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ADAPTIVE PARAMETER ESTIMATION FOR AN ENERGY MODEL OF BELT CONVEYOR WITH DC MOTOR

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ABSTRACT

In practice, design parameters of a belt conveyor likely drift away from their design values by maintenance, readjustment, retrofit, abrasion, and circumstance change. For the purpose of energy optimization, these parameters should be estimated through experiments. In this paper, a new energy model of a DC motor driven belt conveyor is presented. Then, based on an adaptive observer, a parameter estimation algorithm is derived. In addition, under a persistent excitation condition, the convergence of the parameters to the desired values can also be concluded. Compared with the existing methods, our methods can be implemented by measuring only the feed rate of the belt conveyor and the angular velocity of the rotor of the DC motor.

Key Words: Parameter estimation, energy model, belt conveyor, observer, DC motor.

I. INTRODUCTION

Belt conveyors have a high transfer capacity and a long transfer distance. They are widely used to transfer bulk material in the mining, metallurgical, and coal industries. According to the report in [1], about 10% of the total maximum power demand in South Africa is to handle materials, where up to 40% of the energy cost is borne by the operational cost of the belt conveyor systems [2]. Therefore, improving energy efficiency of belt conveyors by reducing the energy consumption of material handling is of great significance.

There are four levels where energy efficiency of a belt conveyor can be improved: performance, operation, equipment, and technology [3]. It is easy to achieve higher energy efficiency by introducing highly efficient equipment [4–8]. Nevertheless, extra investment is needed to retrofit or replace the equipment. At the operation level, many methods have been proposed to improve energy efficiency for the belt conveyors [9–18]. For example, the authors in [17] proposed an optimal switching control and a variable speed drive based optimal control to reduce the energy consumption of belt conveyors. In [18], an analytical energy model was proposed. This model had four coefficients that could be estimated through algorithms, such as least square (LSQ) [19] and recursive least square (RLSQ) [20]. After obtaining the energy model, an optimization was also done at the operational level with two performance indicators, energy cost and energy consumption. In order to estimate these four coefficients, however the power of the motor P_M, feed rate T, and belt speed V should be measured. Recently, in order to estimate unknown states and unknown constant parameters, adaptive observers have made great progress [21–30].

In this paper, a new energy model of belt conveyor with a DC motor is introduced. Then, an adaptive observer is designed for the model. In order to identify the four coefficients of the energy model , the feed rate T of the belt conveyor and the angular velocity w_m of the rotor of the DC motor should be measured on-line. Then, based on the adaptive observer, a parameter estimation algorithm is derived. In addition, under a persistent excitation condition, the convergence of the parameters to the desired values can also be concluded. Simulation results show the validity of our methods.

This paper is organized as follows. The analytical energy model of belt conveyors in [18], the model of a DC motor, and the adaptive observers design are reviewed in Section II. In Section III, we present a new energy model of belt conveyors with DC motors, an adaptive observer for this model, and a parameter estimation algorithm. In Section IV, an example is given to show the validity of our new methods. Section V presents the conclusion.

II. PRELIMINARIES

2.1 An analytical energy model of belt conveyors

A typical belt conveyor is shown in Fig. 1. As in [18], an analytical energy model of the belt conveyor is given as follows:

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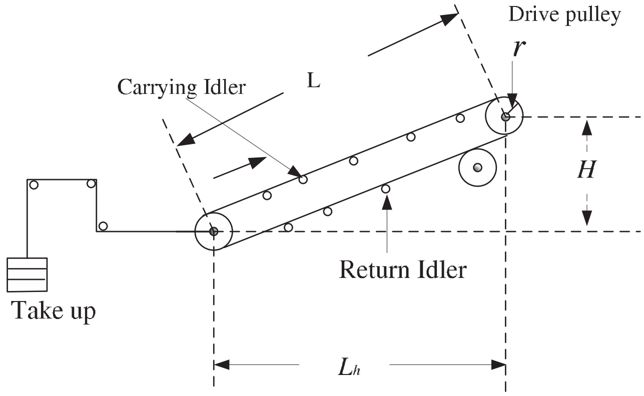

Fig. 1. Typical profile of belt conveyors.

$$P_T - \frac{V^2 T}{3.6} = \bar{\theta}_1 T^2 V + \bar{\theta}_2 V + \bar{\theta}_3 \frac{T^2}{V} + \bar{\theta}_4 T, \tag{1}$$

where P_T is the mechanical power; V denotes the the belt speed (m/s); T is the feed rate (t/h); $\bar{\theta}_1$, $\bar{\theta}_2$, $\bar{\theta}_3$, and $\bar{\theta}_4$ are four parameters. In practice, the four parameters often drift away through maintenance, readjustment, retrofit, abrasion, and circumstance change. For the purpose of energy optimization, these four parameters are estimated by both off-line and on-line parameter estimation schemes, based on P_T, V, and T measured on-line and off-line, respectively, in [18].

2.2 A DC motor

The dynamics of a DC motor are given by [31]:

$$\begin{cases} J_m \dfrac{d\omega_m}{dt} + b_m \omega_m = k_p i_f i_a - T_L, \\ L_f \dfrac{di_f}{dt} + R_f i_f = e_f, \\ L_a \dfrac{di_a}{dt} + R_a i_a = e_a - k_c i_f \omega_m, \end{cases} \tag{2}$$

where J_m denotes the mass moment of inertia of the motor, ω_m is its angular velocity, k_p is the torque constant, b_m is the damping coefficient, T_L is the presence of some external load, L_a and L_f are its inductances, R_a and R_f are its resistances, i_a and i_f are its currents, k_c is a proportional constant to the flux and the angular velocity of the motor, and e_a and e_f are two separate potentials are used to power the armature and field, respectively. The corresponding circuit is shown in Fig. 2.

2.3 Adaptive observers

Adaptive observers can be used to estimate unknown parameters. Now, let us review the adaptive observers' design. Consider the following system in adaptive observer form [26]:

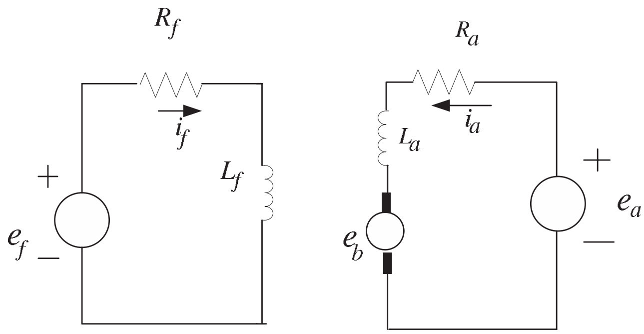

Fig. 2. Circuit diagram of separately excited DC motor.

$$\begin{cases} \dot{z} = \bar{A}_0 z + \gamma(y, u) + \bar{b}\beta^T(y, u, t)\theta, \\ y = \bar{C}_0 z, \end{cases} \tag{3}$$

where $z \in \mathcal{R}^n$, $y \in \mathcal{R}$, $u \in \mathcal{R}^m$, $\theta \in \mathcal{R}^p$, and γ (y, u) are smooth function mapping $\mathcal{R} \times \mathcal{R}^m \to \mathcal{R}^n$; $\bar{A}_0 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}$; $\bar{b} = [\bar{b}_1, \cdots, \bar{b}_n]^T \in \mathcal{R}^n$ is given such that the polynomial $\bar{b}_1 s^{n-1} + \cdots + \bar{b}_n$ is Hurwitz and $\bar{b}_1 > 0$; and $\bar{C}_0 = [1 \quad 0 \quad \cdots \quad 0]$, $\beta^T(y, u, t)\theta = \sum_{i=1}^{p} \theta_i \beta_i(y, u, t)$, $\beta_i(y, u, t)$ ($i = 1, \ldots, p$) are continuous functions and uniformly bounded for every (y, u) bounded. A global adaptive observer with parameter convergence is designed in [26] as follows:

$$\begin{cases} \dot{\hat{z}} = \bar{A}_0 \hat{z} + \bar{K}\bar{C}_0 e + \gamma(y, u) + \bar{b}\beta^T(y, u, t)\hat{\theta}, \\ \dot{\hat{\theta}} = \Gamma\beta(y, u, t)\bar{C}_0 e, \end{cases}$$

where Γ is any symmetric positive definite matrix, $e = z - \hat{z}$, $\bar{K} = \frac{1}{\bar{b}_1}(\bar{A}_0 \bar{b} + \lambda \bar{b})$ with λ being an arbitrary positive real. Since $(\bar{A}_0, \bar{b}, \bar{C}_0)$ satisfies the strictly positive real condition, then, for any symmetric definite matrix $\bar{Q}$, there exist a symmetric positive definite matrix $\bar{P}$, a positive real $\bar{d}$ such that [26]:

$$(\bar{A}_0 + \bar{K}\bar{C}_0)^T \bar{P} + \bar{P}(\bar{A}_0 + \bar{K}\bar{C}_0) < -\bar{d}\bar{Q}, \quad \bar{P}\bar{b} = \bar{C}^T.$$

III. A NEW ENERGY MODEL

In this section, we make the following assumption: the belt is non-slip. T and ω_m are measured on-line. We shall also assume a constant potential e_f and assume that the circuit is operating at steady state so that $e_f = i_f R_f$, yielding a constant field current i_f. Therefore, we have:

$$\begin{cases} J_m \dfrac{d\omega_m}{dt} + b_m\omega_m = k_T i_a - T_L, \\ L_a \dfrac{di_a}{dt} + R_a i_a = e_a - k_b\omega_m, \end{cases} \tag{4}$$

where $k_T = k_p i_f$, $k_b = k_c i_f$.

When the DC motor is employed to drive the conveyor belt, then,

$$T_L = F_U r, \quad V = 2\pi r w_m, \tag{5}$$

where r is the radius of the rotor and F_u is the peripheral driving force of the belt conveyor and can be calculated by the following equation [18]:

$$\begin{aligned} F_U = {} & \frac{VT}{3.6} + \frac{T^2}{6.48\rho b_1^2} \\ & + \left\{ gfQ\left[L\cos\delta + L(1-\cos\delta)\left(1 - \frac{2Q_B}{Q}\right)\right] + k_3 + C_{Ft} \right\} \\ & + k_1\frac{T^2}{V^2} + \left(\frac{gL\sin\delta + gfL\cos\delta}{3.6} + k_2 \right)\frac{T}{V}, \end{aligned} \tag{6}$$

where f is the artificial friction factor; L is the center-to-center distance (m); $Q = Q_{RO} + Q_{RU} + 2Q_B$; Q_{RO} is the unit mass of the rotating parts of carrying idler rollers (kg/m), and Q_B is the unit mass of the belt of rotating parts of the return idler rollers (kg/m), Q_B is the unit mass of the belt (kg/m); δ is the inclination angle (^{o}); ρ is the bulk density of material (kg/m^3); b_1 is the width between the skirt boards (m); k_1, k_2, and k_3 are constant coefficients that relate to the structural parameters of the belt conveyor; and C_{Ft} is a constant. From (5) and (6), we have:

$$\begin{aligned} T_L - \frac{2\pi r^2 T w_m}{3.6} = {} & \frac{rT^2}{6.48\rho b_1^2} \\ & + r\left\{ gfQ\left[L\cos\delta + L(1-\cos\delta)\left(1 - \frac{2Q_B}{Q}\right)\right] + k_3 + C_{Ft} \right\} \\ & + k_1 r\frac{T^2}{4\pi^2 r^2 w_m^2} + \left(\frac{gL\sin\delta + gfL\cos\delta}{3.6} + k_2 \right)\frac{T}{2\pi r w_m}. \end{aligned} \tag{7}$$

Let $\theta_1 = \dfrac{r}{6.48\rho b_1^2}$, $\theta_2 = r\left\{ gfQ\left[L\cos\delta + L(1-\cos\delta)\left(1 - \dfrac{2Q_B}{Q}\right)\right] + k_3 + C_{Ft} \right\}$, $\theta_3 = \dfrac{k_1 r}{4\pi^2 r^2}$, $\theta_4 = \left(\dfrac{gL\sin\delta}{3.6} \dfrac{gfL\cos\delta}{3.6} + k_2 \right)\dfrac{1}{2\pi r}$, $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$, and

$$\psi(T, w_m) = \left[T^2 \quad 1 \quad \frac{T^2}{w_m^2} \quad \frac{T}{w_m} \right]^T.$$

Then, a new energy model of the belt conveyor with a DC motor is given as follows:

$$\begin{cases} J_m \dfrac{d\omega_m}{dt} + b_m\omega_m = k_T i_a - \dfrac{2\pi r^2 T w_m}{3.6} - \psi^T(T, w_m)\theta, \\ L_a \dfrac{di_a}{dt} + R_a i_a = e_a - k_b\omega_m, \end{cases} \tag{8}$$

or

$$\dot{z} = \tilde{A}z + \tilde{B}u + \tilde{b}\frac{\psi^T(T, w_m)}{J_m}\theta, \tag{9}$$

where $\tilde{A} = \begin{bmatrix} -\dfrac{b_m}{J_m} & \dfrac{k_T}{J_m} \\ -\dfrac{k_b}{L_a} & -\dfrac{R_a}{L_a} \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\tilde{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $z = \begin{bmatrix} w_m \\ i_a \end{bmatrix}$, $u = \begin{bmatrix} -\dfrac{2\pi r^2}{3.6J_m} T w_m \\ \dfrac{1}{L_a} e_a \end{bmatrix}$.

For the conveyors with permanent instruments for T and w_m, the real-time data can be accessed through the supervisory control and data acquisition system (SCADA). Therefore, let w_m be the output of the system (8), *i.e.*,

$$y = w_m = Cz, \tag{10}$$

where $C = [1\ 0]$. We obtain that

$$\operatorname{rank}\begin{bmatrix} C \\ C\tilde{A} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 1 & 0 \\ -\dfrac{b_m}{J_m} & \dfrac{k_T}{J_m} \end{bmatrix} = 2,$$

which means that ($\tilde{A}$, C) is observable. Then, the coordinate transformation

$$\tilde{z} = Q_1 z = \begin{bmatrix} 1 & 0 \\ -\dfrac{b_m}{J_m} & \dfrac{k_T}{J_m} \end{bmatrix} z$$

can transform (9), (10) into the following canonical form [32]:

$$\begin{cases} \dot{\tilde{z}}_1 = \tilde{z}_2 + a_1 y - \dfrac{2\pi r^2}{3.6J_m} T w_m + \dfrac{1}{J_m}\psi^T(T, y)\theta, \\ \dot{\tilde{z}}_2 = a_2 y - \dfrac{2R_a \pi r^2}{3.6L_a J_m} T w_m + \dfrac{k_T}{J_m L_a} e_a + \dfrac{R_a}{L_a J_m}\psi^T(T, y)\theta, \\ y = \tilde{z}_1, \end{cases} \tag{11}$$

where $a_1 = -\dfrac{b_m}{J_m} - \dfrac{R_a}{L_a}$, $a_2 = -\dfrac{R_a b_m}{L_a J_m} - \dfrac{k_T k_b}{J_m L_a}$. Let $b = [1,\ b_2]^T$ be a vector such that

$s + b_2$

is a Hurwitz polynomial. Consider the following filter transformation [26]:

$$\begin{cases} x_1 = \tilde{z}_1, \\ x_2 = \tilde{z}_2 - \sum_{i=1}^{4} \eta_i \theta_i, \end{cases} \tag{12}$$

where

$$\dot{\eta}_i = -b_2\eta_i - \frac{b_2}{J_m}\psi_i(T, \omega_m) + \frac{R_a}{L_a J_m}\psi_i(T, \omega_m), \tag{13}$$

where $\eta_i(t_0) = 0$, $\psi_i(T, \omega_m)$ is the ith component of $\psi(T, \omega_m)$ ($i = 1, \dots, 4$). This transforms (11) into to the following system:

$$\begin{cases} \dot{x}_1 = x_2 + a_1\omega_m - \dfrac{2\pi r^2}{3.6 J_m} T w_m + \left(\dfrac{1}{J_m}\psi(T, \omega_m) + \eta\right)^T \theta, \\ \dot{x}_2 = a_2\omega_m - \dfrac{2R_a \pi r^2}{3.6 L_a J_m} T w_m + \dfrac{k_T}{J_m L_a} e_a \\ \qquad + b_2\left(\dfrac{1}{J_m}\psi(T, \omega_m) + \eta\right)^T \theta, \end{cases} \tag{14}$$

where $\eta = [\eta_1, \eta_2, \eta_3, \eta_4]^T$. For the system (14) in adaptive observer form, an adaptive observer can be designed as follows [26]:

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + k_1 e_1 + a_1\omega_m - \dfrac{2\pi r^2}{3.6 J_m} T w_m + \left(\dfrac{1}{J_m}\psi(T, \omega_m) + \eta\right)^T \hat{\theta}, \\ \dot{\hat{x}}_2 = k_2 e_1 + a_2\omega_m - \dfrac{2R_a \pi r^2}{3.6 L_a J_m} T w_m + \dfrac{k_T}{J_m L_a} e_a \\ \qquad + b_2\left(\dfrac{1}{J_m}\psi(T, \omega_m) + \eta\right)^T \hat{\theta}, \end{cases} \tag{15}$$

and

$$\dot{\hat{\theta}} = \Gamma\left(\frac{1}{J_m}\psi(T, \omega_m) + \eta\right) e_1, \tag{16}$$

where $e_1 = x_1 - \hat{x}_1$, and $k_1 = b_2 + \lambda$, $k_2 = b_2\lambda$, $\lambda > 0$, Γ is any symmetric positive matrix. From (14) and (15), (16), it follows that

$$\begin{cases} \dot{e}_1 = e_2 - k_1 e_1 + \left(\dfrac{1}{J_m}\psi(T, \omega_m) + \eta\right)^T \tilde{\theta}, \\ \dot{e}_2 = -k_2 e_1 + b_2\left(\dfrac{1}{J_m}\psi(T, \omega_m) + \eta\right)^T \tilde{\theta}, \end{cases} \tag{17}$$

and

$$\dot{\tilde{\theta}} = -\Gamma\left(\frac{1}{J_m}\psi(T, \omega_m) + \eta\right) e_1, \tag{18}$$

where $e = x - \hat{x}$, $\tilde{\theta} = \theta - \hat{\theta}$.

Let us now state and prove the main results of this paper.

Theorem 1. For the energy model of belt conveyor with a DC motor (8), there exists a filter transformation (12), (13) to transform (8) into (14). Moreover, for the system (15), (16), if k_1 and k_2 are selected such that (A, b, C) satisfies the strictly positive real condition, where $A = A_0 + KC$, $K = [k_1, k_2]^T$, then $\left\|\hat{\theta}(t) - \theta\right\|$ is uniformly bounded.

Proof. Using the same method as in [26], we can obtain the result.

In order to ensure that $\hat{\theta}(t)$ converges to the desired value, the following result is needed.

Lemma 1. Consider the following system:

$$\begin{cases} \dot{\eta} = A_1\eta + B_1\dfrac{\psi(T, \omega_m)}{J_m}, \\ \bar{y} = C_1\eta + D_1\dfrac{\psi(T, \omega_m)}{J_m}, \end{cases} \tag{19}$$

where $A_1 = diag\{-b_2, -b_2, -b_2, -b_2\}$, $B_1 = diag\left\{-b_2 + \dfrac{R_a}{L_a}, -b_2 + \dfrac{R_a}{L_a}, -b_2 + \dfrac{R_a}{L_a}, -b_2 + \dfrac{R_a}{L_a}, -b_2 + \dfrac{R_a}{L_a}\right\}$, $C_1 = I$, $D_1 = I$. If there exist $T_0 > 0$, $k_p > 0$ such that $\psi(T, \omega_m)$ satisfies the following persistence excitation condition

$$\int_t^{t+T_0} \psi(T(\tau), \omega_m(\tau))\psi^T(T(\tau), \omega_m(\tau))d\tau > k_p I, \tag{20}$$

then, there exists $k_p' > 0$ such that

$$\int_t^{t+T_0} \bar{y}(\tau)\bar{y}^T(\tau)d\tau > k_p' I. \tag{21}$$

Proof. It follows from (19) that

$$C_1(SI - A_1)B_1 + D_1 = \left(\frac{s + \dfrac{R_a}{L_a}}{s + b_2}\right) I,$$

which implies that the system (19) is stable and minimal phase. By lemma 2.6.7 in [33], we obtain the result.

Theorem 2. For the energy model of the belt conveyor with a DC motor (8), there exists a filter transformation (12), (13) to transform (8) into (14). Moreover, for the system (15), (16), if k_1 and k_2 are selected such that (A, b, C) satisfies the strictly positive real condition, and there exist $T_0 > 0$ and $k_p > 0$ such that the condition (20) holds, then, we have $\lim_{t\to\infty}\|\theta - \hat{\theta}(t)\| = 0$.

Proof. From (13), we have:

$$\eta_i(t) = e^{-b_2 t}\int_{t_0}^{t} e^{b_2\tau}\left(-\frac{b_2}{J_m} + \frac{R_a}{L_a J_m}\right)\psi_i(T(\tau), \omega_m(\tau))d\tau.$$

One can see that $\psi(T, \omega_m)$ is bounded for every (T, w_m) bounded. Therefore, $\eta_i(t)$ $(i = 1, 2, 3, 4)$ are bounded for every (T, w_m), which implies that $\left(\frac{1}{J_m}\psi(T, \omega_m) + \eta\right)$ is bounded for every (T, w_m) bounded. Along the trajectory of the system (17), (18), calculate the derivative of the following Lyapunov function:

$$V(e) = e^T Pe + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}. \tag{22}$$

We have

$$\left.\frac{dV(e)}{dt}\right|_{(17),(18)} = e^T(A^T P + PA)e + 2e^T Pb\left(\frac{1}{J_m}\psi(T, y) + \eta\right)^T\tilde{\theta} - \tilde{\theta}^T\left(\frac{1}{J_m}\psi(T, y) + \eta\right)e_1 < -de^T Qe.$$

Thus, $\|e(t)\|$ and $\|\tilde{\theta}(t)\|$ are uniformly bounded for any $t > t_0$. Moreover, $\left(\frac{1}{J_m}\psi(T, y) + \eta\right)$ is uniformly bounded, then, $\|\dot{e}(t)\|$ is also uniformly bounded. Since $V(e)$ is a uniformly bounded non-increasing function

$$\lim_{t\to\infty}\int_{t_0}^{t} e^T(\tau)Qe(\tau)d\tau < V(t_0) - V(\infty) < V(\infty),$$

By Barbalat's Lemma [26], we have:

$$\lim_{t\to\infty}\|e(t)\| = 0. \tag{23}$$

From (18) and (23), we have:

$$\lim_{t\to\infty}\dot{\tilde{\theta}}(t) = 0.$$

Then, there exits a constant $\tilde{\theta}^*$ such that

$$\lim_{t\to\infty}\tilde{\theta}(t) = \tilde{\theta}^*.$$

Therefore, for any $\varepsilon > 0$, there exists $t_1 > 0$ such that

$$\|\tilde{\theta}(t) - \tilde{\theta}^*\| < \varepsilon, \forall t > t_1. \tag{24}$$

Now, we will prove that $\tilde{\theta}^* = 0$ by contradiction. Assume that $\tilde{\theta}^* \neq 0$.

Consider the following function

$$\varphi(\tilde{\theta}(t), t) = \frac{1}{2}[\tilde{\theta}^T(t + T_0)\Gamma^{-1}\tilde{\theta}(t + T_0) - \tilde{\theta}^T(t)\Gamma^{-1}\tilde{\theta}(t)],$$

which is bounded. The time derivative of $\varphi(\tilde{\theta}(t), t)$ is given as

$$\begin{aligned}
\frac{d\varphi(\tilde{\theta}(t), t)}{dt} &= \tilde{\theta}^T(t + T_0)\Gamma^{-1}\dot{\tilde{\theta}}(t + T_0) - \tilde{\theta}^T(t)\Gamma^{-1}\dot{\tilde{\theta}}(t) \\
&= \int_t^{t+T_0}\frac{d}{d\tau}(\tilde{\theta}^T(\tau)\Gamma^{-1}\dot{\tilde{\theta}}(\tau))d\tau \\
&= -\int_t^{t+T_0}\frac{d}{d\tau}(\tilde{\theta}^T(\tau)\bar{y}(\tau)e_1(\tau))d\tau \\
&= \int_t^{t+T_0} e_1(\tau)\bar{y}^T(\tau)\Gamma\bar{y}(\tau)e_1(\tau)d\tau \\
&\quad - \int_t^{t+T_0}\tilde{\theta}^T(\tau)\dot{\bar{y}}(\tau)e_1(\tau)d\tau \\
&\quad - \int_t^{t+T_0}\tilde{\theta}^T(\tau)\bar{y}(\tau)(e_2(\tau) - k_1 e_1(\tau))d\tau \\
&\quad - \int_t^{t+T_0}\tilde{\theta}^T(\tau)\bar{y}(\tau)\bar{y}^T(\tau))\tilde{\theta}(\tau)d\tau.
\end{aligned}$$

Note that $\dot{\bar{y}}(\tau) = \left(\frac{1}{J_m}\dot{\psi}(T(\tau), y(\tau)) + \dot{\eta}(\tau)\right)$ is uniformly bounded for every bounded (T, w_m), and, if $\lim_{t\to\infty} e(t) = 0$ and (21), (24) hold, then, there exists $M > 0$ such that, when $t > t_1$, we have

$$\begin{aligned}
\frac{d\varphi(\tilde{\theta}(t), t)}{dt} &< M\int_t^{t+T_0}(e_1^2(\tau) + e_2(\tau)^2)d\tau \\
&\quad - \int_t^{t+T_0}\tilde{\theta}^{*T}\bar{y}(\tau)\bar{y}^T(\tau)\tilde{\theta}^* d\tau \\
&\quad - 2\int_t^{t+T_0}\tilde{\theta}^{*T}\bar{y}(\tau)\bar{y}^T(\tau)(\tilde{\theta}(\tau) - \tilde{\theta}^*)d\tau \\
&\quad - \int_t^{t+T_0}(\tilde{\theta}(\tau) - \tilde{\theta}^*)^T\bar{y}(\tau)\bar{y}^T(\tau)(\tilde{\theta}(\tau) - \tilde{\theta}^*)d\tau \\
&< M\int_t^{t+T}(e_1^2(\tau) + e_2(\tau)^2)d\tau \\
&\quad - \frac{k_p'}{2}\tilde{\theta}^{*T}\tilde{\theta}^* < -\frac{k_p'}{4}\tilde{\theta}^{*T}\tilde{\theta}^*, \forall t > t_1,
\end{aligned}$$

which contradicts the boundedness of $\varphi(\tilde{\theta}(t), t)$. Therefore, $\lim_{t\to\infty}\tilde{\theta}(t) = 0$. The proof is completed.

IV. SIMULATION RESULTS

We test the proposed adaptive parameter estimation (13), (15), (16) by simulation with parameters $b_2 = 4.0$, $\lambda = 3$, and $\Gamma = \text{diag}\{180, 180, 180, 180\}$ and with four coefficients $\theta_1 = 1$, $\theta_2 = 0.3$, $\theta_3 = 3.5$, and $\theta_4 = 2.1$, for a DC motor, whose parameters are: $k_T = 0.1$, $k_b = 0.1$, $b_m = 0.4$, $J_m = 0.05$ Kgm^2, $R_a = 15$ Ohm, $L_a = 1.0$ H, $r = 0.01$ m, and $e_a = 380$ V. The initial conditions of (14), (15), and (16) are given by (0.01, 0.7), (0.8, 0.2), and (0.1, 0.1, 0.1, 0.1), respectively. It should be noted that it is difficult to check that inequality (20) holds. In practice, if the feed rate T does not change much, a complete determination of all of the parameters is impossible. In order to estimate all of the parameters, one should sufficiently disturb the feed rate T during the period of estimation. In this example, we choose the feed rate $T = 0.09(8 + 5\sin(10t + 1) + 2\cos(-5t + 2) + \sin(20t)) + 0.3(8.6 + 2\cos(-5t + 2) + \sin(15t + 0.4) + \sin(20t) + 4\sin(t))\,|\,(\sin(4t + 0.5))\,|$ kg/s $(0 \le t \le 60s)$. The simulation results are shown in Fig. 3–Fig. 6.

To test the algorithm against measurement noise, a band limited white noise is added to y_1. The results are demonstrated in Fig. 7–Fig. 10.

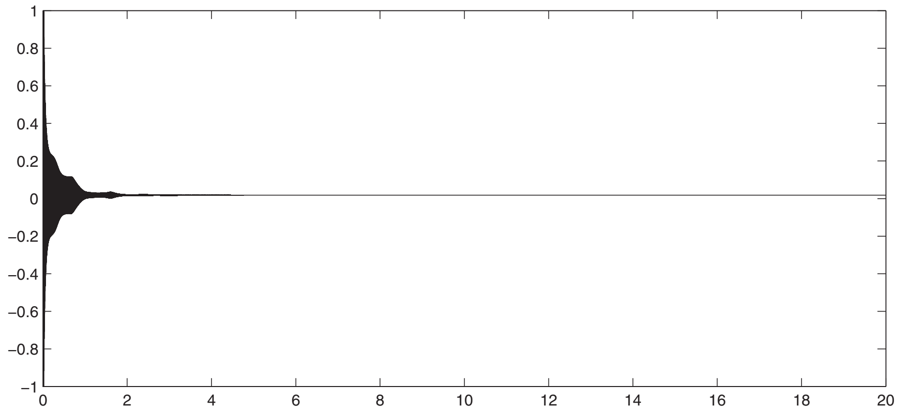

Fig. 3. Trajectory of $\tilde{\theta}_1$.

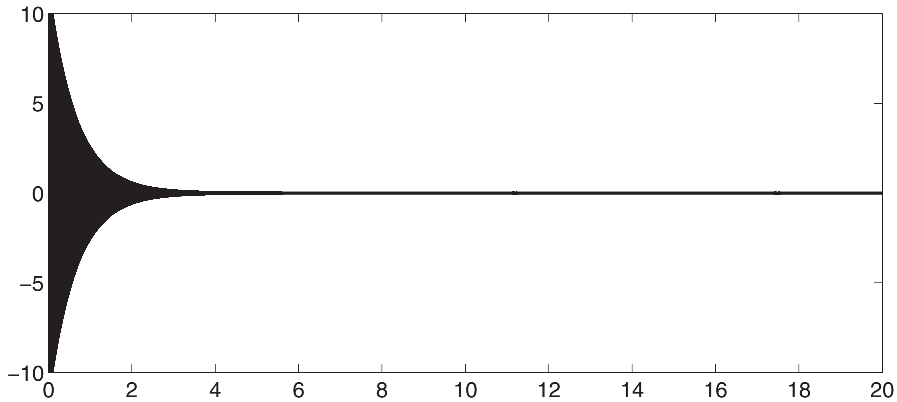

Fig. 4. Trajectory of $\tilde{\theta}_2$.

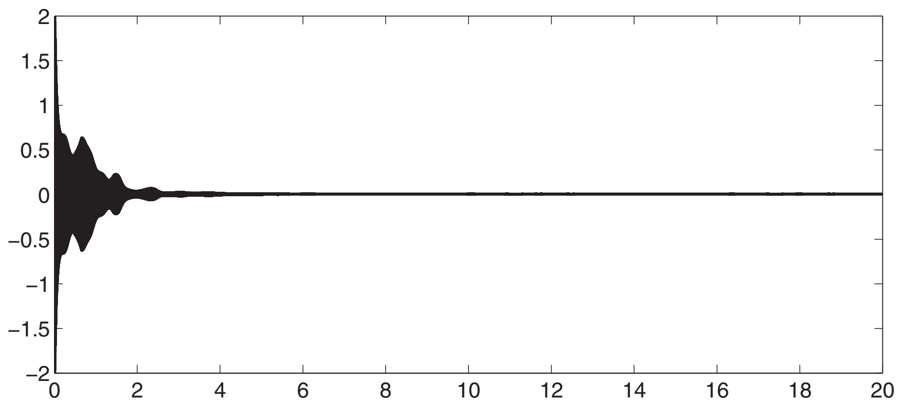

Fig. 5. Trajectory of $\tilde{\theta}_3$.

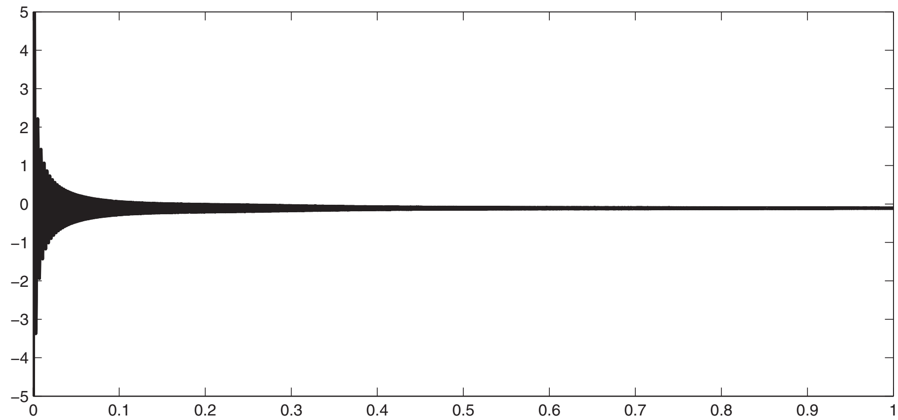

Fig. 6. Trajectory of $\tilde{\theta}_4$.

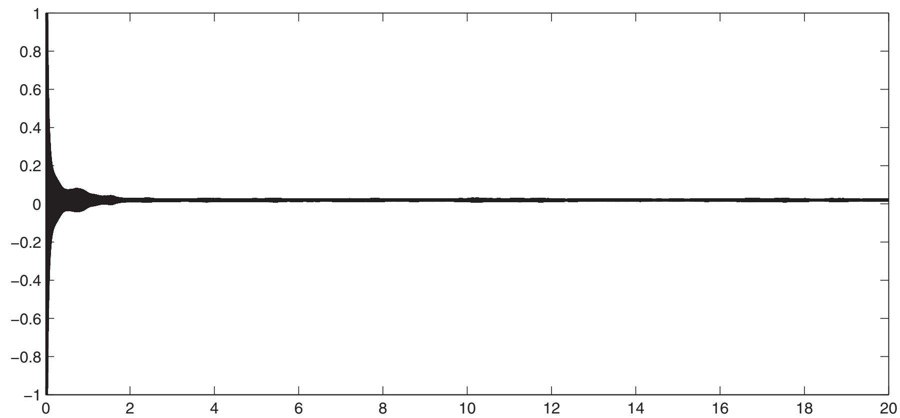

Fig. 7. Trajectory of $\tilde{\theta}_1$ with band limited white noise.

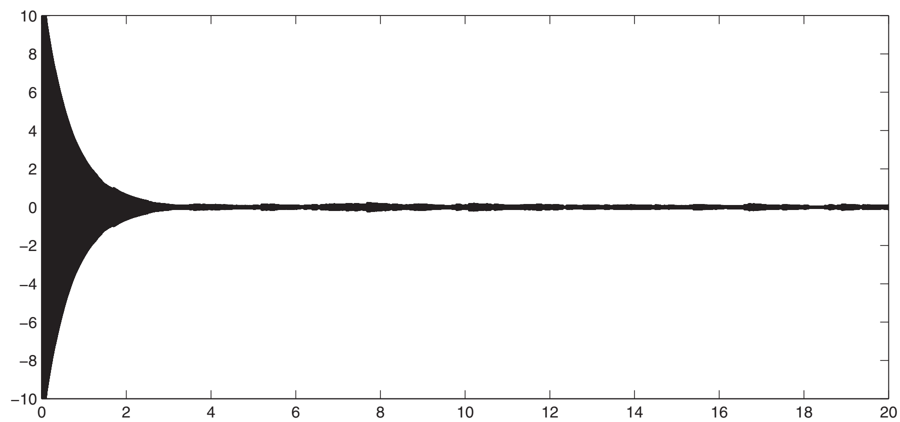

Fig. 8. Trajectory of $\tilde{\theta}_2$ with band limited white noise.

Practically, the parameters may drift away during the conveyor belt operation. For example, $\theta_1 = 1.2$, $\theta_2 = 0.3$, $\theta_3 = 3.5$, $\theta_4 = 2.3$, using the same initial conditions and the feed rate T, we implement the adaptive identifer for 30s. Figs 11–14 show the simulation results.

V. CONCLUSION

In this paper, a new energy model of a conveyor belt driven by a DC motor was presented, which lumped all the parameters into four coefficients. Then, an

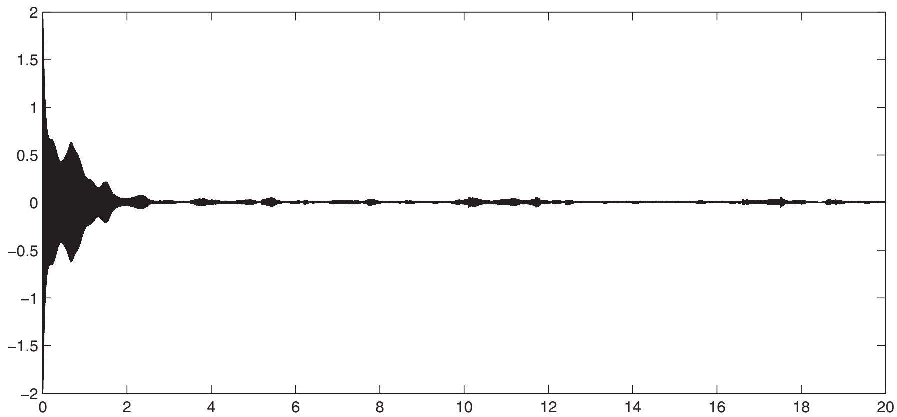

Fig. 9. Trajectory of $\tilde{\theta}_3$ with band limited white noise.

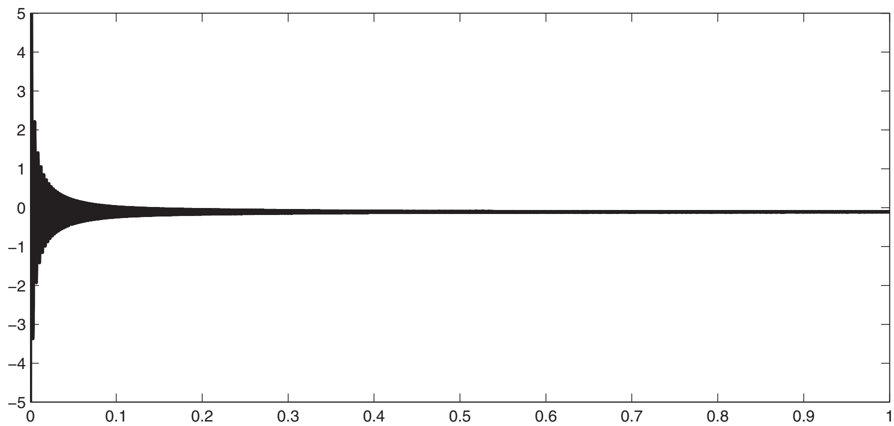

Fig. 10. Trajectory of $\tilde{\theta}_4$ with band limited white noise.

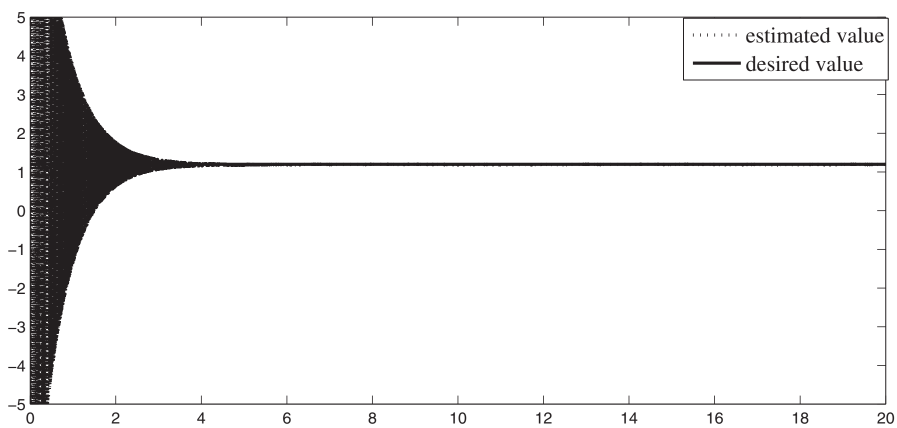

Fig. 11. Trajectories of θ_1 and $\hat{\theta}_1$.

adaptive observer was designed to estimate the unknown parameters. In addition, under a persistent excitation condition, the convergence of the parameters to the desired values could also be concluded. Compared with the existing methods, our methods could be implemented by measuring only the feed rate of the conveyor belt and the angular velocity of the rotor of the DC motor.

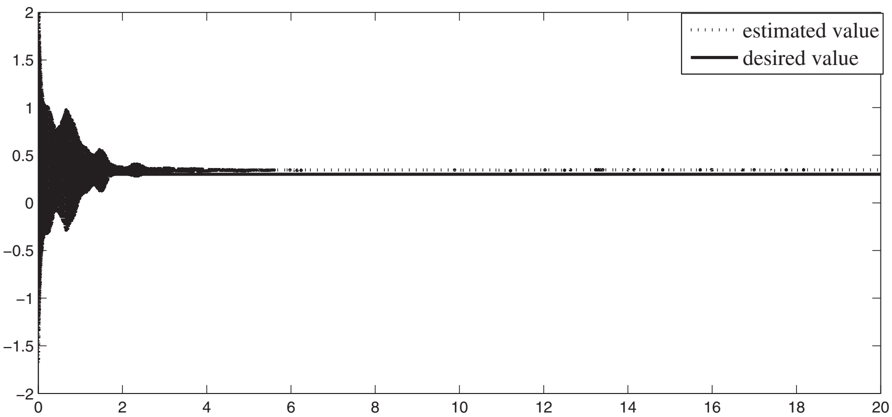

Fig. 12. Trajectories of θ_2 and $\hat{\theta}_2$.

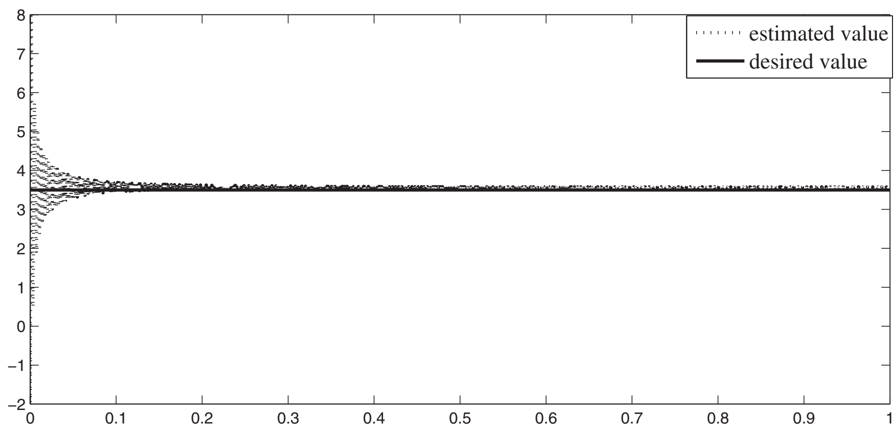

Fig. 13. Trajectories of θ_3 and $\hat{\theta}_3$.

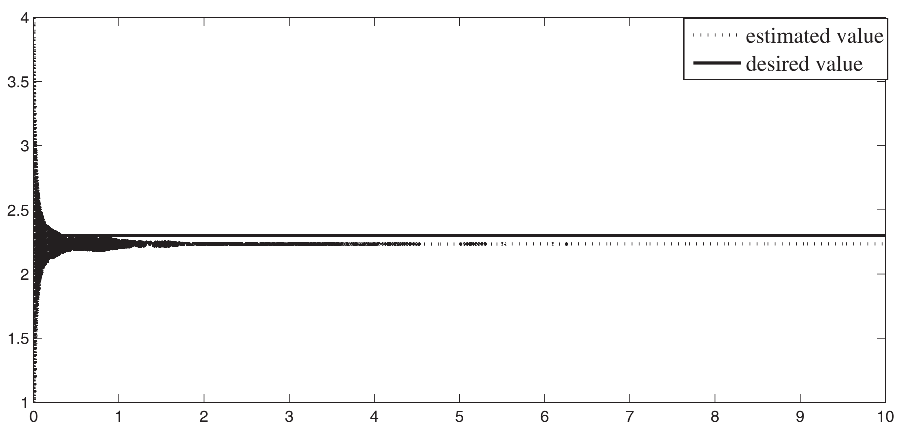

Fig. 14. Trajectories of θ_4 and $\hat{\theta}_4$.

REFERENCES

1. Marais, J., E. Mathews, and R. Pelzer, “Analysing DSM opportunities on mine conveyor systems,” Industrial and Commercial Use of Energy Conference, Cape Town, South Africa, pp. 28–30 (2008).
2. Hager, M. and A. Hintz, “The energy saving design of belts for long conveyor systems,” *Bulk Solids Handl*, Vol. 13, No. 4, pp. 749–758 (1993).
3. Xia, X. and J. Zhang, “Control systems and energy efficiency from the POET perspective,” IFAC Conference on Control Methodologies and Technology for Energy Efficiency, Vilamoura, Portugal, pp. 29–31 (2010).
4. Reicks, A. V., “Belt conveyor idler roll behavours,” In M. A. Alspaugh (Ed.) *Bulk material handling by conveyor belt 7*, Littleton, Colorado, pp. 35–40 (2008).
5. Staniak, K. and J. França Jr, “Energy saving in long distance conveyors-Novel idler technology,” In W. T. Hennies, L. A. Ayres da Silva, and A. P. Chaves (Eds) *Mine planning and equipment selection*, Balkema, Rotterdam, Netherlands, pp. 479–486 (1996).
6. Tapp, A. G., “Energy saving troughing idler technology,” *Bulk Solids Handl*, Vol. 20, No. 4, pp. 437–449 (2000).
7. Jansen, M., “The development of energy-optimized conveyor belts-a joint project of the conveyor belt group of ContiTech AG and RWE power AG,” *World Min*, Vol. 60, No. 2, pp. 83–87 (2008).
8. De Almeida, A. T., P. Fonseca, and P. Bertoldi, “Energy-efficient motor systems in the industrial and in the services sectors in the European Union: characterisation, potentials, barriers and policies,” *Energy*, Vol. 28, pp. 673–690 (2003).
9. Middelberg, A., J. Zhang, and X. Xia, “An optimal control model for load shifting—with application in the energy management of a colliery,” *Appl Energy*, Vol. 86, pp. 1266–1273 (2009).
10. Marx, D. J. L. and J. E. Calmeyer, “An integrated conveyor energy model methodology,” *Trans South African Inst Electr Eng*, Vol. 95, pp. 256–264 (2004).
11. Marx, D. J. L. and J. E. Calmeyer, “A case study of an integrated conveyor belt model for mining industry,” In IEEE AFRICON 2004; Gaborone, Botswana, pp. 15–17 (2004).
12. Lauhoff, H., “Speed control on belt conveyors-dose is really save engery?” *Bulk Solids Handl*, Vol. 26, No. 6, pp. 68–77 (2005).
13. Van Den Heuvel, B., “Investigations dealing with variable-speed drives of belt conveyor systems,” *World Min*, Vol. 58, No. 2, pp. 105–113 (2006).
14. Daubs, W., S. Köerber, and N. Becher, “Raw coal loading and belt conveyor system at Nochten opencast mine—a new conveying and loading system based on drives controlled and adjusted by frequency converter,” *Braunkohle Surf Min*, Vol. 50, No. 2, pp. 117–130 (1998).
15. Köhler, U., M. Sykulla, and V. Wuschek, “Variable-speed belt conveyors gaining in importance,” *Braunkohle Surf Min*, Vol. 53, No. 1, pp. 65–72 (2001).
16. Koopmann, K., D. Weber, C. Keller, W. Daus, and S. Köber, “Conveying device for opencast mines,” *US Patent*, 6 209 711 B1 (2001).
17. Zhang, S. and X. Xia, “Optimal control of operation efficiency of belt conveyor systems,” *Appl. Energy*, Vol. 87, No. 6, pp. 1929–1937 (2010).
18. Zhang, S. and X. Xia, “Modeling and energy efficiency optimization of belt conveyors,” *Appl. Energy*, Vol. 88, No. 9, pp. 3061–3071 (2011).
19. Ljung, L., *System Identification Theory For The User*, 2nd ed. Prentice-Hall Inc., New Jersey (1999).
20. Koubaa, Y., “Recursive identification of induction motor parameters,” *Simul Modell Pract Theory*, Vol. 12, pp. 363–381 (2004).
21. Lüders, G. and K. S. Narendra, “An adaptive observer and identifier for a linear system,” *IEEE Trans. Autom. Control*, Vol. 18, pp. 496–499 (1973).
22. Kreisselmeier, G., “Adaptive observers with exponential rate of convergence,” *IEEE Trans. Autom. Control*, Vol. 22, pp. 2–8 (1977).
23. Narendra, K. S. and A. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall Inc., New Jersey (1989).
24. Zhang, Q., “Adaptive observers for mimo linear time-varying systems,” *IEEE Trans. Autom. Control*, Vol. 47, pp. 1202–1204 (2002).
25. Bastin, G. and M. Gevers, “Stable adaptive observers for nonlinear time varying systems,” *IEEE Trans. Autom. Control*, Vol. 33, pp. 650–658 (1988).
26. Marino, R. and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive and Robust*, Prentice-Hall Inc., New Jersey (1995).
27. Marino, R. and P. Tomei, “Adaptive observers with arbitrary exponential rate of convergence for nonlinear systems,” *IEEE Trans. Autom. Control*, Vol. 40, pp. 1300–1304 (1995).
28. Xu, A. and Q. Zhang, “Nonlinear system fault diagnosis based on adaptive estimation,” *Automatica*, Vol. 40, 1183–1193 (2004).
29. Zhang, X. and L. Yan, “An adaptive output feedback dynamic surface control for a class of nonlinear systems with unknown backlash-like hysteresis,” *Asian J. Control*, Vol. 15, No. 2, 489–500 (2013).
30. Sang, Q. and G. Tao, “Adaptive Control of Piecewise Linear Systems with State Feedback for Output Tracking,” *Asian J. Control*, Vol. 15, No. 4, 933–943 (2013).
31. Stadler, W., *Analytical Robotics and Mechatronics*, McGraw Hill, New York (1995).

32. Tsay, Y. and L. Shieh, “Block decompositions and block modal controls, of multivariable control systems,” *Automatica*, Vol. 19, pp. 29–40 (1983).
33. Sastry, S. and M. Bodson, *Adaptive Control, Stability, Convergence, Robustness*, Prentice-Hall Inc., New Jersey (1989).

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