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An optimal control model for load shifting – With application in the energy management of a colliery

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ABSTRACT

This paper presents an optimal control model for the load shifting problem in energy management and its application in a South African colliery. It is illustrated in the colliery scenario that how the optimal control model can be applied to optimize load shifting and improve energy efficiency through the control of conveyor belts. The time-of-use electricity tariff is used as an input to the objective function in order to obtain a solution that minimizes electricity costs and thus maximizes load shifting. The case study yields promising results that show the potential of applying this optimal control model to other industrial Demand Side Management initiatives.

1. Introduction

Load shifting is a basic problem in Demand Side Management (DSM), which is a topic of increasing importance in South Africa, where the main electricity supplier Eskom is trying hard to supply the growing peak time electricity demand.

There are plenty of references solving energy management problems by various techniques. For example, fuzzy logic is introduced in [10] and [4] to solve the load shifting problem of electric water heaters and the energy management of a domestic photovoltaic panel respectively; an artificial neural network regression model is used in [12] for a petrochemical plant; integer programming is applied in [6,1] for mid-term management of a thermal and electricity supply system of an industrial consumer and the peak-load management of a steel plant respectively. Based on some new progress in switched optimal control in [2], an optimal control model is introduced in [13] for the energy management of hot water cylinders. The aim of this paper is to apply the switched optimal control idea suggested in [13] to give an optimal control model for load shifting problems; and to apply this model in the conveyor belt systems of a South African colliery.

The kind of load shifting problems in which the optimal control model works well are those common in industrial systems and processes which can be controlled by on–off switching functions. The optimal control model suggested here can also be applied to problems such as unit commitment, economic dispatch, etc. These are easily formulated into switched optimal control problems, and the solutions obtained by optimal control algorithms are globally optimal. In this instance, the optimal control idea is adopted for the load shifting problem.

In the application of the optimal control model in the South African colliery scenario, Eskom's time-of-use (TOU) active energy tariff structure is used to obtain a suitable objective function for the control problem. This ensures a means of reducing the total operational electricity costs as well as a means for maximizing the load that is shifted from *peak* TOU periods to the less expensive *standard* and *off-peak* periods.

This case study illustrates formulation of an optimal control model for a complex industrial energy management problem. The challenge in the system modeling of the colliery is that the whole conveyor belt system is complex. It is quite difficult to identify which conveyor belts should be considered and which conveyor belts can be neglected. To resolve this difficulty, the conveyor belt system was divided into several groups; only one group was found to make a large percentage contribution to the total energy consumption of the colliery. Therefore this group was selected for further study of the optimal control model.

Note that solution algorithms of an optimal control problem are often based on Pontryagin Principle, and many practical energy problems can not meet the smooth requirements therein.

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Therefore we discretize the optimal control model for the colliery into a binary integer programming problem. This is very useful and convenient since there are several software tools available for solving such problems iteratively. For example, the built-in binary integer programming function, called "*bintprog*", provided by the Mathworks in the Matlab Optimization toolbox can be used. This Matlab binary integer programming function also allows the user to select between several different branch-and-bound techniques which helps to minimize the objective function in the shortest possible time.

The numerical results given in Section 3.9 show that the optimal discrete time scheduling of the conveyor belt system, obtained through the binary integer programming technique, reduces the cumulative active energy costs by up to 49% for 5 weekdays in a high-demand season, which is quite satisfactory.

The rest of this paper is organized as follows. In Section 2 the general optimal control model is presented and the solution algorithm is discussed in Section 2.1. Section 3 gives an application of the optimal control model in the conveyor belt system at the colliery. Concluding remarks are given in Section 4.

2. Optimal control model

Consider a system with n modules and let the power of the ith module be $P_i(t)$ Watt when it is working in its usual way before any DSM strategy. The switching on/off status of the ith module in a proposed DSM strategy can be represented by the following switching function

$$u_i(t) = \begin{cases} 1, & \text{when switched on,} \\ 0, & \text{when switched off.} \end{cases} \tag{1}$$

That is, $u_i(t) = 1$ denotes that the ith module is working with the power $P_i(t)$ at time t as usual, while $u_i(t) = 0$ means the ith module has been switched off. Note that $u_i(t)$ is a binary integer and can not be any value in the interval $(0, 1)$. The actual power of the ith module at time t is now $P_i(t)u(t)$. Let $p(t)$ be the time-of-use (TOU) electricity tariff function. Assuming that the load shifting problem is considered over the time period $[t_0, t_f]$, and take a partition of this interval such that $t_0 < t_1 < t_2 < \cdots < t_{N-1} < t_N = t_f$. Then the total electricity cost on the interval $[t_0, t_f]$ is the summation of the electricity cost over each small interval $[t_j, t_{j+1}], j = 0, 1, \cdots, N-1$. Now the electricity cost of the ith module over the interval $[t_j, t_{j+1}]$ can be approximated as $P_i(t)u(t)p(t)(t_{j+1} - t_j)$, thus the total electricity cost J for all the n modules over $[t_0, t_f]$ is $J = \sum_{i=1}^{n}\sum_{j=1}^{N} P_i(t)u(t)p(t)(t_{j+1} - t_j)$. In other words, one has

$$J = \int_{t_0}^{t_f} \sum_{i=1}^{n} P_i(t)u_i(t)p(t)\mathrm{d}t. \tag{2}$$

There are some constraints for the minimization of J. For convenience, we consider the following general form of constraints to include as many as possible cases

$$g(u(t), t) \leqslant 0, \tag{3}$$

where $u = (u_1, u_2, \cdots, u_n)^{\mathrm{T}}$, and g can be a vector valued function of time and many other process variables, such as flow rates, energy, power, mass transferred, losses, etc. Now an optimal control problem is obtained: Find an optimal solution u of the following problem

$$\begin{aligned} &\min \quad J = \int_{t_0}^{t_f} \sum_{i=1}^{n} P_i(t)u_i(t)p(t)\mathrm{d}t, \\ &\text{subject to} \quad g(u(t), t) \leqslant 0. \end{aligned}$$

The optimal solution u is also called an optimal controller.

2.1. Discretization of the optimal control problem

For a theoretical background on optimal control theory, particularly focussing on the formulation of the optimal control problem and Pontryagin's maximum principle, the reader is referred to the likes of [8,3,7,11]. The general methods for solving the optimal solutions, which are time-varying functions, in optimal control problems are usually based on Pontryagin's Maximum Principle or its variations. These methods often depend on some smooth conditions and the solutions of some differential equations which restrict their applications in practical problems.

Therefore it is reasonable to discretize the optimal control problem to obtain an ordinary optimization problem, where the optimal solution is not a time-varying function but a fixed point. Before applying the discretization process, two things are worthy to note. The first thing is that the time interval of the optimal control problem should be divided as many as possible so that the resulted ordinary optimization problem is a good approximation of the optimal control problem, the second thing is that when the number of divided sub-intervals increases, the number of variables in the optimization problem increases, and the computational complexity increases accordingly. Therefore this kind of discretization idea is applicable only if the total number of obtained variables are not too big so that computer algorithms can solve it quickly.

In our case study for the South African colliery, the discretization process leads to a binary integer programming problem whose solution is quite standard and can even be solved directly by a Matlab function. This helps much to solve the optimal control problem. The case study also shows that although the discretization would result in a coarser or less accurate solution in general, choosing discretization instead of Pontrayagin Maximum Principle can be acceptable in many practical problems.

Now consider the discretization of (2). Since the time interval is $[t_0, t_f]$, divide this interval equally into N sub-intervals so that each subinterval has the length $T_s := \frac{t_f - t_0}{N}$. Then the optimal control problem (2) and (3) can be approximated by

$$\begin{aligned} &\min \quad J = \sum_{i=1}^{n} \sum_{j=1}^{N} P_i^j u_i^j p^j T_s, \\ &\text{s.t.} \quad g(u^j, (j-1)T_s) \leqslant 0, \end{aligned} \tag{4}$$

where $P_i^j = P_i((j-1)T_s)$, $u_i^j = u_i((j-1)T_s)$, $p^j = p((j-1)T_s)$, and $u^j = (u_1^j, \cdots, u_n^j)^{\mathrm{T}}$. This is an ordinary optimization problem with nN number of variables $\{u_i^j : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant N\}$. Therefore various solution algorithms from linear and nonlinear programming can be tried to solve this simplified problem.

3. A South African Colliery case study

3.1. Colliery overview

The South African colliery studied in this paper has two identical Dense Media Separation (DMS) plant modules, *E12* and *G12* which are responsible for processing the ore material from an open cast mine, whose name has been omitted and is denoted by X mine here. The ore material from X mine is delivered to the colliery by train and is dumped in rail bins before being transported to either the run of mine (ROM) stockpile or directly to the DMS feed bin by the upstream D-group conveyor belt system. Fig. 1 gives an overview of the flow of coal, conveyor belts and coal storage silos and stockpiles at the colliery, as well as the variable names and abbreviations used in this paper. Note that the symbol r_* denotes the transmission rate (ton/h) of some conveyor belt, for example r_{N10} denotes the transmission rate (ton/h) of the conveyor belt $N10$.

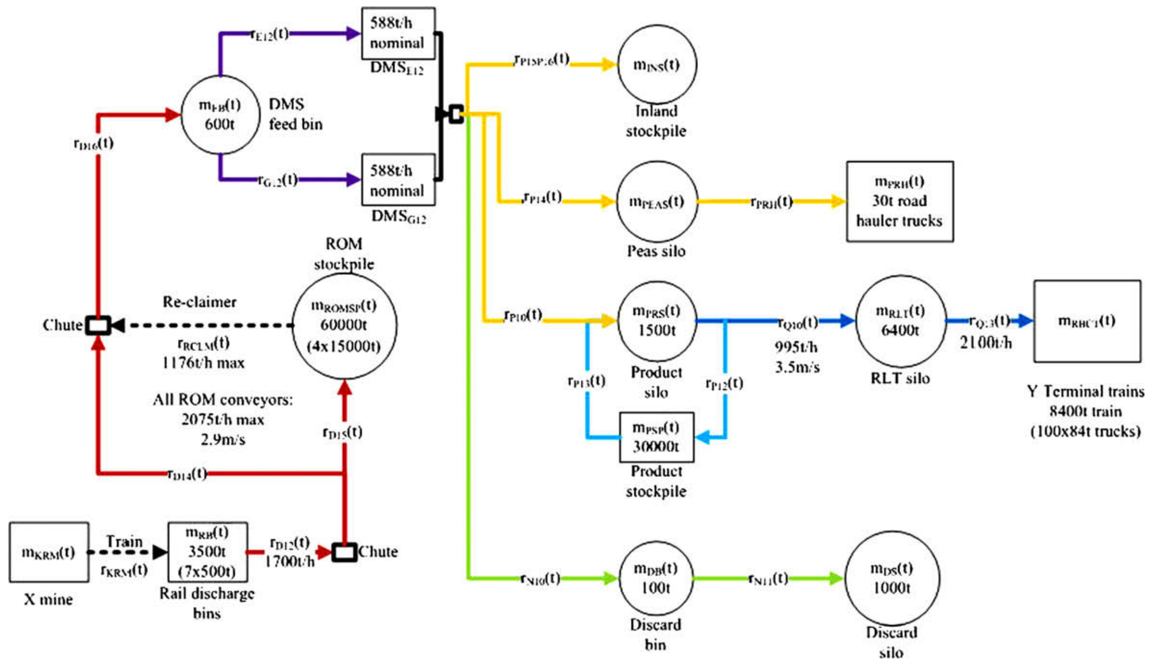

Fig. 1. Process flow diagram showing conveyor belts, coal silos and stockpiles and the flow of coal throughout the plant.

After passing through the DMS plant modules, the processed material follows one of four paths according to the size and quality of the material:

- Discarded material is transported to the Discard Silo, m_{DS}, via the $N10$ and $N11$ conveyor belts.
- Export quality coal is transported to the Product Stockpile, m_{PRS}, via the $P10$ conveyor belt.
- Coal classified as inland product is transported to the *inland stockpile*, m_{INS}, via the $P15$ and $P16$ conveyor belts.
- Product material that falls within the PEAS category sizes is transported to the *PEAS silo*, m_{PEAS}, via the $P14$ conveyor belt.

From the *product silo*, the export quality coal is either stacked on the *product stockpile* or transported, via the Q10 overland conveyor belt, to the *Rapid Loading Terminal* (RLT) silo. The *RLT silo* is used as a central base for loading the trains that transport export quality coal to the Y terminal. The trains are named RBCT trains, and the mass of the coal in an RBCT train at time t is denoted by $m_{RBCT}(t)$.

As shown in Fig. 2, the downstream *Q-group* makes the largest percentage (26%) contribution to the overall power consumption of all the conveyor belts at the colliery. The *Q-group* conveyor belts are also most suitable for load shifting energy management because this system can be isolated to be controlled independently from the rest of the colliery as shown in Fig. 3. For these reasons the *Q-group* conveyor belt system will form the main focus of the rest of this paper and will be used as an example for illustrating the optimal control model in the previous section.

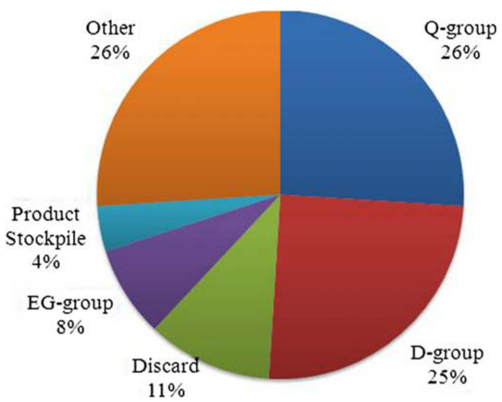

Fig. 2. Percentages of power consumptions of different belt groups.

3.2. Modeling of control problem

The isolation of the *Q-group* conveyor belts, discussed above, is possible because of the relatively large capacity of the *Product Stockpile* (PSP), m_{PSP}, which makes it possible to model the product silo and stockpile as one lumped system with the larger capacity of the product stockpile, $m_{PSP\_MAX} = 30\,000$ (tons).

It should be noted that the colliery is a *key-customer* under the Eskom Megaflex time-of-use (TOU) tariff plan. The electricity tariff is the Eskom Megaflex *Active Energy Charge*. That is,

$$p(t) = \begin{cases} p_o & \text{if } t \in [0,6) \cup [22,24) \\ p_s & \text{if } t \in [6,7) \cup [10,18) \cup [20,22)\,, \\ p_p & \text{if } t \in [7,10) \cup [18,20) \end{cases} \tag{5}$$

where t is the time of any weekday in hours (from 0 to 24); p_o, p_s and p_p are the *off-peak*, *standard* and *peak* TOU active energy tariffs in R/kW h; and R and c are the South African currency Rand and cent respectively. Note that one Rand, denoted by $R1$, equals 100 cents. The values for p_o, p_s and p_p vary according to the time of day, the

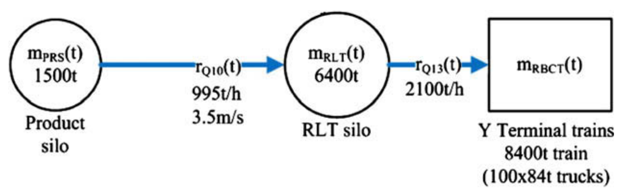

Fig. 3. Isolated Q-group conveyor belt system.

day of the week as well as the season. The seasonal TOU active energy charges are given in Table 1.

3.3. Control horizon

The time interval for which the load shifting problem is considered is referred to here as the *control horizon* and consists of the time period $[t_{t0}, t_{t2})$, as illustrated in Fig. 4.

This control horizon is formulated to coincide with the arrival times of the trains because the main objective of the Q-group conveyor belts is to supply the trains with the correct amount of coal as soon as they arrive. Since the trains do not arrive at fixed time intervals, each control horizon has to be adapted according to the train schedule of the specific time period concerned. The control horizons are also independent of the time of day which is useful, since the trains can arrive at any time of day and the number of trains per day is not constant.

The control horizon extends over a time period of two train arrivals i.e. from the time of arrival of the current train (train 0) at time t_{t0}, to the time of arrival of the second train after train 0, namely train 2 at time t_{t2}.

The idea is that once train 1 of the kth control horizon arrives at RLT at time t_{t1}, this time becomes the initial time t_{t0} of the next control horizon, i.e, the $(k+1)$th control horizon. The control horizon thus shifts forward each time a train arrives at RLT. This approach prevents short-sightedness of the control algorithm in the sense that it always optimizes for at least one extra train interval after the current one.

3.4. Basic system assumptions

The following assumptions are made in order to model Q-group conveyor belt system as a simplified optimal control problem:

(1) The product stockpile always has enough coal ($m_{\mathrm{PRS}}(t)$) at any time t to supply the Q10 conveyor belt with coal, via the product silo, at its nominal rate of $r_{\mathrm{Q10}}(t)$.

Table 1
Eskom's active energy charges for the Megaflex key-customer tariff structure, 2007–2008 [5]

Eskom's active energy charge		
TOU period	High-demand season (June–August) Incl. VAT [c/kW h]	Low-demand season (September–May) Incl. VAT [c/kW h]
Peak (p_p)	63.04	17.89
Standard (p_s)	16.67	11.10
Off-peak (p_o)	9.06	7.87

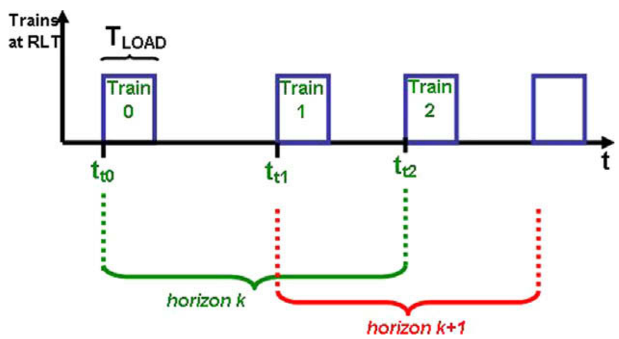

Fig. 4. Graphical illustration of the control horizon.

(2) The power $P_i(t)$ is a function of the feed rate of the ith conveyor belt, $r_i(t)$, (ton/h). However, since conveyor belts are most power efficient when operated at their maximum, or nominal design feed rate [9], it will be assumed that all conveyor belts always operate at their maximum design feed rates and in turn they also operate at their maximum power consumption when switched on and at a feed rate of 0 [ton/h] and power of 0 Watt when switched off. Thus the power consumption, $P_i(t)$, is a constant, P_{i_MAX}, for the ith conveyor belt when it is switched on. This assumption must be validated by constructing the constraints of the objective function in such a way that the conveyor belts can always operate at their maximum feed rates.

(3) The time delay associated with coal being transported from the beginning to the end of a conveyor link is ignored.

(4) The trains are loaded at a fixed rate from the instant that they arrive until they are full.

(5) The train schedule already takes into account the minimum time between trains that the colliery can deal with.

(6) All train arrival times coincide with the discrete time intervals mT_s for some integer m, that is, the difference between the arriving times of any two trains is an integer multiple of T_s.

(7) The discrete time period T_s (h) is never greater than 1 h, which corresponds with the shortest time period of the change in the price function $p(t)$.

(8) The discrete time period is chosen such that T_s is an integer fraction of 1 h, i.e. $nT_s = 1$ h where n is an integer.

(9) The start-up energy consumption of the conveyor belts is not taken into account. Although start-up power could be a significant factor, the energy consumption of long term operations of the conveyor belt is more significant.

3.5. More hypotheses for optimal control modeling

The hypotheses for the control problem can be laid out as follows:

(1) Assume:
- Train 0 arrives at $t = t_{t0}$, and leaves at $t = t_{t0} + T_L$ (h),
- Train 1 arrives at $t = t_{t1}$, and leaves at $t = t_{t1} + T_L$ (h),
- Train 2 arrives at $t = t_{t2}$, and leaves at $t = t_{t2} + T_L$ (h).

with

$$t_{t0} + T_L \leqslant t_{t1}, \tag{6}$$
$$t_{t1} + T_L \leqslant t_{t2}, \tag{7}$$

while t_{t0}, t_{t1} and t_{t2} are known, the control horizon is defined as the interval $[t_{t0}, t_{t2})$, and T_L (h) is the train loading period and can be calculated as:

$$T_L = \frac{m_{\mathrm{RBCT\_MAX}}}{r_{\mathrm{Q13\_MAX}}} = \frac{8400}{2100} = 4. \tag{8}$$

(2) Q10 and Q13 conveyor feed rates (ton/h):

$$r_{\mathrm{Q10}}(t_{t1}) = r_{\mathrm{Q10\_\,MAX}} \cdot u_{\mathrm{Q10}}(t), \tag{9}$$
$$r_{\mathrm{Q13}}(t_{t1}) = r_{\mathrm{Q13\_\,MAX}} \cdot u_{\mathrm{Q13}}(t), \tag{10}$$

where

$$r_{\mathrm{Q10\_\,MAX}} = 995 \qquad (\mathrm{ton/h}), \tag{11}$$
$$r_{\mathrm{Q13\_\,MAX}} = 2100 \qquad (\mathrm{ton/h}), \tag{12}$$

u_{Q10} and u_{Q13} are the switching functions of Q10 and Q13, respectively.

(3) Train arrival times, t_{t0}, t_{t1} and t_{t2} coincide with discrete time intervals (see Assumption 6 of Subsection 3.4).

(4) Q13 conveyor switching function is always defined as

$$u_{Q13}(t) = \begin{cases} 1, t \in [t_{t0}, t_{t0} + T_L] \cup [t_{t1}, t_{t1} + T_L] \\ 0, t \in (t_{t0} + T_L, t_{t1}) \cup (t_{t1} + T_L, t_{t2}) \end{cases} \tag{13}$$

(5) The mass, in tons, contained by the RLT silo can be modeled as

$$m_{RLT}(t) = m_{RLT}(t_0) + \int_{t_0}^{t} [r_{Q10}(\tau)u_{Q10}(t) - r_{Q13}(\tau)u_{Q13}(t)]d\tau,$$

where t_0 is the initial time and can be assumed to be $t_0 = 0$.

3.6. Objective function

Since u_{Q13} is already known, there is no need for it to be included in the objective function for the Q-group conveyor control problem. In fact, by omitting any unnecessary variables from the objective function the computing time for the obtained optimization problem should be greatly reduced. By simplifying (2) the objective function for the Q-group conveyors can be formulated as the cost of operating the Q10 conveyor belt as follows:

$$J = \int_{t_{t0}}^{t_{t2}} P_{Q10\_MAX}\, p(t)u_{Q10}(t)dt \tag{14}$$

When represented in terms of the discrete sampling time, T_s, the objective function to be minimized can be represented as:

$$\min\ J = T_s \sum_{i=1}^{N-1} P_{Q10\_MAX}\, p(t_i)u_{Q10}(t_i), \tag{15}$$

where $N = \frac{t_{t2} - t_{t0}}{T_s}$.

In this case, (15) can be further simplified by removing the constant elements T_s and P_{Q10_MAX}, which do not affect the minimum objective function, to obtain the objective function J in its simplest form:

$$\min\ J = \sum_{i=1}^{N-1} p(t_i)u_{Q10}(t_i) \tag{16}$$

3.7. Constraints

The following constraints are considered for the RLT silo:

(1) Minimum constraint on m_{RLT}:

$$m_{RLT} \geqslant 0 \qquad \forall t \in [t_{t0}, t_2] \tag{17}$$

(2) Maximum constraint on m_{RLT}:

$$m_{RLT} \leqslant 6400 \qquad \forall t \in [t_{t0}, t_2] \tag{18}$$

(3) Threshold constraint on $m_{RLT}(t_{t1})$ at time t_{t1}:

$$m_{RLT}(t_{t1}) \geqslant m_{RLT\_THR} \tag{19}$$

(4) Threshold constraint on $m_{RLT}(t_{t2})$ at time t_{t2}:

$$m_{RLT}(t_{t2}) \geqslant m_{RLT\_THR} \tag{20}$$

The last two constraints in Eqs. (19) and (20) are required to ensure that the RLT silo never runs empty while a train is being loaded, i.e. while conveyor Q13 is switched on. This will ensure that the trains will always be loaded at the maximum feed rate of conveyor Q13 which is greater than the maximum feed rate of conveyor Q10. The derivation of the threshold value, m_{RLT_THR}, is shown below.

$$m_{RLT}(t) \geqslant 0$$

$$m_{RLT}(t_{t1} + T_L) = m_{RLT}(t_{t1}) + \int_{t_{t1}}^{t_{t1}+T_L} [r_{Q10\_MAX} - r_{Q13\_MAX}]dt \geqslant 0$$

$$m_{RLT}(t_{t1}) + [r_{Q10\_MAX} - r_{Q13\_MAX}] \int_{t_{t1}}^{t_{t1}+T_L} 1dt \geqslant 0$$

$$m_{RLT}(t_{t1}) \geqslant -[r_{Q10\_MAX} - r_{Q13\_MAX}](t_{t1} + T_L - t_{t1}) = m_{RLT\_THR}$$

3.8. Solving binary integer programming problem in Matlab

The Matlab optimization toolbox *bintprog* function requires the objective function to be in the form of

$$\min \mathbf{f}^{\mathbf{T}} \cdot \mathbf{x}, \tag{21}$$

subject to equality constraints in the form of

$$\mathbf{Aeq} \cdot \mathbf{x} = \mathbf{beq}, \tag{22}$$

and inequality constraints in the form of

$$\mathbf{A} \cdot \mathbf{x} \leqslant \mathbf{b}, \tag{23}$$

where **f**, **b** **beq** are vectors and **A** and **Aeq** are matrices. The solution is **x**, which is a binary integer vector, i.e. its entries can only take on the values of 0 or 1.

For the Q-group conveyor problem at the colliery, the binary vector **x** is made up of the discrete time switching function vector of the Q10 conveyor belt as represented by

$$\mathbf{x} = [u_{Q10}(t_0), u_{Q10}(t_1), \ldots, u_{Q10}(t_{N-1})]^{T},$$

and the objective function coefficients vector **f** is defined as:

$$\mathbf{f} = [p(t_0), p(t_1), \ldots, p(t_{N-1})]^{T}.$$

The formulation of the constraint matrices will now be discussed. The threshold constraint on m_{RLT} at time t_{t1} is dealt with first, and is formulated as follows:

$$m_{RLT}(t) = m_{RLT}(t_{t0}) + T_s \sum_{i=0}^{N-1} r_{Q10\_MAX} \cdot u_{Q10}(t_i) - T_s \sum_{i=0}^{N-1} r_{Q13\_MAX} \cdot u_{Q13}(t_i).$$

So for the threshold constraint to hold, the following must be true:

$$m_{RLT}(t_{t0}) + T_s \sum_{t_i=t_{t0}}^{(t_{t1}-T_s)} r_{Q10\_MAX} \cdot u_{Q10}(t_i) - T_s \sum_{t_i=t_{t0}}^{(t_{t1}-T_s)} r_{Q13\_MAX} \cdot u_{Q13}(t_i) \geqslant m_{RLT\_THR},$$

which results in:

$$-\sum_{t_i=t_{t0}}^{(t_{t1}-T_s)} u_{Q10}(t_i) \leqslant -\left(\frac{m_{RLT\_THR} - m_{RLT}(t_{t0})}{T_s \cdot r_{Q10\_MAX}}\right) - \left(\frac{T_s \sum_{t_i=t_{t0}}^{(t_{t1}-T_s)} r_{Q13\_MAX} \cdot u_{Q13}(t_i)}{T_s \cdot r_{Q10\_MAX}}\right). \tag{24}$$

Similarly, for the threshold constraint at time t_{t2} the following must hold:

$$-\sum_{t_i=t_{t0}}^{(t_{t2}-T_s)} u_{Q10}(t_i) \leqslant -\left(\frac{m_{RLT\_THR} - m_{RLT}(t_{t0})}{T_s \cdot r_{Q10\_MAX}}\right) - \left(\frac{T_s \sum_{t_i=t_{t0}}^{(t_{t2}-T_s)} r_{Q13\_MAX} \cdot u_{Q13}(t_i)}{T_s \cdot r_{Q10\_MAX}}\right). \tag{25}$$

The right hand side of the inequality in (24) represents the constant that is placed in the first element of the **b** vector in (23). The left hand side of the inequality represents the values of the row vector that would fill the first row of the **A** matrix in (23). The values in the **A** matrix will thus be either 0 or −1, depending on the switching function and the **b** vector will contain rational constants.

In a similar manner the inequalities in (26) and (27) can be used to populate the remainder of the **A** matrix and **b** vector according to the minimum and maximum inequality constraints on m_{RLT} in (17) and (18), respectively.

$$-\sum_{t_i=t_{t0}}^{(t_n)} u_{\mathrm{Q10}}(t_i) \leqslant -\left(\frac{0 - m_{\mathrm{RLT}}(t_{t0}) + (T_s \cdot r_{\mathrm{Q13\_MAX}})\sum_{t_i=t_{t0}}^{(t_n)}[u_{\mathrm{Q13}}(t_i)]}{T_s \cdot r_{\mathrm{Q10\_MAX}}}\right), \tag{26}$$

$$\sum_{t_i=t_{t0}}^{(t_n)} u_{\mathrm{Q10}}(t_i) \leqslant \left(\frac{6400 - m_{\mathrm{RLT}}(t_{t0}) + (T_s \cdot r_{\mathrm{Q13\_MAX}})\sum_{t_i=t_{t0}}^{(t_n)}[u_{\mathrm{Q13}}(t_i)]}{T_s \cdot r_{\mathrm{Q10\_MAX}}}\right). \tag{27}$$

3.9. Results

Since the operation of the Q13 conveyor belt depends entirely on the schedule of the trains arriving at RLT, the controller cannot shift any of the Q13 conveyor belt's load to other time periods. The only way to improve load shifting for this conveyor is to schedule the trains to arrive shortly after peak times. However the arrival times of the trains are actually irregular: they can arrive in any time of a day and the number of arrived trains are not constant each day. Due to this difficulty and the lack of new data on the arrival times, the following arrival times of the trains from 00h00 of 04 November 2006 to 23h00 of 08 November 8 2006 in Table 2 are taken as the arrival times of the trains in five consecutive weekdays of a week in a high-demand season (June to August) in the years 2007 and 2008. This kind of assumption is reasonable because the aim of the case study is simply to illustrate the potential savings in a high-demand season.

Therefore the minimization time interval is chosen to be from 04:00 of 4 November 2006 to 23:00 of 8 November 2006, which is 115 h and approximately 5 days. There are seven trains arrived and to be loaded during this period. Now the corresponding high-demand season electricity tariff in Table 1 is applied to compute the optimal control problem. This computed result is compared to the actual electricity consumption of the same period. When the two energy costs are compared, the same 2007 and 2008 TOU tariff is applied to both the cases with and without optimal control.

The load shifting results for the Q10 conveyor belt are analyzed below.

Table 3 is the comparison of the simulated load shifting results before and after the controller algorithm had been implemented to the base-line load shifting values. It shows that the controller decreases the total amount of energy used during peak times from 25% (without the controller) to 8% (with the controller).

The energy consumed by the Q10 conveyor during peak periods is reduced from 12 375 kW h to 3300 kW h. This means that a total of 9075 kW h of energy is shifted form the peak periods to the standard and off-peak periods during the simulation interval of approximately 115 h.

The controller also causes the total cumulative active energy cost of operating the Q10 conveyor during the simulation interval to be reduced from R12526 to R6424, which is a 49% reduction in cost for the simulation period. This is also evident in Fig. 7.

The percentage of the total operational cost incurred during peak periods is reduced from 62% to 32% when the controller is implemented.

Fig. 5 shows the load shifting effect of the Q-group controller. The bottom figure shows that the number of instances where the Q10 conveyor had to be operated during the TOU peak time peri-

Table 2
Train arrival times at RLT

Train	Date	Time
1	4 November 2006	04:00
2	4 November 2006	22:00
3	5 November 2006	07:00
4	6 November 2006	04:00
5	6 November 2006	15:00
6	7 November 2006	01:00
7	8 November 2006	03:00
8	8 November 2006	23:00

Table 3
Comparison of load shifting before and after control

	Q10 conveyor		
	Intervals switched on	Energy [kW h]	Cost [*R*]
	Before controller implementation		
Peak/total ratio	25%	25%	62%
Sum during peak hours	38	12375	7801
Sum total of all hours	151	49830	12526
	After controller implementation		
Peak/total ratio	8%	8%	32%
Sum during peak hours	10	3300	2080
Sum total of all hours	120	39600	6424

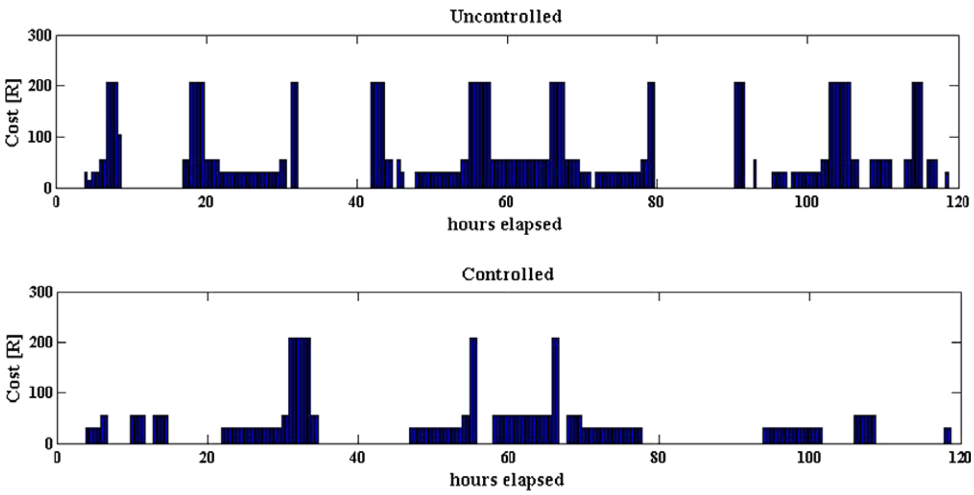

Fig. 5. Energy costs of Q10 in the controlled and uncontrolled states.

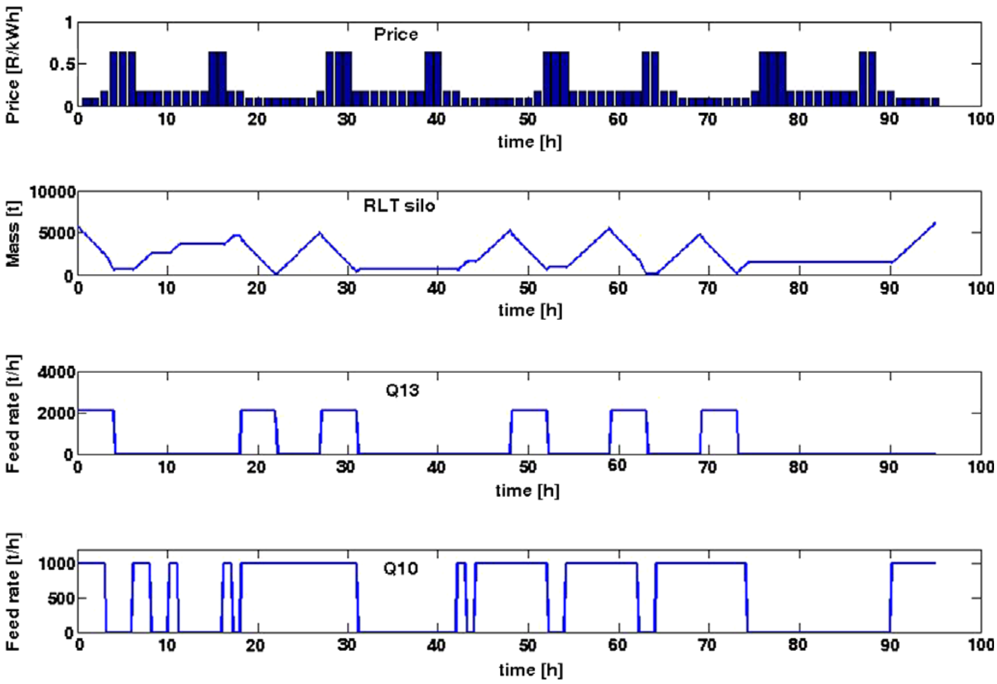

Fig. 6. RLT silo, Q10 and Q13 after optimal control.

ods was reduced from 10 to only 3 for the duration of the test, which extended over approximately 5 full days.

Fig. 6 shows the simulation results for the RLT silo, Q10 and Q13 conveyor belts after the controller had been implemented and simulated in *Simulink*. The first graph in Fig. 6 gives a reference of prices for the rest three graphs. The second graph is on the mass of the RLT silo. The third graph is on the switching status of the Q13 conveyor belt, which is determined by the arrival times of the trains. The fourth graph is the switching status of Q10 which is obtained by the solution of the optimal control problem. The second graph indicates that the mass of the RLT silo is between 0 ton and 6400 tons, thus the capacity constraints (17) and (18) are satisfied. By comparing the second and the third graph, it is obvious that the RLT silo never runs empty when the train is being loaded. The third graph shows that the Q13 conveyor belt is always operated at its nominal feed rate and its operating time is completely determined by the train arrival times in Table 2. It shows also that the arrival time of the train is uncontrolled and sometimes it

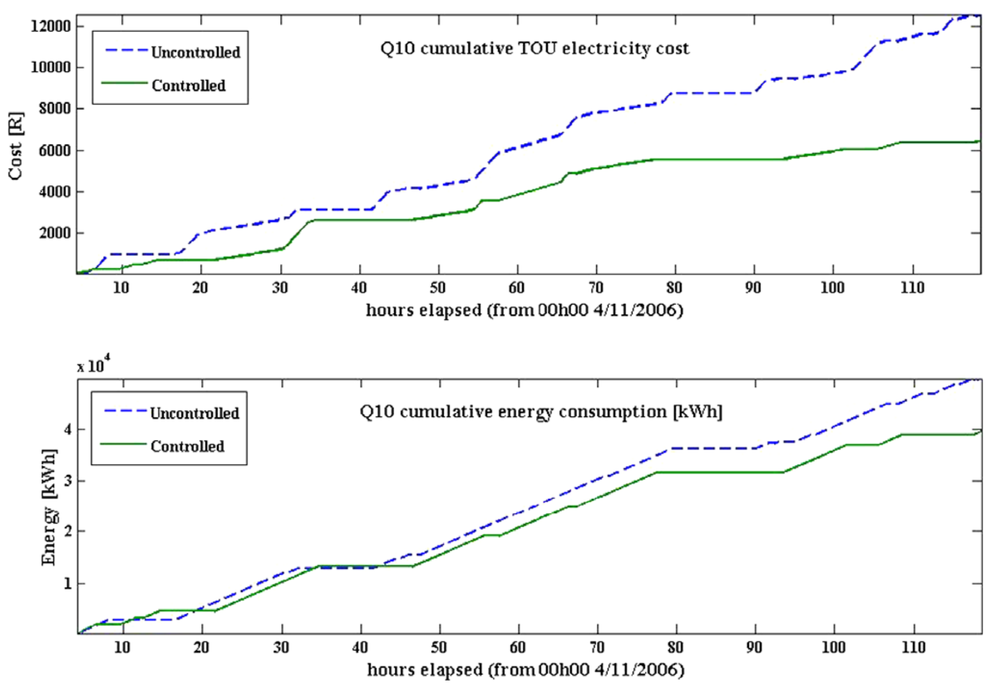

Fig. 7. Cumulative energy cost and energy of Q10 over the test period.

makes Q13 working at peak time high price period. By the comparison of the fourth graph with the first graph, it is clear that the operating time of Q10 has avoided most of the peak time high price periods, and this explains the reason of cost savings.

Fig. 7 shows the cumulative cost of active energy (top graph) and the cumulative energy consumption (bottom graph) for the Q10 conveyor belt for the duration of the test period. In both these graphs the controlled colliery is compared to the uncontrolled colliery represented by the base case test data. It is clearly shown that the controller reduces the cumulative active energy costs by up to 49%, as shown in Table 3., by the end of the test period. The bottom graph in Fig. 7 shows only a small deviation in the overall energy consumed after the controller is implemented. This is expected, since roughly the same amount of coal was transported to the RLT silo by the Q10 conveyor belt in both cases. In the uncontrolled case the data showed that the Q10 conveyor belt was not always operating at its maximum feed rate, resulting in the conveyor belt having to operate for slightly longer periods, which resulted in the slightly higher energy consumption when compared to the simulation results of the controlled conveyor.

It should be noted that the execution time of the Matlab binary integer programming algorithm can increase exponentially with an increase in the number of variables that have to be calculated. However, should such a controller be implemented in practice, this problem can be overcome by using adequately fast computer hardware readily available today.

Remark 1. The computation results on the colliery indicate a great potential in cost savings due to load shifting and energy optimization. The reason for the feasibility of load shifting is that the mine has storage silos which can be loaded during off-peak time. Since storage silos are common in mine industry, load shifting and the corresponding cost savings in terms of a TOU tariff are always possible. The same idea is also applicable to other industry such as water pumping systems. In fact, a water pumping system often consists of pumps and reservoirs, and the reservoirs have similar storage capacity, this makes the load shifting possible. We have applied this idea to study the water pumping system of a South African water supplier ([14]). The case study gives the potential cost savings for the colliery based on the high-demand season electricity tariff. In a low-demand season, the difference between peak time price and off-peak time price is significantly less than the case of high-demand seasons. Therefore the corresponding cost savings for low-demand seasons will be obviously less than that of high-demand seasons. Although the cost savings for high-demand seasons is quite promising, this colliery has stopped to implement the project because the electricity supplier did not honor DSM programmes with special incentives but load-shed unselectively all major industrial end users at the beginning of 2008 to curtail the extreme energy shortage situation. However the colliery is still considering the load shifting plan as industrial customers are required to reduce 10% of their usual energy consumption in the nationwide Power Conservation Programme in 2008.

4. Conclusion

This paper gives an optimal control model for load shifting problems and discusses its applications in the scenario of a South African colliery. The obtained optimal controller for this colliery reduces the cumulative active energy costs by up to 49% during 5 weekdays in a high-demand season. The percentage of total amount of energy used during peak time is also reduced from 25% to 8%. This case study shows the potential of using optimal control as a starting point for developing controllers to facilitate both load shifting and process optimization. Furthermore, the ease with which approximated optimal solutions can be obtained by discretizing these problems as ordinary optimization problems is very convenient for some other practical problems such as the load shifting in pumping processes and irrigation on farms, etc.

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