Optimization of mine ventilation fan speeds according to ventilation on demand and time of use tariff

Arnab Chatterjee *, Lijun Zhang, Xiaohua Xia

Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa

HIGHLIGHTS

- DSM techniques are applied to an underground mine ventilation network.
- A minimization model is solved to find the optimal speeds of the main mine fan.
- Ventilation on demand (VOD) leads to a saving of USD 213160.
- The optimal mining schedule, together with VOD, leads to a saving of USD 277035.
- According to a case study, a maximum of 2540035 kW h can be saved per year.

ARTICLE INFO

Article history:
Received 4 August 2014
Received in revised form 10 January 2015
Accepted 30 January 2015

Keywords:
Energy efficiency
Load management
Mine ventilation
Optimization
Time of use tariff
Ventilation on demand (VOD)

ABSTRACT

In the current situation of the energy crisis, the mining industry has been identified as a promising area for application of demand side management (DSM) techniques. This paper investigates the potential for energy-cost savings and actual energy savings, by implementation of variable speed drives to ventilation fans in underground mines. In particular, ventilation on demand is considered in the study, i.e., air volume is adjusted according to the demand at varying times. Two DSM strategies, energy efficiency (EE) and load management (LM), are formulated and analysed. By modelling the network with the aid of Kirchhoff’s laws and Tellegen’s theorem, a nonlinear constrained minimization model is developed, with the objective of achieving EE. The model is also made to adhere to the fan laws, such that the fan power at its operating points is found to achieve realistic results. LM is achieved by finding the optimal starting time of the mining schedule, according to the time of use (TOU) tariff. A case study is shown to demonstrate the effects of the optimization model. The study suggests that by combining load shifting and energy efficiency techniques, an annual energy saving of 2540035 kW h is possible, leading to an annual cost saving of USD 277035.

1. Introduction

With growing concerns about the environment and energy security across the globe, these areas of research also become popular. Carbon emissions trading has been a common area of interest with regard to environmental studies [1,2]; and demand side management (DSM) programmes are introduced around the world as an effective and quick solution to the increasing energy concerns. DSM programmes seek to reduce the gap between power supply and demand broadly by energy efficiency improvement, energy conservation interventions, fuel switching, load expansion and self-generation, among which energy efficiency (EE) and load management (LM) are the most popularly used methods in the industrial sector [3]. The LM approach aims to reduce electricity demand at peak periods by giving monetary incentive to shift load to off-peak periods, e.g., in the form of the time of use (TOU) tariff or demand response [4,5]. The EE approach aims to reduce overall electricity consumption by installing energy efficient equipment and/or optimizing industrial processes, e.g., by making use of variable speed drives (VSDs).

DSM techniques are applied in various industries as can be found in literature. Some applications consider load shifting by finding the optimal switching times of equipment or processes according to the TOU tariff, e.g., controlling conveyor belts [6,7], pumps [8,9], geysers [10] and equipment that form part of a batch process [11]. These examples show significant cost saving compared to the baseline case, but actual energy saving is very little because the objective is focused on load shifting from peak periods. Considerable attention has also been drawn to EE improvement of
industrial processes by the application of VSDs. In particular, the application of VSDs to fans and pumps have been studied [12–14], reasons being that VSDs offer more energy saving than a simple switching strategy [7,12] and a small reduction in the speed can result in large energy savings [15–17]. However, these examples do not consider LM under the TOU tariff in order to shift load from peak time. Thus very few cases consider obtaining energy saving and peak load shifting with application of DSM techniques, simultaneously [18]. In particular, little evidence is found of application of VSDs to achieve both LM and EE in mining ventilation networks.

It is known that the industrial sector consumes about 37% of the world’s total delivered energy, and out of this, the mining industry contributes about 9% [19]. Mine ventilation is crucial for safety reasons, because it is responsible for clearing out noxious and flammable gasses; and also provides a comfortable working environment underground. The rating of the fans that provide ventilation, depending on its use, can range from 100 kW [20] to about 3000 kW [21]. Thus, contribution by the ventilation fans towards the total power consumption is quite significant. Research suggests that, depending on the type of mine, up to 40% [22] of the total electricity used, and up to 60% [23] of mining operating cost can be attributed to ventilation underground.

Existing studies on mine ventilation networks vary in objectives. Some consider making use of computer programmes in conjunction with survey data to model the changes in the structures of the ventilation network to achieve safe and economic solutions [24,25]. These cases do not consider LM or EE in any way. Other studies are performed with the objective of running the ventilation system more efficiently to reduce costs. e.g., in [26] a review is done at the beginning of every week to determine which levels will be inactive for the next week, such that the auxiliary fans in those levels can be switched off. This study considers EE but not by application of VSDs. Optimization techniques are sometimes employed, e.g., [27,28] discuss the use of nonlinear programming methods to find the optimal flow rates in the branches such that the rating of the fan, required to supply the network, is minimized; [29] shows the use of genetic algorithms (GA) to decide on the optimum number, location, and size of booster fans/regulators in the network; [21] makes use of mixed integer programming (MIP) to find the optimal angles of the auxiliary fan blades, in order to match the varying flow rates throughout the different stages in a mine. These studies are all performed with the objective of minimizing energy consumption and/or related costs, but there is no consideration for the TOU or the application of VSDs.

Literature suggests that it is common practice around the world, especially in older mines, to run ventilation fans continuously, regardless of production needs. Also, they are sized to meet the maximum volume requirements over their operating period. Examples of this can be found in Slovenia [30], Canada [31] and South Africa [32]. As such, the concept of ventilation on demand (VOD) is explored [33,34]. The idea is to allow variable air flow by adjusting the speed of the fan over time (based on the demand), as opposed to running them at full capacity at all times. These studies make use of VSDs to perform EE measures, but there is no consideration for shifting load from peak times according to the TOU tariff. Another limitation is that only auxiliary fans are selected to be part of the VOD programmes, as opposed to the main mine fan. This is done because changing the speed of the main fan will affect the entire network and is more complicated to control than the auxiliary fans.

The novelty of this paper is that it focuses on applying both DSM strategies together (load shifting and energy efficiency) to the main fan in an underground mine ventilation network. Also, no study has been found that performs optimization which incorporates the fan laws together with the ventilation network. Thus, energy efficiency is achieved by determining an optimal speed profile for the main fan according to VOD. In addition, LM is introduced, by optimizing the mine schedule according the TOU tariff, such that further energy cost reduction can be achieved.

The ventilation network is first mathematically modelled by extending the work shown in [27,28], whose main feature is modelling the network using Kirchhoff’s current and voltage law. Then, the optimal schedule of a set of given mining processes is found by solving an optimization problem formulated. Thereafter, the fan speed optimization problem is solved to determine the optimal fan speed profile throughout the mining processes that matches the VOD and reduces energy consumption. For the fan speed optimization problem, the fan’s operating point is correctly determined by incorporating the fan laws and the network’s system curve to get a realistic figure of the savings [18,35]. A case study is presented to demonstrate and verify the effectiveness of the optimization model built. The results show that by combining load shifting and energy efficiency techniques, an annual energy saving of 2540035 kWh is possible, leading to an annual cost saving of USD 277035.

The rest of the paper is organized as follows. Section 2 presents the formulation of the ventilation problem. A case study is given in Section 5. Section 6 concludes this study.

### 2. Problem formulation

The problem is divided in two parts, EE and LM. For the EE case, the fan speed is adjusted according to the varying demand of air flow rate throughout the day. In the case of the LM, the optimal starting time of the mining schedule is found (according to the TOU tariff) that leads to minimum cost; and then the fan speed is adjusted accordingly.

For either case, the ventilation network needs to be modelled. To help understand the model better, an example of a simple network is presented. A path in a network can be defined as a direct chain from the inlet node to the outlet node. In addition, an assumption is made to simplify the model that says, the airflow in all branches contained in the path must have the same direction. The total number of paths in a network is given by 

\[ n_p = n - m + 1, \]

where \( n \) is the number of branches and \( m \) is the number of nodes. In Fig. 1, there is 1 fan, 13 branches, 9 nodes, and thus, 5 paths.

Tellegen’s theorem states that “the power provided to the system and the power consumed in the network must be equal” [36]. From a ventilation network point of view, it can be concluded that the sum of air power supplied by the fans must be equal to the sum of power losses incurred in the branches of the network.

The analogy of circuit theory is often used in modelling flow distribution networks by comparing the flow rate to current, and the voltages to the pressure [27,28]. Thus Tellegen’s theorem, along with Kirchhoff’s laws form the basis to modelling an airflow distribution network.

The adaptation of Kirchhoff’s current law is given by

\[ \sum_{j=1}^{n} B_{ij} Q_i = 0, \quad \text{for } i = 1, \ldots, m, \]

where \( j \) is the branch number, \( n \) is the total number of branches, \( Q_i \) is the flow rate of branch \( i \), \( n \) is the node number, \( m \) is the total number of nodes, \( B_{ij} \) is the element of an \( m \times n \) incidence matrix \( B = [B_{ij}] \) that describes the node-to-branch incidence

\[
B_{ij} = \begin{cases} 
1, & \text{if flow in branch } j \text{ enters node } i; \\
-1, & \text{if flow in branch } j \text{ leaves node } i; \\
0, & \text{if branch } j \text{ is not incident with node } i.
\end{cases}
\]
The adaptation of Kirchhoff’s voltage law is applied to the paths in this case;

\[ \sum_{j \in F} L_{pj} H_j - \sum_{j = 1}^{n_p} L_{pj} h_j = 0, \quad \text{for } p = 1, \ldots, n_p, \]  

where \( F \) is the set of branches containing a fan, \( H_j \) is the fan pressure in branch \( j \), \( L_{pj} \) is an element of the path matrix \( L = [L_{pj}] \), \( h_j \) is the pressure loss over the \( j \)th branch, \( p \) is the path number, and \( n_p \) is the total number of paths.

The path matrix \( L \) is an \( n_p \times n \) matrix that describes the branch-to-path incidence and defined by

\[ L_{pj} = \begin{cases} 1, & \text{if path } p \text{ contains branch } j; \\ 0, & \text{if path } p \text{ does not contain branch } j. \end{cases} \]

### 2.1. Atkinson resistance

Unlike ohm’s law, the relationship between the pressure drop over a branch/pipe and the flow rate through it is not linear. It is derived from the Darcy–Weisbach equation [37] and is given by [27,28]

\[ h = R Q^2, \]  

where \( h \) is the pressure drop over the pipe, \( Q \) is the flow rate through the pipe, and \( R \) is known as Atkinson resistance, given by

\[ R = \frac{\rho l}{2DA}, \]  

where \( \rho \) is the density of air, \( l \) is the length of the pipe, \( D \) is the diameter of the pipe, \( A \) is the cross-sectional area of the pipe, and \( \mu \) is the Darcy friction factor, which is dependent on the material of the pipe.

This is a theoretical determination of the branch resistances. In practice, however, the resistance of a branch would be found by using data for the flow rate and pressure differences obtained from meters inside the mine, and solving Eq. (3) for \( R \).

### 2.2. Fan laws and performance curves

A particular fan can be characterized by its fan curve, i.e., how much pressure the fan needs to develop to supply a particular flow rate. A simple example of fan curves is shown in Fig. 2. It can be seen that a fan running at a particular speed has its corresponding fan curve, therefore changing the speed will result in a new curve. It can also be seen that as the speed reduces, a lower head/pressure is required to achieve the same flow rate. This means less power is needed to supply the fan, since the air power is given as the product of the pressure and the flow rate. Thus less power is consumed at lower running speeds [15]. However, the question arises as to how to characterize the relationship between flow rate and power. This is solved by the Fan Laws, also known as the Affinity Laws.

The Fan Laws are a set of governing equations that define the relationship between the speed of the fan (\( N \)), the pressure developed by the fan (\( H \)), and the input power to the fan (\( P \)). These laws are given as [38]

\[ \frac{Q_1}{Q_2} = \frac{N_1}{N_2}, \]  

\[ \frac{H_1}{H_2} = \left( \frac{N_1}{N_2} \right)^2, \]  

\[ \frac{P_1}{P_2} = \left( \frac{N_1}{N_2} \right)^3. \]

In these equations, the variables with subscript 1 are those at the original condition, and the ones with subscript 2 are the resulting variable due to the change in speed. E.g., using the first formula, the new flow rate (\( Q_2 \)) can be found, due to the change in speed, if the initial flow rate (\( Q_1 \)) is known. The same idea can be applied to the other formulae regarding the pressure and power, respectively.

### 2.3. System curves

The system curve represents the combination of all the aerodynamic resistances in the system, by simplifying the entire network into a single airway with an equivalent resistance, \( R_{eq} \).

For instance, in Fig. 1, the system curve can be found by finding the equivalent resistance seen from the fan’s point of view. Ideally, the system curve is characterized by a second order function in the same form as Eq. (3). Practically, however, Eq. (8) is taken as the general form to cater for offsets and disturbances.

\[ H = b_2 Q^2 + b_1 Q + b_0. \]  

In order to find the total equivalent resistance, some equivalent resistance simplifications are necessary. With the analogy from circuit theory, the following equations are derived.

For pipes in series [39]

\[ R_{eq} = R_1 + R_2. \]  

For parallel pipes,

\[ \frac{1}{\sqrt{R_{eq}}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}. \]

For a delta to wye conversion, with reference to Fig. 3, the following equations are derived from first principles

\[ 2R_1 = \frac{R_1(R_0 + R_b)}{R_c + R_b + 2 \sqrt{R_b (R_c + R_b)}} + \frac{R_b (R_c + R_b)}{R_c + R_b + 2 \sqrt{R_b (R_c + R_b)}} + \frac{R_c (R_b + R_c)}{R_c + R_b + 2 \sqrt{R_b (R_c + R_b)}}. \]
2Rq = \frac{Rb(Ra + Rc)}{Ra + Rb + Rc + 2\sqrt{Ra(Rb + Rc)}} + \frac{Rc(Ra + Rb)}{Ra + Rb + Rc + 2\sqrt{Rc(Rb + Ra)}} - \frac{Ra(Rb + Rc)}{Ra + Rb + Rc + 2\sqrt{Ra(Rb + Rc)}}. 
\tag{12}

2Rf = \frac{Rf(Ra + Rb)}{Ra + Rf + Rb + 2\sqrt{Rf(Rb + Ra)}} + \frac{Rb(Rf + Ra)}{Ra + Rf + Rb + 2\sqrt{Rb(Ra + Rf)}} - \frac{Ra(Rf + Rb)}{Ra + Rf + Rb + 2\sqrt{Ra(Rf + Rb)}}. 
\tag{13}

Thus, the equivalent resistance as seen from the fan’s point of view in Fig. 3 is calculated by

R_{eq} = R_f + R_i + ((R_f + R_i)(R_b + R_c) + R_q. 
\tag{14}

Therefore, the coefficient b_2 in Eq. (8) is found by the above calculations to be b_2 = R_{eq}. The other coefficients are considered to be 0 in this study, because an ideal case is assumed.

2.4. Fan operating point

When analyzing flow rate/pressure/power changes of a fan with respect to speed changes, often the fan/affinity laws are used. However, this is only valid when the fan is being analysed by itself. When considering the fan as part of a network, using only the fan laws will result in incorrect analysis, i.e., there is no guarantee that the fan will operate at the calculated flow rate/pressure. More importantly, a reduction in fan speed will often result in an overestimate of power savings [35]. When formed part of a network, the fan will operate at a steady state condition, which is known as the operating point of the fan. The operating point of the fan is given by the intersection point between a fan curve and system curve.

A simple example of this is shown in Fig. 2. It can be seen that if the fan is running at 1000 RPM, it will operate at point 1, and if it is running at 250 RPM, it will operate at point 2. This shows that every time the speed changes, there will be a new operating point.

Therefore, it is imperative that the affinity laws are used in conjunction with the system curve, such that the correct operating point is found.

In Section 3, three equations are used as part of the objective and constraint functions, i.e., P_f(N_{f}(t)), Q_f(N_{f}(t)), and H_f(N_{f}(t)). These equations express the power, the flow rate, and the pressure as a function of the fan speed, respectively. The fan operating point \(H_f(N_{f}(t)), Q_f(N_{f}(t))\) is time varying for VSDs and determines the power consumption of the fan \(P_f(N_{f}(t)).\)

It is noted that for ease of notation, the subscript \(k\) in the functions \(P_f(N_{f}(t)), Q_f(N_{f}(t)), H_f(N_{f}(t))\) is omitted in the remaining of this section. For each fan \(k\), the same procedure as given here applies. To form the flow-speed function \(Q(N(t))\), the fan curve of the highest speed is required. This curve can be described by a second order polynomial given by the generic form

\[H_{f,\text{full}} = a_2Q_{f,\text{full}}^2 + a_1Q_{f,\text{full}} + a_0,\]
\tag{15}

where \(H_{f,\text{full}}\) is the pressure developed by the fan at full speed, \(Q_{f,\text{full}}\) is the flow rate delivered by the fan at full speed; \(a_2, a_1,\) and \(a_0\) are constant parameters derived from the full speed fan curve.

By making use of the fan laws, and replacing Eqs. (5) and (6) into Eq. (15), the following equation is obtained

\[H = a_2Q^2 + a_1Q \left(\frac{N}{N_{f,\text{full}}}\right) + a_0 \left(\frac{N}{N_{f,\text{full}}}\right)^2\]
\tag{16}

where \(N_{f,\text{full}}\) is the rated full speed.

By equating Eq. (16) to the system curve Eq. (8) and solving for \(Q\), a function is obtained for the flow rate only in terms of the normalized speed \((N/N_{f,\text{full}})\). By curve fitting, this function can be approximated to a linear function given by [18]

\[Q(N(t)) = c_1 \frac{N(t)}{N_{f,\text{full}}} + c_0,\]
\tag{17}

where \(c_1\) and \(c_0\) are constant parameters.

Thus, Eq. (17) can be used to find the flow rate at the operating point corresponding to any given fan speed \(N(t)\).

The pressure-speed function is obtained in a similar method, i.e., the same procedure is followed until Eq. (16) is reached. Therefore, Eqs. (16) and (8) are each expressed in terms of the flow rate \(Q\). These are then equated to each other, and then the resulting equation is solved for \(H\). This will result in a function for the pressure developed by the fan expressed only in terms of the normalized speed. Again, by curve fitting, this function can be approximated to a second order polynomial given by the generic form [18]

\[H(N(t)) = d_2 \left(\frac{N(t)}{N_{f,\text{full}}}\right)^2 + d_1 \frac{N(t)}{N_{f,\text{full}}} + d_0.\]
\tag{18}

Lastly, with the fan operating point found by Eqs. (17) and (18), the power of the fan at the operating point is obtained by following a similar process to the flow-speed formulation. First, the power curve for the highest speed is expressed as a 3rd order polynomial; into which, Eqs. (5) and (7) are replaced to yield the following equation

\[P = e_1Q^3 + e_2Q^2 \left(\frac{N}{N_{f,\text{full}}}\right) + e_1Q \left(\frac{N}{N_{f,\text{full}}}\right)^2 + e_0 \left(\frac{N}{N_{f,\text{full}}}\right)^3.\]
\tag{19}

Thereafter, by replacing Eq. (17) into Eq. (19), a function is obtained for the power only in terms of the normalized speed. By curve fitting, this function can be approximated to a 3rd order polynomial given by the generic form [18]

\[P(N(t)) = f_3 \left(\frac{N(t)}{N_{f,\text{full}}}\right)^3 + f_2 \left(\frac{N(t)}{N_{f,\text{full}}}\right)^2 + f_1 \frac{N(t)}{N_{f,\text{full}}} + f_0.\]
\tag{20}

After Eq. (20) is obtained, the power of the fan operating at different speeds can be calculated accurately with the fan’s operating point accounted for.

3. Energy efficiency

To find the optimal fan speeds that result in minimum energy cost, while adhering to the flow rate requirements, a generic mathematical formulation of the optimization model is given in this section.

The objective is to minimize the following discrete cost function

\[J = \sum_{k=1}^{K} \sum_{t=1}^{T} P_k(N_k(t))C(t)\Delta t,\]
\tag{21}

where \(K\) is the total number of fans, \(T\) is the total number time steps, \(C(t)\) is the time of use tariff at time step \(t\), and \(\Delta t\) is the length of one time step in hours.

\(P_k(N)\) is a 3rd order polynomial that describes the power of the fan \(k\) as a function of its speed \(N_k(t)\). It is detailed in Section 2.4 and given in Eq. (20).
Since the optimization variable is chosen to be the fans’ speeds, the flow rates of the branches that contains a fan are defined by flow rates of that fan. This forms the first system constraint, given by

\[ Q_j(t) = Q^N_j(N_k(t)), \text{ for } j \in F. \]  

(22)

where \( Q_j(t) \) and \( Q^N_j(N_k(t)) \) are the flow rate of branch \( j \) and the flow rate of fan \( k \) that lie in branch \( j \) at time \( t \), respectively.

\( Q^N_j(N_k(t)) \) is a linear equation that describes the flow rate of a fan \( k \), as a function of its speed \( N_k(t) \). It is given by Eq. (17) in Section 2.4.

The second constraint is based on the ventilation load of the mine. This is where the concept of VOD is used, i.e., the demand for airflow rate at each branch, at each time step forms the load to the system. This can be written as

\[ Q^{\text{min}}_j(t) \leq Q_j(t) \leq Q^{\text{max}}_j(t), \]  

(23)

where \( Q^{\text{min}}_j(t) \) and \( Q^{\text{max}}_j(t) \) are the minimum and maximum allowable airflow in branch \( j \) at time \( t \). For the case where the demand is a fixed amount, \( Q^{\text{min}}_j(t) = Q^{\text{max}}_j(t) \).

The rest of the constraints are formed based on the principles of conservation of mass and energy. The principle of conservation of mass is stated in Eq. (1), but is shown here again including the time variable.

\[ \sum_{i=1}^{n} B_i Q_j(t) = 0 \quad \text{for} \; i = 1, \ldots, m. \]  

(24)

The principles of conservation of energy is expanded from Eq. (2) to give

\[ \sum_{j \in F} L_{pj} H_j(t) - \sum_{j \in F} L_{pj} Q_j^2(t) = 0, \; \text{for} \; p = 1 \ldots n_p, \]  

(25)

where \( H_j(t) \) is the fan pressure in branch \( j \) at time \( t \). For a specific fan \( k \), this is a second order polynomial that describes the pressure across the fan as a function of its speed \( N_k(t) \) at time \( t \) and denoted as \( H_k(N_k(t)) \), which is derived from the fan laws, and is presented in Eq. (18) in Section 2.4.

Thus an optimization model can be completely described by the objective function, given by Eq. (21), and the constraints, given by Eqs. (22)–(25).

4. Load management

To further investigate the potential of peak shaving and cost minimization by implementing LM strategy, the LM problem of the ventilation system is formulated. It is assumed that a schedule consists of \( N \) processes that follow each other. Therefore, the main idea of LM is to find an optimal starting point of these processes in view of the TOU tariff.

To do so, the following cost function is minimized

\[ J = \sum_{i=1}^{t_{\text{Start}}} P_1 C(i) + \sum_{i=1}^{t_{\text{Start}}} P_2 C(i) + \cdots + \sum_{i=1}^{t_{\text{End}}} P_{N} C(i), \]  

(26)

where \( t_\text{Start} \) is the starting time of the schedule and the decision variable; \( t_{\text{End}}, C(1), \ldots, C(N) \) are the durations (hours) of the processes 1 to \( N \); \( P_1, \ldots, P_N \) are the power consumptions (kW) of processes 1 to \( N \), at the specified flow rates, respectively; and \( C(i) \) is the electricity cost (cents/kWh) at time instant \( i \), according to the TOU tariff; and

\[ t_y = t_x + \sum_{z=1}^{y} t_{p(z)}, \quad \text{for } y = 1, \ldots, N. \]  

(27)

5. Case study

5.1. Problem description

For demonstration purposes, a case study is presented. The example from Fig. 1 is used, and it is assumed that a schedule is followed over 24 h with regard to the mining processes. The schedule is shown in chronological order, in Table 1, i.e., the processes have to follow each other as shown. For this example it is assumed that only branch 7 and 8 are being actively mined, and thus only these branches are required to follow the schedule. Thus, there is no particular requirements for the other branches of the mine. Again, the problem is divided in two parts: energy efficiency and load shifting. For the first case, it is assumed that the schedule shown in Table 1 starts at the beginning of the day, i.e., no consideration is made towards the optimal starting time according to the TOU tariff. For the second case, the optimal starting time of the schedule is found that leads to the minimum cost, and then the optimal fan speeds are found accordingly. For both cases, the optimization period is for one day, 24 h.

For either case, the system curve is needed. It is assumed that the resistance of each branch is known (see Table 2). Thus the procedure described in Section 2.3 is followed, and it is found that the equivalent resistance seen by the fan is \( R_{eq} = 0.2122 \text{ N s}^2/\text{m}^4 \). Assuming no initial offset, the system curve is described by

\[ H = 0.2122Q^2. \]  

(28)

The data from the fan curves (given in Fig. 4) are used to obtain the power-speed, pressure-speed, and flow-speed functions. The data for this fan curve is obtained from an operating mine in South Africa.

The power-speed, pressure-speed, and flow-speed functions are given by Eqs. (29)–(31) respectively.

\[ P(N) = 1796 \left( \frac{N}{750} \right)^3, \]  

(29)

\[ H(N) = 7973 \left( \frac{N}{750} \right)^2, \]  

(30)

\[ Q(N) = 194 \left( \frac{N}{750} \right). \]  

(31)

The incidence matrix and path matrix can be structured according to Eqs. (1) and (2). They are not presented here to save space.

### Table 1

<table>
<thead>
<tr>
<th>Process</th>
<th>Duration (h)</th>
<th>Min flow required (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling</td>
<td>8</td>
<td>15.2</td>
</tr>
<tr>
<td>Explosive charge-up</td>
<td>3</td>
<td>10.3</td>
</tr>
<tr>
<td>Blasting</td>
<td>6</td>
<td>none</td>
</tr>
<tr>
<td>Cleaning and Mucking</td>
<td>7</td>
<td>25.5</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Branch no.</th>
<th>( R ) (N s²/m⁴)</th>
<th>Branch no.</th>
<th>( R ) (N s²/m⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0225</td>
<td>8</td>
<td>1.35</td>
</tr>
<tr>
<td>2</td>
<td>0.1104</td>
<td>9</td>
<td>0.225</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>10</td>
<td>0.0551</td>
</tr>
<tr>
<td>4</td>
<td>0.168</td>
<td>11</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>3.6</td>
<td>12</td>
<td>0.0385</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>13</td>
<td>0.0385</td>
</tr>
<tr>
<td>7</td>
<td>0.072</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.2. Results and discussion

As per the requirements, only branches 7 and 8 are limited to specific flow rates throughout the day. The flow rates for the rest of the branches are limited between 3 m\(^3\)/s and an infinite amount, i.e., the model is allowed to choose any value that will minimize the cost, while maintaining the constraints. The tariff structure shown in Table 3 is used to calculate the energy cost.

5.2.1. Case 1: Energy efficiency

The problem is solved using the \textit{fmincon} solver in Matlab’s optimization toolbox. The \textit{SQP} algorithm was found to give the best results. The objective function, linear, and non-linear constraints were coded as separate functions, and then passed through the \textit{fmincon} solver. The control variables were chosen to be the flow rate in each branch and the speed of the fan, both at each time step. The tolerance on the non-linear equality constraints, given by Eq. (25), was set to be \(1 \times 10^{-4}\) such that the possible solution for the flow rates are within the acceptable range.

The schedule from Table 1 is followed, and started at 00:00. A safety margin of 3 m\(^3\)/s is kept on the lower limit, and no upper bounds of the flow rates are considered. Therefore the demand constraints for branches 7 and 8 are given by

\[
\begin{align*}
Q_{7,8}(t) &\geq 18.2 \quad \text{for} \quad 00:00 \leq t < 08:00, \\
Q_{7,8}(t) &\geq 13.3 \quad \text{for} \quad 08:00 \leq t < 11:00, \\
Q_{7,8}(t) &\geq 3.00 \quad \text{for} \quad 11:00 \leq t < 17:00, \\
Q_{7,8}(t) &\geq 28.5 \quad \text{for} \quad 17:00 \leq t < 24:00.
\end{align*}
\]

The results include flow rates of branches 7 and 8, optimal fan speeds found, and the power profile of the fan. They are presented in Figs. 5–8, respectively. It is noted that the solutions obtained might not lead to a global minimum, and therefore they may be referred to as sub-optimal solutions. As can be seen from Figs. 5 and 6, the minimum required flow rates of branches 7 and 8 are achieved over the 24 h period, and the constraints are all satisfied. It is noted that the optimal flow rates of the branches, as deter-
The model shown is followed and the optimal fan speeds are found. Theoretically, the power of a fan can be reduced if the flow rates of the branches exactly equal to the minimum required values. However, that cannot be always satisfied due to physical limits. In other words, there is no possible solution that can satisfy exactly the minimum requirements for both branches 7 and 8.

Fig. 7 shows that the fan speed is varied throughout the day, and this means that the power consumption will vary as well, reflected in Fig. 8. The non-linear relationship is clearly visible, whereby dropping the speed from 300 rpm to about 225 rpm reduces the power consumption from 125 kW to about 50 kW.

The optimized fan speed, when the TOU tariff is followed, results in a daily energy cost of USD 431. Comparing this to a situation where the fan needs to be run at full capacity at all times to supply the maximum flow rate of 28.5 m$^3$/s to branches 7 and 8, the cost of energy would be USD 1015 per day. This leads to a cost saving of USD 584 per day, which adds up to USD 213160 cost saving per year (assuming the same cycle throughout the year).

In terms of energy savings, keeping a constant speed leads to daily energy consumption of 11499 kW·h; whereas varying the speed results in the daily consumption to be 4540 kW·h. This leads to daily savings of 6959 kW·h, which translates to savings of 2540035 kW·h per year.

5.2.2. Case 2: Energy efficiency and load management

For this case, the optimal starting time is first found by solving the model shown in Section 4. The control variable was chosen to be the starting time of the schedule (in hours), thus it had to be limited to integer values from 1 to 24. Hence, the genetic algorithm (GA) solver in Matlab is used to solve this problem. An initial population of a starting time of 00:00 was chosen. The TOU tariff that is followed is shown in Table 3.1 Thereafter, the schedule shown in Table 1 is followed and the optimal fan speeds are found.

After minimization of the objective, it is found that the minimum cost over a 24 h period occurs when the cycle is started at 05:00. It is noted that changing the schedule will result in a change in the working hours for the workers. Since this is a preliminary study, the social effects vs. economic benefits of the change are not considered.

Keeping a safety margin of 3 m$^3$/s, the demand constraints are given by

\[
Q_{7,8}(t) \geq 18.2 \text{ for } 05:00 \leq t \leq 13:00
\]
\[
Q_{7,8}(t) \geq 13.3 \text{ for } 13:00 \leq t \leq 16:00
\]
\[
Q_{7,8}(t) \geq 3.00 \text{ for } 16:00 \leq t \leq 22:00
\]
\[
Q_{7,8}(t) \geq 28.5 \text{ for } 22:00 \leq t \leq 05:00
\]

The results are presented in Figs. 9–12. Again, the minimum required flow rates are achieved for branches 7 and 8. However this time it can be seen that the higher demand has been shifted to the off-peak times, and thus less energy is consumed during peak times, as shown in Fig. 12. If the optimal speed profile is followed, the energy cost (according to the TOU tariff) in 24 h results to USD 255.6. Compared to case 1, this leads to a further USD 175 saving per day. Thus the annual energy-cost saving, when taking into account energy efficiency and load shifting, is USD 277035.

It must be noted that when compared to case 1, the energy savings in this case stays the same, i.e., 2540035 kwh per year. This is because the load is just shifted according to the TOU tariff, and the total demand still remains the same.

5.2.3. Analysis according to a flat tariff

In the previous 2 sections, all cost analysis was done using the TOU tariff only. This section presents an analysis according to a flat tariff, as well as compares the optimization results of using a flat tariff against the TOU tariff.

![Figure 9](https://via.placeholder.com/150)

**Fig. 9.** Flow rate in branch 7 for case 2.

![Figure 10](https://via.placeholder.com/150)

**Fig. 10.** Flow rate in branch 8 for case 2.

---

When the fan is run at a constant speed of 28.5 m\(^3\)/s to supply the maximum flow rate to branches 7 and 8; and a flat rate of 6.65 Cents/kW h is followed, the cost of energy per day would be USD 301.9. This leads to a daily saving of USD 764.7. Keeping to the flat tariff, if only EE is performed, the daily energy cost would be USD 462.7 or a yearly saving of USD 168886.

It is noted that, in the cases considered here, following the flat tariff is actually cheaper than following the TOU tariff. This however depends on the rates set for the flat tariff and the TOU tariff being compared with. In addition, TOU tariffs are developed and widely used for energy management in underground mine ventilation networks, and this paper provides a foundation for further research which will consider factors such as multiple fans, leakages, and efficiencies of the drives/motors.

6. Conclusion

Different activities in a mine throughout the day lead to varying airflow requirements for the ventilation system. In view of this, ventilation on demand (VOD) is introduced; which involves supplying the ventilation air quantity as required at a specific location and time. To realize VOD, fan speeds in the ventilation network must be adjusted accordingly to save energy and cost. An optimization model is built in this study to determine the optimal fan speeds that lead to minimum energy cost. Energy efficiency and load management strategies are investigated. It has been shown in a case study that a combination of energy efficiency and load shifting can lead to an annual energy saving of 2540035 kW h and an annual cost saving of USD 277035. It must be noted that the 74.8% cost saving and 60.5% energy saving may seem impressive, but this is only an indication of the maximum possible savings that can be achieved for this particular case. Various factors can affect this figure, such as the resistances of the branches, the characteristics of the fan, the TOU tariff, and the requirements of the flow rates. Thus, it has been shown that there is significant opportunity for energy and energy-cost savings in an underground mine ventilation network, and this paper provides a foundation for further research which will consider factors such as multiple fans, leakages, and efficiencies of the drives/motors.

Acknowledgements

This work is supported by the Centre for New Energy Systems (CINES) and the National Hub for the Postgraduate Programme in Energy Efficiency and Demand Side Management at the University of Pretoria.

References


