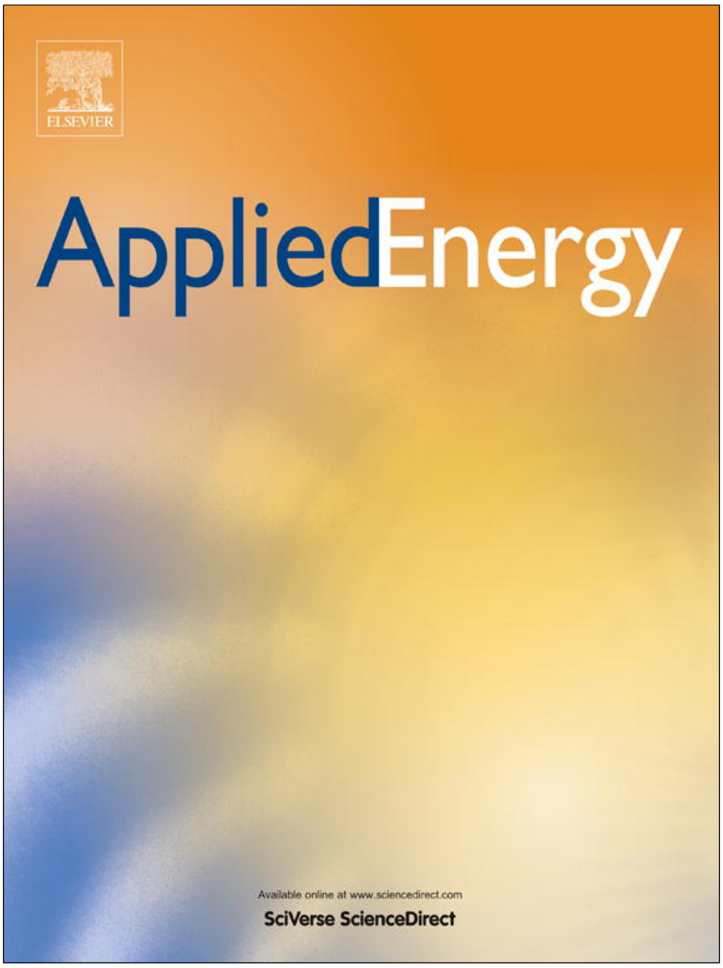
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Mathematical description for the measurement and verification of energy efficiency improvement ☆

Xiaohua Xia *, Jiangfeng Zhang

Centre of New Energy Systems, Department of Electrical, Electronic and Computer Engineering, University of Pretoria, South Africa

HIGHLIGHTS

- A mathematical model for the measurement and verification problem is established.
- Criteria to choose the four measurement and verification options are given.
- Optimal measurement and verification plan is defined.
- Calculus of variations and optimal control can be further applied.

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ABSTRACT

Insufficient energy supply is a problem faced by many countries, and energy efficiency improvement is identified as the quickest and most effective solution to this problem. Many energy efficiency projects are therefore initiated to reach various energy saving targets. These energy saving targets need to be measured and verified, and in many countries such a measurement and verification (M&V) activity is guided by the International Performance Measurement and Verification Protocol (IPMVP). However, M&V is widely regarded as an inaccurate science: an engineering practice relying heavily on professional judgement. This paper presents a mathematical description of the energy efficiency M&V problem and thus casts into a scientific framework the basic M&V concepts, propositions, techniques and methodologies. For this purpose, a general description of energy system modeling is provided to facilitate the discussion, strict mathematical definitions for baseline and baseline adjustment are given, and the M&V plan development is formulated as an M&V modeling problem. An optimal M&V plan is therefore obtained through solving a calculus of variation, or equivalently, an optimal control problem. This approach provides a fruitful source of research problems by which optimal M&V plans under various practical constraints can be determined. With the aid of linear control system models, this mathematical description also provides sufficient conditions for M&V practitioners to determine which one of the four M&V options in IPMVP should be used in a practical M&V project.

1. Introduction

With the current economic growth, energy supply cannot meet the increasing demand in many countries. To solve the energy supply problem and also to protect the environment, renewable energy sources are developed, and many energy efficiency projects are also implemented across the world. These energy projects are often started with specific energy saving targets, and the success of these projects need to be determined by checking whether the relevant energy saving targets have been reached. This kind of checking process is called measurement and verification (M&V), and is often carried out by project developers or an independent third party inspection body. The M&V inspection body will undertake a monitoring process and deliver the corresponding energy saving assessment. These energy saving M&V activities are usually guided by the International Performance Measurement and Verification Protocol (IPMVP) [1]. There are also some other energy saving M&V guidelines which are essentially similar to IPMVP, and these guidelines include, but are not limited to, the M&V Guideline for the Federal Energy Management Program [2]; the M&V Guideline of the American Society of Heating, Refrigeration and Air Conditioning Engineers (ASHRAE) [3]; the South African M&V guideline for Demand Side Management projects [4]; and the Australian best practice guideline [5].

Helpful M&V methodologies and examples are given in the above energy saving M&V guidelines. These M&V methodologies from different guidelines are essentially the same with what is

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* Corresponding author. Tel.: +27 12 4202165; fax: +27 12 3625000.
E-mail addresses: xxia@up.ac.za (X. Xia), zhang@up.ac.za (J. Zhang).

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proposed in the IPMVP, in which four M&V methods, Option A, Option B, Option C, and Option D, are given. The first two methods, Options A and B, are applicable to energy subsystems which can be isolated from the whole energy system, where the notion of energy system refers to a system consisting of all energy related facilities and factors under consideration. The later two methods, Options C and D, are applicable to the whole energy system level and do not consider subsystems independently. Option A is defined as partially measured isolated retrofit and only key system parameters are monitored. Option B is applied to the isolated retrofit with full measurement, and all the system parameters are monitored. Option C is designed for monitoring at the whole facility level, and interactions within the system are often ignored. Option D is a comprehensive calibrated simulation, whereby computer simulations for the system performance is performed to calculate energy savings. Although these M&V methods are discussed in these existing M&V guidelines, it is still difficult to find a proper M&V method or plan for a complex energy project so that the reported performance is accurate enough. It is therefore interesting to find out how these general M&V guidelines can be applied in various specific energy projects. Ref. [6] discusses the M&V method for a motor sequencing control of a conveyor belt system, [7] gives a general method for calculating plant-wide industrial energy savings, [8,9] propose a bottom-up approach to energy saving calculations; [10–14] study the uncertainties in M&V, [15] considers the Louisiana home energy rebate offer program, [16] proposes general guidelines for energy modeling in M&V, [17] provides the M&V strategies for energy savings certificates, [18] discusses the M&V for demand response, [19,20] describe the M&V experiences in the United States and South Korea, [21] gives an M&V system design for buildings, [22] provides a case study for a underground pumping system in a mine, and [23] discusses the general M&V process in South Africa.

As defined in [1], the concept Energy Conservation Measure (ECM) is "used to mean measures to improve efficiency or conserve energy or water, or manage demand". All the existing ECM M&V studies compare the energy/power consumption after an ECM with the baseline energy/power consumption to find the corresponding savings. The baseline consumption is assumed to be the corresponding energy/power consumption at the post-implementation period if the ECM was not implemented so that the baseline consumption and actual consumption during the post-implementation period will have the same exact ambient environment such as temperature, and production. However, the baseline consumption at the post-implementation period is never measurable. Therefore, it is either assumed to be the same as the baseline measured or calculated at the pre-implementation period, or adjusted to the post-implementation period based on the pre-implementation baseline consumption data. There is no theoretical analysis to explain how the post-implementation baseline consumption can be obtained from the pre-implementation consumption. In practical ECM M&V projects, the selection of the IPMVP M&V Options A, B, C, and D is usually determined by experience. An M&V plan is also obtained by the experience of M&V professionals, and as such an M&V plan may be far from optimal when there are particular requirements on accuracy and M&V cost. Therefore, scientific ways to select IPMVP M&V options and optimize M&V plans need to be addressed.

This paper aims to provide a mathematical description for ECM M&V problems so that scientific rules behind existing M&V practices are discovered, and M&V option selection and M&V plan development in M&V practices are also guided by scientific principles. In this way, M&V becomes a rigorous branch of science. To this end, general energy system modeling and ECM M&V modeling processes from existing M&V practices are summarized, the concepts of baseline and optimal M&V plan are further defined. The notions of exogenous functions and service level functions are introduced so that baseline at the post-implementation stage can be characterized as functions of exogenous and service level functions. The criteria to select the four M&V Options A, B, C, and D are discussed from a control system point of view. With the above mathematical description, the optimal M&V plan problem is formulated as a calculus of variation or optimal control problem. Since M&V cost and/or M&V uncertainty can be put as objectives or constraints in the M&V plan optimization model, M&V cost and M&V uncertainty can be minimized.

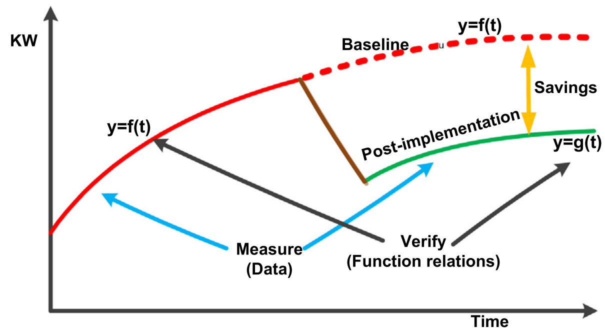

Fig. 1. What is M&V.

The paper is organized as follows. A mathematical description on energy modeling, M&V modeling, and the corresponding applications are given in Section 2. The mathematical formulation of optimal M&V plan is introduced in Section 3, and conclusions are made in Section 4.

2. A mathematical description of M&V for ECM projects

2.1. What is M&V

The general principle of M&V is illustrated in Fig. 1. The power consumption before the implementation of any ECM project is called the baseline power consumption. This baseline power consumption is marked in red[1] and expressed as the function $y = f(t)$ in Fig. 1, where y is the power consumed at time t. If the ECM was not implemented, the power consumption could still be represented by the function $y = f(t)$ (see the red dotted line). With the implementation of the ECM, power consumption level becomes lower at the post-implementation period, and this post-implementation power consumption can be characterized by another function, $y = g(t)$. The difference between $f(t)$ and $g(t)$ gives the savings from the ECM. However, the determination of the savings $f(t) - g(t)$ is not straightforward since $f(t)$ at the post-implementation stage does not physically exist and therefore cannot be measured. The determination of $f(t)$ at the post-implementation stage becomes the most tricky part in M&V, and it will be discussed in the following subsections.

2.2. Energy system modeling

There are plenty of study on various energy system modeling in literature (see, for example, [24–27]). This section introduces general energy modeling notations and terminologies to facilitate the discussions on M&V modeling. Consider the performance of an energy system over a given time period $[t_0, t_f]$. Let $z(t) = (z_1(t), \ldots, z_n(t))^T$ be an n-dimensional vector denoting all variables in the energy system, and $p(t) = (p_1(t), \ldots, p_m(t))^T$ an m-dimensional

[1] For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

vector representing parameters in this energy system. The variables and parameters in $(z(t),p(t))$ will be properly chosen to fully describe the energy system over the period $[t_0,t_f]$. For example, $(z(t),p(t))$ could consist of variables and parameters from power, energy, current, voltage, impedance, magnetic flux, frequency, time, etc., in an electrical system; or water flow rates, steam pressure and temperature in a thermal system. The difference between $z(t)$ and $p(t)$ is that $p(t)$ represents given information of the system over $[t_0,t_f]$, while $z(t)$ does not, and its values over $[t_0,t_f]$ needs to be identified through studying the underlying physical process. For simplicity, we often abuse the notation and call $(z(t),p(t))$ variables of the system. In this energy system, z and p are usually interrelated and constrained such as

$$\phi(z(t), p(t)) = 0, \tag{1}$$

where $\phi(z) = (\phi_1(z),\ldots,\phi_s(z))^T$ is an s-dimensional function representing constraints such as energy balance and physical limitations, and the component functions $\phi_1(z),\cdots,\phi_s(z)$ are elements of a set of functions under consideration. To be more specific, let $\mathbb{C}(z,p)$ denote the set of all the functions of the variables $(z,p) = (z_1,\ldots,z_n,p_1,\ldots,p_m)^T$ under consideration, where $\mathbb{C}$ indicates that constants in the functions are real or complex numbers. For example, $\mathbb{C}(z,p)$ can be defined in strict mathematical terminologies as the field of analytic functions or meromorphic functions over the complex field $\mathbb{C}$, or the fraction field of the polynomial ring generated by multivariable complex coefficient polynomials in variables (z,p). Note that the interrelations between $z(t)$ and $p(t)$ are taken as the equality constraint in (1), which will cover almost all the practical interrelations. This is because most of the practical interrelations between $z(t)$ and $p(t)$ can be written mathematically as equality or inequality relations, while inequality relations can always be written as equivalent equalities with the introduction of extra variables. For instance, the relation $z_1(t) - p_1(t) \geqslant 0$ is equivalent to $z_1(t) - p_1(t) - z_{n+1}^2(t) = 0$, where $z_{n+1}(t)$ is a new variable.

A performance indicator of the energy system (1) is a time function $y(t)$ evaluating a chosen energy efficiency performance of the system. If a given performance indicator of a system over a period of time can be expressed by a mathematical function incorporating the variables and parameters that fully describe the energy system in question, this function can be called the baseline - mathematically this can be expressed as below. The performance indicator $y(t)$ is said to have a baseline function $\alpha(z,p)$ over $[t_0,t_f]$ if the function $\alpha(z,p) \in \mathbb{C}(z,p)$ satisfies

$$y(t) = \alpha(z(t), p(t)), \quad t \in [t_0, t_f]. \tag{2}$$

Note that an ECM will usually change the values of the performance indicator $y(t)$ after project implementation, and thus $y(t)$ needs to be characterized by a new function different from $\alpha(z,p)$ after the ECM implementation. However, the baseline function relation $\alpha(z,p)$ is not affected by the ECM since it will provide a benchmark to check the impact of the ECM; in other words, the baseline function $\alpha(z(t),p(t))$ remains unchanged after the implementation of the ECM, and the comparison of this $\alpha(z(t),p(t))$ with the changed value of $y(t)$ at post-implementation period provides a measure of ECM impact (e.g., energy or power savings). Since the values of the baseline function $\alpha(z,p)$ at the post-implementation stage of the ECM are not physically measurable, therefore, the following hypothesis is made to assume the existence of baseline function over the whole project period.

Hypothesis 1. Given the energy system (1) on $[t_0,t_f]$, assume that the performance indicator $y(t)$ always has a baseline function over $[t_0,t_f]$.

Now assume that an ECM project for this energy system starts installation or retrofitting from the time instant t_1 and completed at t_2, $t_0 < t_1 < t_2 < t_f$, such that $[t_0,t_1]$ is the pre-implementation period, and $[t_2,t_f]$ is the post-implementation period of the ECM. Note that the period $[t_1,t_2]$ is the installation period of the ECM and system variables and performance are usually not stable, therefore this period is often not discussed in M&V. Generally an ECM would cause the following changes to the energy system: to change certain constant variable values to new constants (i.e. the change of system operating point), to change the values of $(z(t),p(t))$ on $[t_2,t_f]$, or change the functional relationships between $(z(t),p(t))$ on $[t_2,t_f]$ such that the performance indicator $y(t)$ on $[t_2,t_f]$ has to be calculated by a new function $\gamma(z,p)$. Under these cases, the values of $(z(t),p(t))$ will often change, and this is described mathematically by the following definition.

Definition 1. An ECM is mathematically defined as a map $\theta : \mathbb{C}(z_B,p_B) \to \mathbb{C}(z_A,p_A)$ such that $\theta(z_B) = z_A$, and $\theta(p_B) = p_A$, where z_B,p_B,z_A,p_A satisfy

$$\begin{aligned} z_A(t) &= z(t), \quad t \in [t_0,t_f], \\ p_A(t) &= p(t), \quad t \in [t_0,t_f], \\ z_B(t) &= z(t), \quad t \in [t_0,t_1], \\ p_B(t) &= p(t), \quad t \in [t_0,t_1], \end{aligned} \tag{3}$$

$(z_A(t),p_A(t))$ over $[t_0,t_f]$ represents the actual value of $(z(t),p(t))$ for which the energy system is impacted by the ECM, and $(z_B(t),p_B(t))$ over $[t_2,t_f]$ equals what the value of $(z(t),p(t))$ would have been if such an ECM was not implemented.

The equations in (3) imply that z_A and p_A are actual values of z and p, respectively, for the overall project period; while z_B and p_B represent only the values of z and p, respectively, over the pre-implementation period. With the change of values of $(z(t),p(t))$ caused by the ECM, the system constraint Eq. (1) holding on $[t_0,t_1]$ will not hold any more in most cases, this means that new mathematical equations need to be found to characterize the interrelations of $(z(t),p(t))$. In the following equation, it is assumed that these new interrelations of $(z(t),p(t))$ are characterized by the function ψ:

$$\psi(z(t), p(t)) = \psi(z_A(t), p_A(t)) = 0, \quad t \in [t_2, t_f]. \tag{4}$$

Assume that $z^m = (z_1^m, \ldots, z_{n'}^m)^T$, where $\{z_1^m, \ldots, z_{n'}^m\}$ is the set of all the variables from $z_A(t)$ such that $z_1^m(t), \ldots, z_{n'}^m(t)$ are measurable on $[t_0,t_f]$. A function $\gamma(z,p) \in \mathbb{C}(z,p)$ is called measurable on $[t_0,t_f]$ if $\gamma(z_A(t),p_A(t))$ can be represented as a function of $(z^m(t),p_A(t))$ for all $t \in [t_0,t_f]$. Since the parameter $p_A(t)$ is always known, the measurable function $\gamma(z,p)$ is also called representable by the measurable variable z^m. Note that the measurable variables defined here refer to those variables which are measurable over both the pre-implementation and post-implementation period, while in practice some variables may be measurable only at the pre-implementation (or post-implementation) period but not at the post-implementation (or pre-implementation) period. These variables can still be discussed in similar fashion, however, to simplify the discussion, these variables are ignored and the full measurability over $[t_0,t_f]$ is assumed for all measurable functions mentioned in this paper.

Definition 2. The performance indicator $y(t)$ is said to have a measurable baseline function for the ECM if $y(t)$ has a baseline function $\alpha(z,p)$ over $[t_0,t_1]$, and there exits a function $\beta \in \mathbb{C}(z,p)$ such that

$$\alpha(z_B(t), p_B(t)) = \beta(z^m(t), p_B(t)), \quad t \in [t_0, t_1] \tag{5}$$

and β is hereditary in the sense that for any $t \in [t_2,t_f]$, the value of $\beta(z_B^m(t),p_B(t))$ can be determined by the value of $\beta(z^m,p_B)$ on $[t_0,t_1]$, where $z_B^m(t)$ on $[t_2,t_f]$ is defined as what the value of the variable of $z^m(t)$ would have been if the ECM was not implemented, and $z_B^m(t) \equiv z^m(t)$ for all $t \in [t_0,t_1]$.

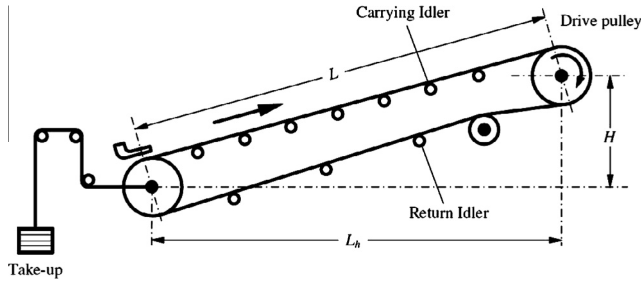

Fig. 2. An illustration of conveyor belt [29].

Eq. (5) implies that the baseline function over the pre-implementation period can be calculated from measurable variables. Note that $z_B^m(t)$ over $[t_2,t_f]$ is not measurable, however, the hereditary property allows the ability to infer its performance on $[t_2,t_f]$ from past values on $[t_0,t_1]$.

For any function $\gamma(z,p) \in \mathbb{C}(z,p)$, the change of values of $(z(t),p(t))$ on $[t_2,t_f]$ after the ECM will cause a possible change of values of $\gamma(z(t),p(t))$, and the following notion of exogenous function will characterize those functions whose values at the post-implementation period are not affected by the ECM.

Definition 3. A function $\gamma(z,p) \in \mathbb{C}(z,p)$ is called an exogenous function to the ECM if it is measurable, and

$$\gamma(z_B(t),p_B(t)) = \gamma(z_A(t),p_A(t)), t \in [t_2,t_f]. \tag{6}$$

Denote $\mathcal{S}_{Ex}$ the set of all the exogenous functions in $\mathbb{C}(z,p)$. Except for exogenous function, there is also another important class of functions in $\mathbb{C}(z,p)$ called service level functions. These are functions that are affected by the ECM, can be determined by measurable variables in z^m on $[t_0,t_f]$, and have acceptable physical meanings so that the baseline function of the performance indicator $y(t)$ can be expressed as a function of exogenous functions and service level functions. Service level functions are often used to adjust the baseline at the post-implementation period. Denote the set of all the service level functions by $\mathcal{S}_{SL}$. In order that a practical energy project can be measured and verified, the baseline function α must be a function of exogenous functions and service level functions, and the following definition follows.

Definition 4. The performance indicator $y(t)$ is said to have an M&V baseline function if it has a measurable baseline function β for the ECM, and there exist integers ℓ and k, measurable $\gamma_i \in \mathcal{S}_{Ex}$, $\delta_j \in \mathcal{S}_{SL}$, $1 \leqslant i \leqslant \ell$, $1 \leqslant j \leqslant k$, and a function F such that

$$\begin{aligned}&\beta(z_B^m(t),p_B(t)) = F(\gamma_1(z_B^m(t),p_B(t)),\cdots,\\&\gamma_\ell(z_B^m(t),p_B(t)),\delta_1(z_B^m(t),p_B(t)),\cdots,\delta_k(z_B^m(t),p_B(t)))\end{aligned} \tag{7}$$

holds for all $t \in [t_0,t_1] \cup [t_2,t_f]$.

Eq. (7) means that the baseline function β can be completely determined by some exogenous functions and service level functions. To characterize the change of the performance indicator after the ECM, the performance indicator in the old system is called baseline performance indicator and is denoted by $y_B(t)$, while the performance indicator in the new system after the ECM is called the actual performance indicator and is denoted by $y_A(t)$.

The following hypothesis is needed in M&V to calculate the impact from the ECM.

Hypothesis 2.

(i) The actual performance indicator $y_A(t)$ is measurable on $[t_2,t_f]$ in the sense that there exists a function $\bar{\beta}(z^m(t),p_A(t))$ on $[t_2,t_f]$, such that $y_A(t) = \bar{\beta}(z^m(t),p_A(t))$ for all $t \in [t_2,t_f]$.

(ii) The energy system has an M&V baseline function as defined in (7).

With the help of the above hypothesis, the impact of the ECM to the performance indicator is calculated below:

$$\begin{aligned}y_B(t) - y_A(t) &= \beta(z_B^m(t),p_B(t)) - \bar{\beta}(z^m(t),p_A(t))\\&= F(\gamma_1(z_B^m(t),p_B(t)),\cdots,\gamma_\ell(z_B^m(t),p_B(t)),\\&\quad \delta_1(z_B^m(t),p_B(t)),\cdots,\delta_k(z_B^m(t),p_B(t)))\\&\quad - \bar{\beta}(z^m(t),p_A(t)), \quad t \in [t_2,t_f].\end{aligned} \tag{8}$$

2.3. M&V modeling

It may happen that only a subset of the variables in z^m is physically measurable in a particular M&V project. This is possible due to the availability of measuring equipment, and the cost of such a measurement. For instance, the cost for the measurement of velocity is higher relative to displacement, etc. Note again that system performance indicator $y_B(t)$ or $y_A(t)$ might not be directly measurable in a particular M&V project, and the purpose of the M&V problem is to establish a model which approximates the performance indicator $y_B(t)$ on $[t_0,t_1] \cup [t_2,t_f]$ and $y_A(t)$ on $[t_2,t_f]$ by the selected measurable variables from z^m, respectively. For this purpose, it is often necessary to remodel the M&V problem.

Definition 5. For any given $\epsilon > 0$, let x be vectors consisting of selected variables from z^m. Then the M&V modeling problem for the energy system defined by (1)–(8) is to find functions $f(x,p)$, $g(x,p)$, $\xi(x,p)$, and $\omega(x,p)$ such that

$$\begin{cases}|f(x(t),p_B(t)) - F(\gamma_1(z_B^m(t),p_B(t)),\cdots,\\\quad\gamma_\ell(z_B^m(t),p_B(t)),\delta_1(z_B^m(t),p_B(t)),\cdots,\\\quad\delta_k(z_B^m(t),p_B(t)))| < \epsilon, t \in [t_0,t_1],\\|g(x(t),p_A(t)) - \bar{\beta}(z^m(t),p_A(t))| < \epsilon, t \in [t_2,t_f],\\\xi(x(t),p(t)) = 0, t \in [t_0,t_1],\\\omega(x(t),p(t)) = 0, t \in [t_2,t_f],\end{cases} \tag{9}$$

where ξ and ω are vector valued functions representing possible constraints that x satisfies before and after the ECM.

In (9), the first two inequalities imply that f and g can be used to approximate the baseline function β and the post-implementation performance function $\bar{\beta}$, respectively; and the last two equalities are the interrelations that variables $(x(t),p(t))$ will satisfy at different time periods. When the M&V model in (9) is built, F and $\bar{\beta}$ can be approximated by $f(x(t),p_B(t))$ and $g(x(t),p_A(t))$ respectively. For simplicity, the M&V model is rewritten as

$$\begin{cases}y_B(t) = f(x_B(t),p_B(t)), & t \in [t_0,t_1],\\y_A(t) = g(x_A(t),p_A(t)), & t \in [t_2,t_f],\\\xi(x_B(t),,p_B(t)) = 0, & t \in [t_0,t_1],\\\omega(x_A(t),p_A(t)) = 0, & t \in [t_2,t_f],\end{cases} \tag{10}$$

where $x_A(t)$ and $x_B(t)$ are defined as follows:

$x_A(t) = x_B(t) = x(t), t \in [t_0,t_1]$;
$x_A(t) = x(t), t \in [t_0,t_f]$; and
$x_B(t)$ equals what the value of $x(t)$ would have been if the ECM was not implemented for $t \in [t_2,t_f]$.

Now the baseline performance $y_B(t)$ on $[t_2,t_f]$ is approximated as

$$y_B(t) = f(x_B(t),p_B(t)), \quad t \in [t_2,t_f], \tag{11}$$

and the impact from the ECM is

$$y_B(t) - y_A(t) = f(x_B(t),p_B(t)) - g(x_A(t),p_A(t)), \ t \in [t_2,t_f]. \tag{12}$$

Table 1
Monthly energy consumption of a laboratory in 2010.

Month	January	February	March	April	May	June	July	August	September	October	November	December
T (°C)	27.1	26.5	24.5	23.1	21.5	19.9	17.4	20.1	23.5	24.9	25	26.8
E (kW h)	3526	3126	2834	3020	2947	2649	2806	2844	2904	2874	3414	3626

2.4. M&V modeling examples

Examples in this subsection are edited from practical M&V projects.

2.4.1. Physical model

The following example shows that an M&V model can sometimes be obtained directly from physical laws.

Example 1. Conveyor belt systems are widely applied in material handling. Fig. 2 is an illustration of a conveyor belt. The power consumption of the conveyor belt system can be calculated as follows [28]:

$$P_T = V(F_H + F_N + F_{st} + F_s), \tag{13}$$

where P_T is the conveyor belt power, V is the belt speed, and F_H, F_N, F_{st}, F_s are, respectively, the primary resistance, secondary resistance, slope resistance and special resistance. Formulae to derive these resistances can be developed from Newton's laws. Many parameters such as frictions are not variable. The above energy model is complex and a simplified M&V model can be built. Ref. [29] builds the following model using only the belt speed V and the mineral feeding rate T.

$$P(V,T) = \frac{V^2 T}{3.6} + \theta_1 T^2 V + \theta_2 V + \theta_3 \frac{T^2}{V} + \theta_4 T, \tag{14}$$

where θ_i are constants, i = 1, 2, 3, 4. Now consider an ECM in which the belt speed is changed from V_0 to $0.5V_0$ at peak time period 16:00–18:00 while the feeding rate T_0 remains constant. Let the conveyor belt work over the period 7:00–22:00 before the ECM, and over the period 7:00–24:00 after the ECM to maintain a fixed amount of production. The purpose of this ECM is to shift the peak time load only and does not aim to reduce the energy consumption. Therefore, the performance indicator is the power consumption. Let $z^m = x = (V,T)$, then $x(t) \equiv (V_0,T_0)$ holds before the ECM, while $x_B(t) \equiv (V_0,T_0)$ and $x_A(t) \equiv (0.5V_0,T_0)$ hold after the ECM. Therefore, $y_B(t) = f(x_B(t)) \equiv P(V_0,T_0), y_A(t) = g(x_A(t)) = \bar{\beta}(z^m(t)) \equiv P(0.5V_0,T_0)$. Then the power savings at peak hours are easily calculated as:

$$\begin{aligned} f(x_B(t)) - g(x_A(t)) &= P(V_0,T_0) - P(0.5V_0,T_0) \\ &= \frac{V_0^2 T_0}{4.8} + \frac{\theta_1 T_0^2 V_0}{2} + \frac{\theta_2 V_0}{2} - \theta_3 \frac{T_0^2}{V_0}. \end{aligned}$$

In this example, the feeding rate $T(t)$ is an exogenous function in $\mathbb{C}(V,T)$, $V(t)$ is neither an exogenous nor a service level function, and the total daily production $Q = \int_{24\text{ h period}} T(t)V(t)\mathrm{d}t$, as a function of $V(t)$ and $T(t)$, is the service level function.

2.4.2. Data model

An M&V data model can be built through methods like regression analysis if there are enough metered data for energy consumption and other major system variables.

Example 2. Table 1 gives the monthly energy consumption (E) of a laboratory.

Let T be the monthly average temperature, then the relation between E and T can be obtained from linear regression: $E = h(T) = 75T + 1297$. Motion sensors are installed in January 2011 as an ECM to control the lights and air conditioners, and this ECM is completed at the end of January 2011. The average ambient temperature and measured energy consumption are 26 °C and 2900 kW h, respectively, in February 2011; and 24 °C and 2767 kW h, respectively, in March 2011. The objective of M&V for this ECM is to determine the savings in February and March 2011. Note that the energy consumption at February and March in 2011 are already known, it is unnecessary to find the function $g(x)$. The savings for February and March 2011 are calculated as y_B(February) − y_A(February) = f(26) − 2900 = (75∗26 + 1297) − 2900 = 347, and y_B(March) − y_A(March) = f(24) − 2767 = (75∗24 + 1297) − 2767 = 330.

2.4.3. Stochastic model

Although the above deterministic models can be accurate enough for many applications, stochastic variables are sometimes not negligible in an energy system, and the M&V model needs to incorporate these stochastic variables. This can be illustrated by the following example, while more involved M&V models based on Gaussian processes can be found in [13].

Example 3. In a coal mine, an ECM project is taken on the production line to save electrical energy while still maintaining normal production. This production line includes the transportation of excavated coal by cars to conveyor belts, then the coal is sent to either a crusher or a spare stockpile; after crushing, the coal is further transported by conveyor belts for screening and recrushing; screened coal will be transported by conveyor belts to a production silo for storage, and coal from this production silo will be transported by conveyor belts to other places for sale. The ECM includes the installation of variable speed drives for the conveyor belts and the crusher, and optimal load shifting for energy cost savings purpose. Meters are installed to measure the total electrical power consumption of the production line. The coal production and the excavation of raw coal are also monitored. Coal production at the ith hour, denoted by C_i, and power consumption at the ith hour, denoted by P_i, are monitored before and after the ECM. To facilitate the discussion, assume that the pre-implementation period includes the hours from i = 1 to i = 1000, and the post-implementation period refers to the hours from i = 1501 to i = 2500. This implies that the variable speed drives and the optimal load shifting systems are installed during the hours from i = 1001 to i = 1500, and during this period the measured data will be ignored in the M&V calculations. A usual idea for the savings calculation is to find a linear relation between production and the power consumption to calculate the baseline, then substitute post-implementation production data to find what the power consumption would have been without the ECM, and the savings will be the difference of this calculated power consumption with the measured power consumption. However, the power consumption of the crusher depends also on the size of the coal excavated which is usually a stochastic process and impossible to measure. Note that the difference in coal sizes will cause the changes of power consumption in the crusher, and thus the change of the total power consumption. The objective of M&V is to calculate the power savings under the post-implementation coal size levels. For simplicity, the following model is assumed for the power consumption.

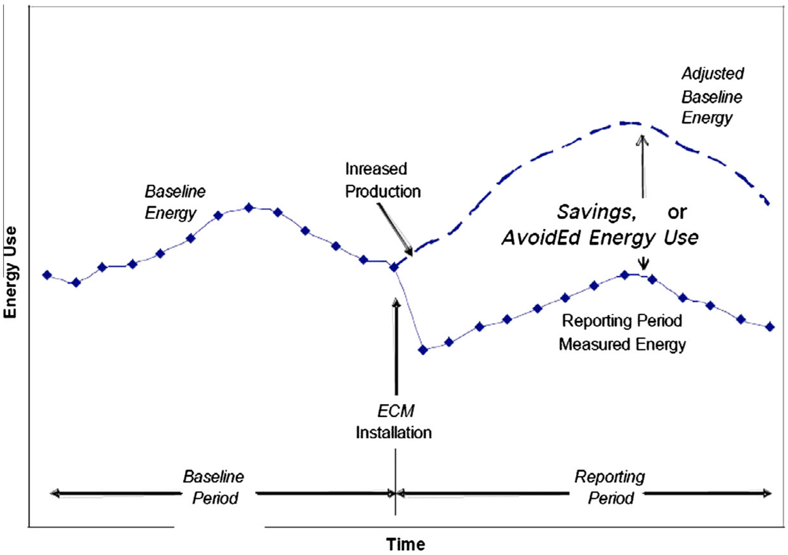

Fig. 3. Baseline and its adjustment [1].

Table 2
Energy related data in a production process.

Month	Energy (kW h)	Temperature (°C)	Production (ton)	Water (ton)
1	55,361	20.5	240	55
2	47,183	22	180	48
3	51,796	20.9	210	50
4	48,015	19	187	46
5	52,366	15	220	53
6	59,409	13	250	58
7	64,096	10.5	255	60
8	56,713	14.7	240	56
9	48,181	20.3	220	47
10	50,737	21.4	200	51
11	46,275	21	175	45
12	45,235	22.8	165	47

Table 3
Comparison of linear regression models.

	x_1	x_2	x_3	x_1 & x_2	x_2 & x_3	x_1 & x_3	x_1 & x_2 & x_3
R^2	0.753	0.848	0.938	0.901	0.951	0.954	0.962
SE_y	3019	2368	1510	2017	1415	1370	1318
C_m	10	100	200	110	300	210	310

$$P_i = \alpha C_i + \beta + \kappa_i, \; i = 1, 2, \ldots, 1000, \\ P_j = \gamma C_j + \delta + \eta_j, \; j = 1501, 1502, \ldots, 2500, \tag{15}$$

where α, β, γ, δ are constants, $\{\kappa_i\}$ and $\{\eta_i\}$ are zero mean stochastic processes, and $\beta + \kappa_i$ and $\delta + \eta_j$ are the impacts from the random coal size to the power consumption before and after the ECM, respectively. By the least squares estimates in [30], the constants α, β, γ and δ can be found. An example is that $\alpha = 180$, $\beta = 65$, $\gamma = 125$, $\delta = 44$, and the saving S_j at time j is calculated by

$$S_j = 180C_j + 65 + \kappa_j - (125C_j + 44 + \eta_j) \approx 55C_j + 21.$$

Note that the above approximation is reasonable since the means of $\{\kappa_i\}$ and $\{\eta_i\}$ are zero.

2.5. Baseline adjustment

Eq. (12) gives the calculation of the savings or impact from the ECM. This calculation indeed coincides with the definition of baseline routine adjustment in [1] as explained below.

Fig. 3 from [1] illustrates that at the post-implementation stage, the increase in the production causes an increase in baseline consumption, and the energy saving is the difference between the adjusted baseline and the reporting period energy consumption after the ECM. [1] further defines the following formula for savings calculation:

$$\text{Savings} = (\text{Baseline} - \text{Period Use or Demand} - \text{Reporting} \\ - \text{Period Use or Demand}) \pm \text{Adjustments}. \tag{16}$$

Baseline adjustments can be classified as *Routine Adjustments* and *Non-Routine Adjustments* [1]. In a routine adjustment, the baselines will be adjusted according to energy-governing factors, such as production volume or weather, which will change routinely during the reporting period. A Non-Routine Adjustment means that the baselines are adjusted according to energy-governing factors, such as facility size, design and operation of equipment, which are not usually expected to change [1]. The energy-governing factors involved in baseline adjustments are called independent variables [1], and the baselines are modeled in terms of the independent variables. Recall that (12) defines the savings as the difference between the baseline performance y_B and the actual performance y_A, where y_B is calculated by $f(x_B(t), p_B(t))$ which equals what the performance would have been over the period $[t_2, t_f]$ if the ECM was not implemented. This is to say, any routine change of the variable x and parameter p on $[t_2, t_f]$ is already included in $f(x_B(t), p_B(t))$, therefore the routine baseline adjustment is automatically included by the savings calculation $f(x_B(t), p_B(t)) - g(x_A(t), p_A(t))$.

Non-routine adjustments are applicable to the cases when (x, p) cannot represent all energy-governing factors, or one of the functions, $f(x, p)$ and $g(x, p)$, cannot adequately and accurately describe the system performance y. This is to say, if the variables in (x, p) and/or the functional relationships of $f(x, p)$ or $g(x, p)$ are changed, then a non-routine adjustment is needed. However, if (x, p) is redefined to include all variables which cause the non-routine changes, then the system can be re-modeled and the newly defined baseline function $f(x, p)$ is still able to characterize the baseline.

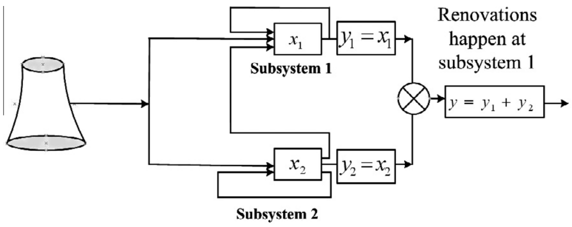

Fig. 4. Selection of Option A or B.

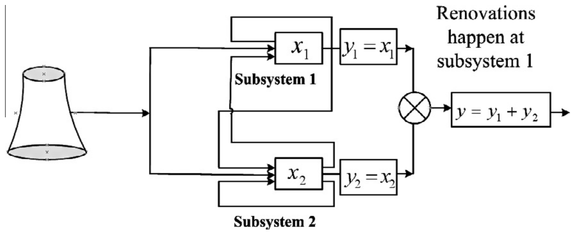

Fig. 5. Selection of Option C or D.

Table 4
Measured data.

x_1	x_2	y_1	y_2
35	28	3197	2264
34	27	3134	2225
31	27	3053	2108
37	29	3287	2342
37	25	3143	2342
36	26	3152	2303
38	24	3134	2381
36	21	2972	2303
32	27	3080	2147
32	28	3116	2147
38	26	3206	2381
36	22	3008	2303

The following example shows that there might be several M&V plans available for an M&V project. Following the requests of constrained budget and M&V precision, a proper M&V plan can be chosen by using the functional relations of $f(x,p)$ and $g(x,p)$ in Definition 5. Routine and non-routine baseline adjustments can also be characterized by this mathematical description.

Example 4. Table 2 gives the 12-month energy consumption baseline data for a production process and the corresponding temperature, production and the amount of water needed for cooling purpose. These data will be needed to build a baseline in order to measure and verify the saving from a future ECM. In order to write the monthly energy consumption (y) as a function of temperature (x_1), production (x_2), and/or the amount of water needed for cooling (x_3), linear regression is applied. The coefficient of determination (R^2) and the standard error of the energy consumption (SE_y) are used to evaluate the regression results. There are several choices for building the linear regression model because the energy consumption data y can be used together with all or any combination of x_1,x_2, and x_3. These different choices are compared in Table 3, where C_m is the cost in US dollars ($) to obtain the data for the corresponding choice. If the budget for the M&V modeling is only $50, then temperature can be used together with energy data for the linear regression. If the budget is increased to $150, with an extra requirement that R^2 must be 0.9 at least, then temperature and production can be measured and the relation of energy consumption with respect to temperature and production can be found. If there is no constraint on budget, but the precision must be as high as possible, then the measurement on all the three variables will be performed. Assume that the M&V inspector eventually measures all the x_1,x_2,x_3 to obtain the model $y = -271.36x_1 + 39.66x_2 + 720.09x_3 + 11747.02$. Assume further that the production line is replaced by an energy efficient one which still uses certain amount of water for cooling. A routine adjusted baseline can be obtained by substituting the measured post-implementation data of x_1,x_2,x_3 into $y = -271.36x_1 + 39.66x_2 + 720.09x_3 + 11747.02$. If, however, the ECM is to replace the production line by an advanced one which does not need any water for cooling, then the non-routine baseline adjustment method is needed, and x_3 is removed from the baseline model, the corresponding model obtained by linear regression is $y = -536.01x_1 + 117.99x_2 + 36996.14$.

2.6. M&V options

The mathematical description of M&V can also be applied to determine whether the IPMVP Option A or B for retrofit isolation needs to be chosen. Assume that a plant consists of two interconnected energy subsystems, subsystem 1 and subsystem 2, and an ECM has been implemented on subsystem 1. For simplicity, assume further the M&V models for the two subsystems are linear and constraints on the variables are ignored:

$$\begin{cases} f_1(x_1,x_2) = a_{11}x_1 + a_{12}x_2 + a_{13}, & t \in [t_0,t_1] \cup [t_2,t_f], \\ g_1(x_1,x_2) = b_{11}x_1 + b_{12}x_2 + b_{13}, & t \in [t_2,t_f], \\ f_2(x_1,x_2) = a_{21}x_1 + a_{22}x_2 + a_{23}, & t \in [t_0,t_1] \cup [t_2,t_f], \\ g_2(x_1,x_2) = b_{21}x_1 + b_{22}x_2 + b_{23}, & t \in [t_2,t_f], \end{cases} \tag{17}$$

where x_1 is the measurable variable of the subsystem 1, f_1 and g_1 on $[t_2,t_f]$ are the baseline and post-implementation performance indicators for subsystem 1 respectively, and similarly x_2,f_2 and g_2 are the corresponding notations for subsystem 2. Then the energy model in (17) can be used to select the IPMVP Options A and B, or C and D as shown in the following proposition.

Proposition 1. *Consider the two energy subsystems in (17), assume $a_{21} = 0$, $b_{21} = 0$, then Option A or B is applicable to subsystem 1, that is to say, the M&V for the whole plant can be done at subsystem 1 and the savings from the whole system equal the saving from subsystem 1.*

When $a_{21} = 0$ and $b_{21} = 0$, functions f_2 and g_2 in Proposition 1 do not contain the variable x_1, hence, subsystem 2 is not affected by the ECM at subsystem 1, and subsystem 1 can be isolated from subsystem 2 which corresponds to Option A or B. This proposition can be illustrated by Fig. 4. Note that if f_2 and g_2 depend on the variable x_1, then subsystems 1 and 2 cannot be isolated, and Option C or D has to be applied, see Fig. 5 for illustration.

The following is an example for Proposition 1.

Example 5. Consider an energy system which consists of two subsystems. The two subsystems at baseline stage are modeled as

$$y_1 = a_{11}x_1 + a_{12}x_2 + b_1u, y_2 = a_{21}x_1 + a_{22}x_2 + b_2u.$$

An ECM is implemented at subsystem 1, and its savings must be found through M&V. At the baseline stage, 12 metered data for (x_1,x_2,y_1,y_2) are available, see Table 4. At the post-implementation stage, there is only one measurement taken. In this case, the savings equal the difference between the baseline and the measured post-implementation system performance $y_1 + y_2$.

It follows from a linear regression that $y_1 = 36x_1 + 27x_2 + 1244$, $y_2 = 3.50734 \times 10^{-15}x_1 + 39x_2 + 899$. The coefficient 3.50734×10^{-15} is approximately 0, therefore, it can be assumed that y_2 is not affected by x_1. Then measurement can be taken only at subsystem 1, which corresponds to the selection of Option A or B so that subsystem 1 is isolated from the whole system.

The following proposition follows directly from the definitions of Options A, B, C, and D.

Proposition 2. *Consider the energy system (1)–(8) and the corresponding M&V model (10).*

(i) *Assume that retrofit isolation, namely, Option A or B, will be the M&V methodology. If x is a proper subset of z^m, i.e., $x \neq z^m$, then Option A is selected; otherwise $x = z^m$ and Option B is selected.*
(ii) *Assume that Option C or D will be the M&V methodology. If the approximated measurable baseline in (11) exists, then Option C can be selected, otherwise Option D is selected.*

3. Main results

With the above discussions on the mathematical descriptions of M&V, the M&V cost and M&V plan can be optimized through the solution of an optimization problem.

M&V cost is usually recovered from the corresponding project savings, and it must be minimized so that project investors can have more energy cost savings. As a guideline, [1] requests that the average annual M&V cost should be less than 10% of the average annual saving being assessed. Ref. [1] shows that M&V cost depends on many factors. These include, but are not limited to, the amount and complexity of the measurement equipment; sampling sizes; M&V options; quantity, complexity and interactions of energy efficiency measures; number and complexity of independent variables; accuracy requirement; and the experience and qualifications of the M&V inspectors. Assume that x in model (10) has included all the variables affecting M&V cost. The measurement cost is usually easy to quantify, while the labor cost is difficult to quantify since it depends on the time spent by the M&V inspectors and some other unmeasurable factors such as knowledge, skills and experience. For simplicity, only measurement cost is considered in this paper.

The purpose of an M&V plan is to select a number of variables from x for measurement, and to use the metered data to approximate the performance indicator y and calculate the impact from the ECM. Note that if two variables x_1 and x_2 need to be measured in an M&V project, then the cost to measure x_1 and x_2 simultaneously, denoted by $C_{\{1,2\}}$, might be different from the total cost to measure x_1 and x_2 separately. This is to say, $C_{\{1,2\}}$ may not equal $C_1 + C_2$, where C_1 and C_2 are the costs to measure x_1 and x_2 respectively. In many examples, $C_{\{1,2\}}$ is less than or equal to $C_1 + C_2$. For an M&V plan, assume that the vector $x_\alpha := (x_{i_1}, x_{i_2}, \cdots, x_{i_k})^T$ with component variables from x is measured, then the corresponding cost for measurement over the period $[t_0,t]$ is denoted by $C_\alpha(t) := C_\alpha(x_\alpha, t)$, where $\alpha = (i_1, i_2, \ldots, i_k)$ is a multi-index with $1 \leqslant i_1 < i_2 < \ldots < i_k \leqslant n''$, and n'' is the dimension of x.

In Example 1, the function $x = (V,T)$, and the cost functions $C_1(t)$ and $C_2(t)$ are, respectively, the costs to measure and process the values of V and T during the time period $[t_0,t]$. The cost function $C_{1,2}(t)$ will be the cost for the measurement of both V and T.

Definition 6. Consider the M&V problem in (10), a given upper bound U for available M&V budget, and any $\epsilon_1 > 0$, this M&V problem is said to have a feasible M&V plan with precision ϵ_1 and an allowable M&V cost if there exist a multi-index $\alpha = (i_1, i_2, \ldots, i_k)$, $1 \leqslant i_1 < i_2 < \ldots < i_k \leqslant n''$, and functions

$$\begin{cases} G(x_\alpha, p), & t \in [t_0, t_1] \cup [t_2, t_f], \\ H(x_\alpha, p), & t \in [t_2, t_f], \\ \Xi(x_\alpha, p), & t \in [t_0, t_1], \\ \Omega(x_\alpha, p), & t \in [t_2, t_f], \end{cases} \tag{18}$$

such that

$$\begin{aligned} &|f(x_B(t), p_B(t)) - G(x_\alpha(t), p_B(t))| < \epsilon_1, \quad t \in [t_0, t_1], \\ &|g(x_A(t), p_A(t)) - H(x_\alpha(t), p_A(t))| < \epsilon_1, \quad t \in [t_2, t_f], \\ &\Xi(x_\alpha(t), p_B(t)) = 0, \quad t \in [t_0, t_1], \\ &\Omega(x_\alpha(t), p_A(t)) = 0, \quad t \in [t_2, t_f], \\ &C_\alpha(t_f) := C_\alpha(x_\alpha, t_f) < U, \end{aligned} \tag{19}$$

where $x_\alpha = (x_{i_1}, x_{i_2}, \ldots, x_{i_k})$, and Ξ and Ω define the constraints that x_α satisfies. Denote this M&V plan by $\mathcal{M}(G, H, \Xi, \Omega, \epsilon_1, \alpha)$.

This definition implies that an M&V plan needs to take advantage of the measurable variable x to construct the functions G, H, Ξ, and Ω in (18) and then approximate the system to a precision of ϵ_1 within the available budget. The first and second inequalities in (19) imply that functions G and H need to be found by the M&V plan to approximate the baseline function f and post-implementation function g, respective. The third and fourth equalities in (19) mean that the measured variables x_α and system parameter $p(t)$ will satisfy necessary constraints defined by Ξ and Ω. The last inequality in (19) ensures that the M&V cost is within the allowable budget.

Definition 7. Given $\epsilon_1 > 0$, let $\mathcal{S}(\epsilon_1, U)$ be the set of all the M&V plan in the form of (18) for the M&V problem (10). An M&V plan $\mathcal{M}^*(G^*, H^*, \Xi^* R, \Omega^*, \epsilon_1^*, \alpha^*)$ is called optimal in the sense of performance precision and measurement cost if it is the optimal solution within $\mathcal{S}(\epsilon_1, U)$ for the following multi-objective minimization problem:

$$\begin{cases} \min & \int_{t \in [t_0, t_1]} |f(x_B(t), p_B(t)) - G(x_\alpha(t), p_B(t))| \mathrm{d}t, \\ \min & \int_{t_2}^{t_f} |g(x_A(t), p_A(t)) - H(x_\alpha(t), p_A(t))| \mathrm{d}t, \\ \min & C_\alpha(x_\alpha, t_f). \end{cases} \tag{20}$$

The first minimization objective in (20) is to minimize the approximation errors of G to the baseline function f over the pre-implementation period $[t_0,t_1]$, the second objective is to minimize the approximation errors of H to the actual performance function g over the post-implementation period $[t_2,t_f]$, and the last objective is to minimize the M&V cost. Note that the variables in the optimization problem (20) are not any usual real variables, but the index α and the functions $G(x_\alpha)$, $H(x_\alpha)$, $\Xi(x_\alpha)$, and $\Omega(x_\alpha)$. Therefore, (20) is a special problem in calculus of variation or optimal control.

After an optimal or near optimal M&V plan is determined, then the baseline and energy savings can be reported, and the saving is $G(x_\alpha(t),p_B(t)) - H(x_\alpha(t),p_A(t))$. The following is an example for the optimal M&V plan modeling.

Example 6. Consider an ECM in an office building which installs intelligent switches to switch off air conditioners and lights when people leave the office rooms for more than 15 min. Lights will be switched on immediately when people come back, and the air conditioners are switched on only if people are back and also the room temperature or humidity is deviated from the set point. The building has 200 office rooms, and all the rooms will install these intelligent switches. In the M&V project, meters will be installed at selected rooms to monitor power consumption. Ambient temperature and humidity data will be obtained from weather services. Through the energy audit, it is found that each room has an air conditioner, and all air conditioners have the same make and technical specifications. The power consumption model for a single air conditioner is:

$$P = f(T, H, u),$$

where P is the power, T is the temperature, H is the humidity, u is the on/off status of the air conditioner, and f is a function. For simplicity, one can assume a simplified linear model as

$$P = au + bT + cH + d,$$

where a, b, c, d are constants. With the measurement on the pre-implementation and post-implementation power consumptions, the savings of a single room can be determined by finding the coefficients a, b, c, d and calculating the corresponding baseline at the post-implementation stage. However, the impact of the 200 intelligent switches cannot be simply estimated by metering only one room because that different room occupants may have different job duties and thus different energy usage profiles. Due to the high cost of metering, it is impossible to install a meter for each of the 200 rooms. Market investigation shows that each meter costs $200. It is obvious that the M&V accuracy depends on meter accuracy, the number of meters installed and also whether the metered rooms are representative enough for the remaining un-metered rooms. For simplicity, the meter inaccuracy is ignored and only the sampling inaccuracy is discussed. Assume that the 200 rooms are classified into Group 1, Group 2, Group 3, and Group 4 in terms of the job duties of the room occupants such as administrative, technical, financial, and managerial. Assume further that there are s_i rooms in Group i with $s_1 = 90$, $s_2 = 80$, $s_3 = 20$, $s_4 = 10$. The M&V saving calculations will be 100% accurate if each room installs a meter. The saving calculations will be at least 0.5% accurate if only 1 m is installed. Now assume that l_i meters are installed in Group i, then the M&V accuracy is $\frac{l_i}{s_i}100\%$ for Group i, and is $\frac{l_1 l_2 l_3 l_4}{s_1 s_2 s_3 s_4}100\%$ for the overall 200 rooms. Since the total metering cost is $200\sum_{i=1}^{4} l_i$, the following multi-objective optimization model for simple M&V plans is obtained.

$$\begin{aligned}
&\min\ 200\sum_{i=1}^{4} l_i \\
&\max\ \frac{l_1 l_2 l_3 l_4}{s_1 s_2 s_3 s_4}\% \\
&\text{subject to } 200\sum_{i=1}^{4} l_i \leqslant B, \\
&\frac{l_1 l_2 l_3 l_4}{s_1 s_2 s_3 s_4} \geqslant 70\%,
\end{aligned}$$

where B is the maximum budget for the M&V project, and 70% is the required sampling accuracy. The optimal solution of this problem will give the number of meters installed in each group of rooms, and thus an optimal M&V plan is obtained.

The above example illustrates an optimization model to determine an optimal M&V plan. This idea has been successfully applied in a large scale lighting retrofit project to find the optimal metering plan [31].

4. Conclusions

By providing a mathematical description of the measurement and verification (M&V) process for energy efficiency improvement, this paper endeavors to alter the perception that M&V is a pure engineering practice, and establish it as a rigorous science. With this mathematical description, the M&V model and M&V plan are defined by mathematical functions. The baseline function is further characterized as a function of the exogenous functions and service level functions. Criteria to select M&V options are given under a control system framework. Optimal M&V plan is also defined as a solution of an optimization problem in calculus of variations and optimal control. Examples show how the M&V models can be derived, and how an optimal M&V plan can be obtained. For future work, uncertainties other than sampling inaccuracy will be included in the optimal M&V plan model, and practical applications will also be studied.

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