned} V(s+1, j) &= V(1,1) + s\bar{a} - \bar{b}\sum_{m=1}^{s}\sum_{i=1}^{N_p} u_i(m) \\ &= V(1,1) + s\bar{a} - (j-1)\bar{b} \end{aligned} \tag{7}$$

where $j = \sum_{m=1}^{s}\sum_{i=1}^{N_p} u_i(m) + 1$.

Because $\sum_{m=1}^{s}\sum_{i=1}^{N_p} u_i(m) \in \{0, 1, 2, \ SN_p\}$, thus it is true that

$$j \in \{1, 2, \ldots, sN_p + 1\}. \tag{8}$$

Considering $V(s+1,j) \in [v^l, v^u]$, part of the above possible values for j are admitted, i.e.,

$$V(1,1) + s\bar{a} - (j-1)\bar{b} \in [v^l, \ v^u].$$

The following is true,

$$\begin{aligned} j &\leqslant \frac{V(1,1) - v^l + s\bar{a}}{\bar{b}} + 1, \\ j &\geqslant \frac{V(1,1) - v^u + s\bar{a}}{\bar{b}} + 1. \end{aligned} \tag{9}$$

With (8) and (9) combined, the domain of j is the set of the natural numbers between k^l and k^u determined by (6). □

Remark 1. With $j = k_{s+1}^l + i$, the Eq. (4) can be written as follows

$$V(s+1, i) = V(1,1) + s\bar{a} - (k_{s+1}^l + i - 1)\bar{b},$$

where i is an element in the set $\left\{1, 2, \ldots, k_{s+1}^u - k_{s+1}^l + 1\right\}$, and

$$\begin{aligned} k_{s+1}^l &= \max\left\{1, \mathrm{ceil}\left(\frac{V(1,1) - v^u + s\bar{a}}{\bar{b}}\right) + 1\right\}, \\ k_{s+1}^u &= \min\left\{sN_p, \mathrm{floor}\left(\frac{V(1,1) - v^l + s\bar{a}}{\bar{b}}\right)\right\} + 1. \end{aligned} \tag{10}$$

Remark 2. From the above remark, it can be seen that there are only $\left(k_{s+1}^u - k_{s+1}^l + 1\right)$ numbers in the practical domain of the water volume at the sampling point $s + 1$.

It can be seen that with (10), the following holds

$$\begin{aligned} k_{s+1}^u - k_{s+1}^l + 1 &= \min\left\{sN_p + 1, floor\left(\frac{V(1,1) - v^l + s\bar{a}}{\bar{b}}\right) + 1\right\} \\ &\quad - \max\left\{1, ceil\left(\frac{V(1,1) - v^u + s\bar{a}}{\bar{b}}\right) + 1\right\} + 1, \\ &\leqslant \min\{sN_p + 1, floor(\cdot) - ceil(\cdot) + 1\} \\ &\leqslant floor(\cdot) - ceil(\cdot) + 1 \ \leqslant \frac{v^u - v^l}{\bar{b}} + 1. \end{aligned}$$

Remark 3. As $T_{sampling}$ gets small, $\bar{b}$ correspondingly decreases and the upper bound $\frac{v^u - v^l}{\bar{b}} + 1$ increases. For a given sampling time period $T_{sampling}$ within the scheduling time period $[t_0, t_f]$, $\frac{v^u - v^l}{\bar{b}} + 1$ is constant, which means that there is an upper bound on the number of the possible values for $v(s+1)$, $\forall s \in (1, \ldots, S)$. As with the intuitive determination, the dimension of the domain of possible values of $v(s+1)$ is $sN_p + 1$, increasing with s and N_p. By contrast, Proposition 1 reduces the dimension of the DP problem. Even if s, N_p get very large, the number of possible values of $v(s+1)$ has an upper bound.

With the reduction of the number of possible values of $v(s)$, the number of the state transfer routes for a specific value of $v(s+1)$ is accordingly reduced. Furthermore, the number of routes can be reduced with the following proposition.

Proposition 2. *Assume the domain of possible values of v(s) is {V(s,k)} as follows,*

$$V(s,k) = V(1,1) + (s-1)\bar{a} - \left(k_s^l + k - 1\right)\bar{b}, \tag{11}$$

where $k = 1, 2, \ldots, k_s^u - k_s^l + 1$. Then for a specific value V(s + 1,j)

$$V(s+1,j) = V(1,1) + s\bar{a} - \left(k_{s+1}^l + j - 1\right)\bar{b}, \tag{12}$$

where $j = 1, 2, \ldots, k_{s+1}^u - k_{s+1}^l + 1$, there exists suitable operation $\sum_{i=1}^{N_p} u_i(s)$ such that

$$V(s+1,j) = V(s,k) + \bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s), \tag{13}$$

where k is a natural number between k_b^l and k_b^u determined by

$$\begin{aligned} k_b^l &= \max\left\{k_{s+1}^l - k_s^l + j - N_p, 1\right\}, \\ k_b^u &= \min\left\{k_{s+1}^l - k_s^l + j, k_s^u - k_s^l + 1\right\}. \end{aligned} \tag{14}$$

It can be seen that with (14), the following holds

$$\begin{aligned} k_b^l - k_b^u + 1 &= \min\left\{k_{s+1}^l - k_s^l + j, k_s^u - k_s^l + 1\right\} \\ &\quad - \max\left\{k_{s+1}^l - k_s^l + j - N_p, 1\right\} + 1, \\ &\leqslant \min\left\{N_p + 1, k_s^u - k_s^l + 1\right\} \\ &\leqslant N_p + 1. \end{aligned} \tag{15}$$

Proof. From (11)–(13), it holds that

$$k = k_{s+1}^l - k_s^l + j - \sum_{i=1}^{N_p} u_i(s). \tag{16}$$

Since $\sum_{i=1}^{N_p} u_i(s) \in \{0, 1, \ldots, N_p\}$,

$$k \geqslant k_{s+1}^l - k_s^l + j - N_p,$$

and

$$k \leqslant k_{s+1}^l - k_s^l + j.$$

With $k \in \left\{1, 2, \ldots, k_s^u - k_s^l + 1\right\}$ considered, k is a natural number between k_b^l and k_b^u determined by (14). □

From Proposition 2, the practical state transfer routes for a specific value $V(s+1,j)$ is limited to several possible values of $V(s,k)$. This domain of k is denoted as $D(s+1,j)$.

Remark 4. It can be seen the above domain is less than $N_p + 1$. Both the number of state transfer routes and the comparison of the cost function at stage $(s+1)$ are significantly reduced from $(sN_p + 1)((s-1)N_p + 1)$ to less than $\left(\frac{v^u - v^l}{\bar{b}} + 1\right)(N_p + 1)$.

Remark 5. With Propositions 1 and 2, the dimension of the DP problem is significantly reduced, which facilitates the fast computation of optimal scheduling process.

Proposition 3. *For the process shown in Fig. 3, if the control $\sum_{i=1}^{N_p} u_i(s)$ is maintained to be constant within the time interval $t \in [T_s, T_{s+1}]$, the water volume $v(t), \forall t \in [T_s, T_{s+1}]$ is constrained within the range $[v^l, v^u]$ when the initial and final water volumes within this time interval are constrained within the range $[v^l, v^u]$.*

Proof. From the reservoir dynamics (1), it holds that

$$v(t) = v(s) + \int_{T_s}^{t}\left(\bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s)\right)dt.$$

Because $\sum_{i=1}^{N_p} u_i(s)$ is constant within the time interval $[T_s, T_{s+1}]$, the water volume $v(t)$ is either monotonically increasing (when $\bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s) > 0$) or monotonically decreasing (when $\bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s) < 0$) or constant (when $\bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s) = 0$).

If $\bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s) = 0$, then $\upsilon(t)$ is independent of the time t, and equal to $\upsilon(s)$. If $\upsilon(s) \in [\upsilon^l, \upsilon^u]$, then $\upsilon(t) \in [\upsilon^l, \upsilon^u], \forall t \in [T_s, T_{s+1}]$.

If $\bar{a} - \bar{b}\sum_{i=1}^{N_p} u_i(s) \neq 0$, then $\upsilon(t) \leqslant \max\{\upsilon(s), \upsilon(s+1)\}$, implying that $\upsilon(t) \in [\upsilon^l, \upsilon^u]$, $\forall t \in [T_s, T_{s+1}]$, when $\upsilon(s)$, $\upsilon(s+1) \in [\upsilon^l, \upsilon^u]$.

In summary, the constraints on the water volume within the time interval are satisfied when the initial and final water volumes are constrained. □

With the above propositions, the DP algorithm is given as follows. At the sampling time $(s+1)$, the minimum cost function for $V(s+1, j)$ is

$$J(s+1, j) = \min_{i \in D(s+1, j)} \{J(s, i) + \delta J(s+1, j, i)\}, \\ = J(s, i^*) + \delta J(s+1, j, i^*) \quad (17)$$

where $\delta J(s+1, j, i)$ is the cost with the state transferred from $V(s, i)$ to $V(s+1, j)$.

With $D_u(s+1, j, i) = \left(k_{s+1}^l - k_s^l + j\right) - i$, the state transfer equation is rewritten as

$$V(s+1, j) = V(s, i) + \bar{a} - b\sum_{n=1}^{N_p} u_n(s), \\ = V(s, i) + \bar{a} - b \times D_u(s+1, j, i). \quad (18)$$

The optimal decision for $V(s+1, j)$ is denoted by $D_u^{opt}(s+1, j) = D_u(s+1, j, i^*)$.

The corresponding cost from $V(s, i)$ to $V(s+1, j)$ is

$$\delta J(s+1, j, i) = P_m \times P_e(s) \times D_u(s+1, j, i) \\ = P_m \times P_e(s) \times \left(k_{s+1}^l - k_s^l + j - i\right). \quad (19)$$

If the cost of pump switches (an indicator for pump maintenance) considered and assumed to be P_{switch} for a pump switch, the above equation is modified as follows, when $D_u(s+1, j, i) \leqslant D_u^{opt}(s, i)$

$$\delta J(s+1, j, i) = P_m \times P_e(s) D_u(s+1, j, i), \quad (20)$$

and when $D_u(s+1, j, i) > D_u^{opt}(s, i)$,

$$\delta J(s+1, j, i) = P_{\mathrm{switch}}(D_u(s+1, j, i) - D_u^{opt}(s, i)) + P_m \\ \times P_e(s) D_u(s+1, j, i). \quad (21)$$

With the above analysis, a reduced dynamic programming algorithm for the scheduling problem can be reached.

4. Simulation

A South African water purification plant [21] is employed as a baseline. The reservoir R1 of the plant could be modeled as a single tank with several identical pumps, of which the pumps G1, G2 are kept 'on' and the pump G3 is kept 'off' (the current configuration) and the operation of the pumps K1, K2 and K3 are required to be optimized.

A simplified model of the plant is shown in Fig. 4. The three identical pumps' (K1, K2, K3) power is 300 kW with the flow capacity 22 ML/day. The inflow of R1 is 45 ML/day and the total outflow of pumps G1 and G2 is 20ML/day. The water volume of R1 is required to be maintained within [0.2, 1.3] ML for the security consideration and requirements of pump operation.

The uncontrollable inflows and outflows of R1 are assumed to be continuously distributed in 24 h, and then the dynamical equation of R1 is

$$\dot{\upsilon}(t) = \frac{45 - 20}{24} - \frac{22}{24}\sum_{n=1}^{3} u_i(t) = \frac{25}{24} - \frac{11}{12}\sum_{n=1}^{3} u_i(t), \quad (22)$$

where the unit of the water volume $\upsilon(t)$ is ML and the unit of the time t is hour.

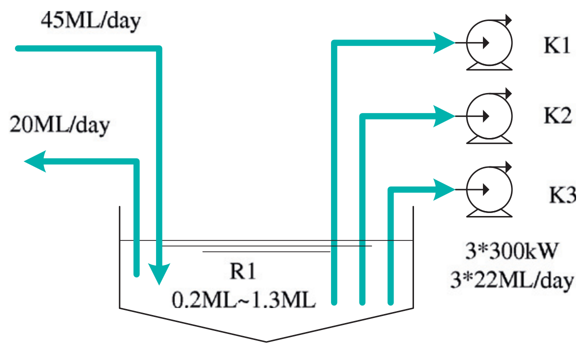

Fig. 4. A simplified model of the water purification plant.

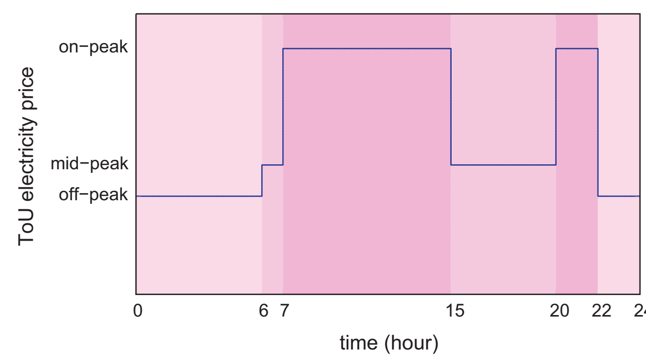

Fig. 5. Time partition of the ToU tariff in Tshwane.

The time partition of the ToU tariff structure of Tshwane city in South Africa is as shown in Fig. 5. The off-peak, mid-peak and on-peak price are 10.49, 13.83 and 26.28 cents/kW h in summer seasons while they are 11.87, 14.11 and 82.05 cents/kW h in winter seasons.

4.1. Scenarios in winter season with $T_{sampling}$ = 0.5 h

The winter season pricing is applied in this scenario and the sampling time period is 0.5 h.

When the cost of pump switches is not considered, an optimization result is shown in Fig. 6. In the figure, the scheduling time length is 48 h. The energy consumption and cost in this scenario are 16,200 kW h and R4909.71, respectively. The number of pump switches is 27.

In Fig. 6, the three pumps are all 'off' within some time intervals, and they are all 'on' within some other time intervals. There is at most one pump operated within the on-peak time intervals. The water volume within the whole time period is bounded in the range [0.2, 1.3].

When the cost of pump switches is considered and the cost per pump switch is assumed to be R1000, simulation is shown in Fig. 7. The energy consumption is the same as in Fig. 6. However the energy cost in this scenario is R5116.89, a little more than that in Fig. 6. This is because the operation of the pumps is more careful to reduce the number of the pump switch. The number of pump switches is eight, which is much less than in Fig. 6. The water volume within the whole time period is bounded in [0.2, 1.3], too.

With different settings for the cost per pump switch, simulation results are summarized in Table 2, where E_{con} and E_{cost} are energy consumption (unit: kW h) and energy cost (unit: Rand) within the time periods respectively. It can be seen that the energy consumption and the corresponding cost when the cost per pump switch is zero are the same as that when the cost per pump switch is R0.1 to

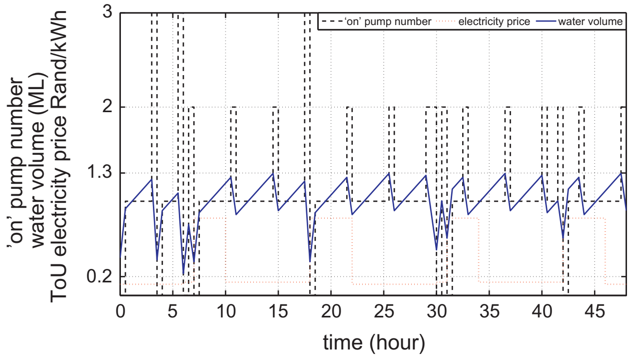

Fig. 6. 48-h Optimal scheduling with pump switch frequency ignored.

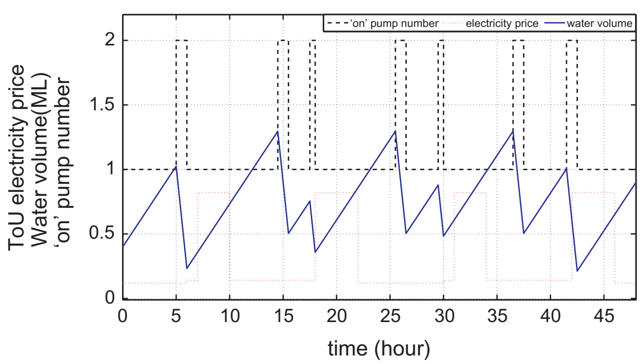

Fig. 7. 48-h Optimal scheduling with pump switch considered.

Table 2
Simulation in the winter season.

P_{switch}	E_{con}	E_{cost}	Switch number
0	16,200	4909.71	27
0.1	16,200	4909.71	10
1	16,200	4909.71	10
10	16,200	4909.71	10
100	16,200	5011.62	9
1000	16,200	5116.89	8
100,000	16,200	5116.89	8

R10. This is because the optimal solution is not unique when the pump switch cost is not considered. Among the solutions, the number of pump switches may be different.

A trend can be seen that with the switch number reduced, the energy cost is increasing although the energy consumption is the same. The increase of the energy cost is the result of increase of energy consumption within the on-peak/mid-peak time intervals. For example, in Fig. 7, two pumps are in operation within [44,44.5] h (on-peak time period) while only one pump is in operation within this period in Fig. 6.

From Table 2, it can be seen that the penalty on a pump switch can reduce the number of pump switches. However, the increase of the penalty weight has a marginal and even no impact when the number of pump switches reaches a certain value.

4.2. Scenarios in winter season with $T_{sampling}$ = 0.25/1 h

Simulation is also done with $T_{sampling}$ = 0.25 h and $T_{sampling}$ = 1 h, respectively.

When $T_{sampling}$ = 1 h and no matter what the cost of pump switches is, simulation is the same and shown in Fig. 8, where

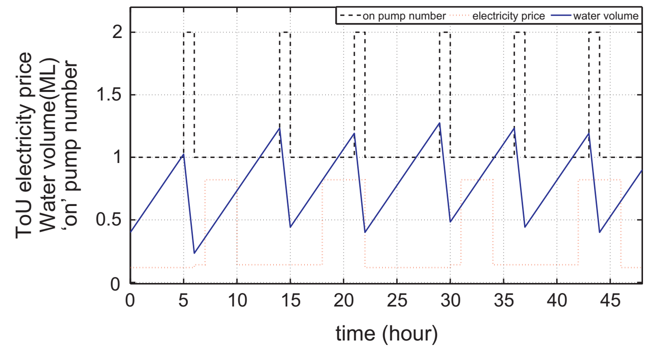

Fig. 8. 48-h Optimal scheduling with $T_{sampling}$ = 1 h.

the energy consumption is 16,200 kW h, energy cost is R5425.98 and the number of pump switches is seven. The energy consumption is the same as those in Table 2, while the energy cost is more than those in Table 2.

This result shows that the increase of the sampling time period could also reduce the number of pump switches and that the capability of the reduction of the number of pump switches is limited.

With the results obtained with $T_{sampling}$ = 0.5 h compared to those obtained with $T_{sampling}$ = 1 h, it can be seen that the increase of the sampling time period can possibly reduce the number of pump switch, but it also results in more energy cost even with the same energy consumption because less load (energy consumption) is shifted from the on-peak (or mid-peak) time intervals to mid-peak and off-peak (or off-peak) time intervals.

When $T_{sampling}$ = 0.25 h, results are shown in Figs. 9–11. In Fig. 9, the cost of pump switches is not considered, while it is considered in Figs. 10 and 11, where the cost per pump switch is assumed to be R10 and R1000 respectively. With different settings of the cost per pump switch, simulation results are shown in Table 3.

From those figures, it can be seen that the consideration of the cost of pump switches can reduce the number of pump switches on one hand, and increase the energy cost even with the same energy consumption on the other hand. The energy cost with the cost per pump switch R1000, is R5008.26, a little more than R4999.40 in the other settings with the same number of pump switch, i.e., with the cost per pump switch R100 and R100000. This is because more water volume is pumped, which can be seen from the comparison of Figs. 10 and 11, and thus more energy is consumed. It can be seen that the difference of the energy consumption is 75 kW h, and the additional cost is 5008.26–4999.4 = R8.86. Therefore the average price of the additional energy consumption is R0.1181, very similar to the off-peak electricity price. When the number of

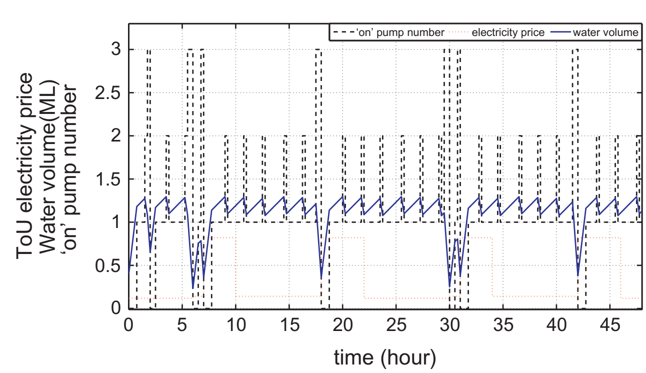

Fig. 9. Optimal scheduling with $T_{sampling}$ = 0.25 h and with cost of pump switches ignored.

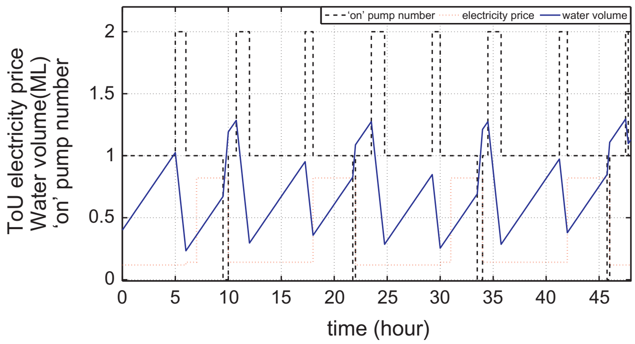

Fig. 10. Optimal scheduling with $T_{sampling} = 0.25$ h and with the cost per pump switch R10.

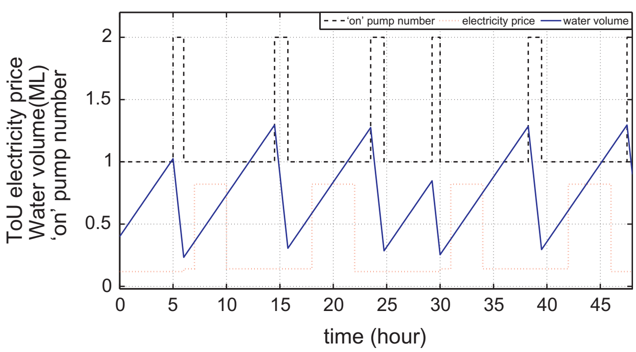

Fig. 11. Optimal scheduling with $T_{sampling} = 0.25$ h and with the cost per pump switch R1000.

Table 3
Simulation in the winter season ($T_{sampling} = 0.25$ h).

P_{switch}	E_{con}	E_{cost}	Switch number
0	16,125	4693.60	42
0.1	16,125	4693.60	13
1	16,125	4693.60	13
10	16,125	4693.60	13
100	16,125	4999.40	7
1000	16,200	5008.26	7
100,000	16,125	4999.40	7

pump switches is the same, the energy cost are the same for the same energy consumption.

With Table 3 compared to Table 2, it is observed that, the smaller the sampling time period is, the less the energy cost is, which implies the better load shifting from on-peak/mid-peak periods to off-peak periods. The penalty on a pump switch can reduce the number of pump switches.

These conclusions can be drawn from the comparison of the results with the summer season pricing, which are shown in Tables 4 and 5. The energy consumption with $T_{sampling} = 1$ h in the summer season is 16,120 kW h, and the corresponding cost is R2657.70. The number of pump switches is 39 when the cost of pump switches is not considered while it is seven when the cost of pump switches is considered (no matter what the cost per pump switch is).

4.3. Comparison of simulation time

A comparison of RDPA with several algorithms for a pumping system (one pump) was given in [19]. Here a further comparison of the extended RDPA proposed in this paper with the conventional DP algorithm is summarized in Table 6. In the conventional DP algorithm, Proposition 1 is applied, while Proposition 2 is not, which means that the admitted domain of the states at stage s is reduced for both algorithms. To further illustrate the time efficiency of the extended RDPA algorithm, a case with a large upper bound ($\vartheta^{\mu} = 6.5$ ML) for the reservoir's volume has been studied. Under such a circumstance, the possible values of the water volume at the sth stage state increases compared with that in the base case ($\vartheta^{\mu} = 1.3$ ML).

From Table 6, it is clear that the computational time of the proposed RDPA is much less than that of the conventional DP algorithm. As $T_{sampling}$ decreases and/or ϑ^{μ} increases, the number of state-transferring routes gets large and thus the computational time of both algorithms increases. Moreover, the larger the number of state-transferring routes is, the more obvious the advantage of the RDPA is. The RDPA is proved to be a good choice for operation scheduling of a pumping station with multiple pumps.

Table 4
Simulation in summer season ($T_{sampling} = 0.25$ h).

P_{switch}	E_{con}	E_{cost}	Switch number
0	16,200	2511.95	39
0.1	16,200	2511.95	13
1	16,200	2511.95	13
10	16,125	2560.1	7
100	16,125	2560.1	7
1000	16,125	2560.1	7
100,000	16,125	2560.1	7

Table 5
Simulation in summer season ($T_{sampling} = 0.5$ h).

P_{switch}	E_{con}	E_{cost}	Switch number
0	16,200	2554.30	25
0.1	16,200	2554.30	10
1	16,200	2554.30	10
10	16,200	2572.98	9
100	16,200	2596.70	8
1000	16,200	2596.70	8
100,000	16,200	2596.70	8

Table 6
A comparison of the time efficiency of the algorithms.

ϑ^{μ} (ML)	$T_{sampling}$ (h)	Conventional DP (s)	RDPA (s)
1.3	0.5	0.0081	0.0070
1.3	0.25	0.0540	0.0289
6.5	0.5	0.0673	0.0292
6.5	0.25	0.3767	0.1659

5. Conclusions

The optimal operation scheduling problem of a pumping station with multiple pumps is studied in the paper. Similar to [19], this problem can be reformulated as a DP problem. The RDPA in [19] is extended here to solve such a problem. In this paper, besides the energy cost, the maintenance cost is considered and expressed in term of the number of pump switches.

Compared with the conventional DP algorithms, the extended RDPA can significantly reduce the computational time because of the reduction of the number of possible state values at each stage and the reduction of the number of the possible state transfer routes from stage s to stage $(s+1)$.

Simulation in open-loop scheduling shows the feasibility of the extended RDPA in the reduction of the energy cost and the reduction of number of pump switches (an indicator for maintenance cost) with the main function achieved.

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