



# An application of model predictive control to the dynamic economic dispatch of power generation

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## ABSTRACT

Two formulations exist for the problem of the optimal power dispatch of generators with ramp rate constraints: the optimal control dynamic dispatch (OCDD) formulation based on control system models, and the dynamic economic dispatch (DED) formulation based on optimization. Both are useful for the dispatch problem over a fixed time horizon, and they were treated as equivalent formulations in literature. This paper first shows that the two formulations are in fact different and both formulations suffer from the same technical deficiency of ramp rate violation during the periodic implementation of the optimal solutions. Then a model predictive control (MPC) approach is proposed to overcome such a technical deficiency. Furthermore, it is shown that the MPC solutions, which are based on the OCDD framework, converge to the optimal solution of an extended version of the DED problem and they are robust under certain disturbances and uncertainties. Two standard examples are studied: the first one of a ten-unit system shows the difference between the OCDD and DED, and possible ramp rate violations, and the second one of a six-unit system shows the convergence and robustness of the MPC solutions, and the comparison with OCDD as well.

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## 1. Introduction

The dynamic dispatch problem of power generation was first considered in the early 1970s in a control system framework in [Bechert and Kwatny \(1972\)](#), motivated by an optimal control formulation and solution done in [Kwatny and Bechert \(1973\)](#). It was argued that a dynamic dispatch solution was more accurate than the static economic dispatching (SED) ([Wood & Wollenberg, 1996](#)), in the sense of its look-ahead capability by solving the optimization problem with the predicted load demand over a time horizon consisting of several time intervals and considering the ramp rate constraints. Ramp rate constraint is a dynamic constraint used to maintain the life of the generators ([Han & Gooi, 2007](#); [Wang & Shahidepour, 1995](#); [Wood, 1982](#)).

The optimal control dynamic dispatch (OCDD) formulation is to model the power generation by means of state equations where the state variables are the electrical outputs of the generators and the control inputs are the ramp rates of the generators. The OCDD problem was originally described in [Bechert and Kwatny \(1972\)](#) and [Kwatny and Bechert \(1973\)](#). In these papers, the optimal feedback controller was synthesized only for the special case of two generators sharing load owing to

computational problems. [Bechert and Chen \(1977\)](#) proposed a multi-pass dynamic programming approach to solve the OCDD problem and obtained the optimal generator output trajectories for up to five generators. The proposed algorithm finds only a local optimal schedule, and the required computer memory and calculation time increase exponentially with the number of generators. The main drawback of the approaches proposed in [Bechert and Kwatny \(1972\)](#), [Kwatny and Bechert \(1973\)](#) and [Bechert and Chen \(1977\)](#) has been the limitation on the problem dimensions.

[Ross and Kim \(1980\)](#) proposed a successive approximation approach with dynamic programming for solving the OCDD problem without limitation of the number of units. The valve-point effects is considered. The large problem with ramping constraints is broken down into smaller subproblems. Each subproblem pairs one unit with an artificial unit and is solved via dynamic programming by discretizing the generation outputs. The feasibility of the problem has been demonstrated on a problem involving 15 units and 16 intervals. However, execution time and problem size increase almost exponentially with the number of units.

It seems that the OCDD approach has been gradually abandoned for almost 20 years until in 1998 the OCDD problem was revisited again by [Travers and Kaye \(1998\)](#). They applied constructive dynamic programming to solve the OCDD problem. Both the generation cost and the ramping up and down cost ([Tanaka, 2006](#)),

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or the ramping cost for short, are included in the objective function as piecewise linear functions. The proposed method provides optimal trajectories for all system states without the need to discretize generator output. However, the dynamic programming method suffers from the “curse of dimensionality”.

Since the 1980s, the dynamic dispatch problem has been formulated as a minimization problem of the total cost over the dispatch horizon, and has been known as the DED problem (see, e.g., Attaviriyunapap, Kita, Tanaka, & Hasegawa, 2002; Barcelo & Rastgoufard, 1997; Han & Gooi, 2007; Han, Gooi, & Kirschen, 2001; Irisarri, Kimball, Clements, Bagchi, & Davis, 1998; Jabr, Coonick, & Cory, 2000; Li, Morgan, & Williams, 1997; Somuah & Khunaizi, 1990; van den Bosch, 1985; Wood, 1982) and the reviews in (Chowdhury & Rahman, 1990; Xia & Elaiw, 2010), and has gained more popularity in the power system community. The DED problem has recently been extended to achieve the objective of optimizing the profit under competitive market conditions (Attaviriyunapap, Kita, Tanaka, & Hasegawa, 2004). Since the formulation of the DED problem, the thrust of research has been focused on various optimization techniques and procedures incorporating extended and complex objective functions or constraints. The early research activities were either mathematical programming based or heuristically based, such as the lambda iterative method (Wood, 1982), gradient projection method (Granelli, Marannino, Montagna, & Silvestri, 1989), Lagrange relaxation (Hindi & Ghani, 1991), linear programming (Somuah & Khunaizi, 1990) and interior point method (Han & Gooi, 2007; Irisarri et al., 1998; Jabr et al., 2000). More recent works have centered around artificial intelligence (AI) methods, on par with the development of AI optimization theories, such as simulated annealing (Panigrahi, Chattopadhyay, Chakrabarti, & Basu, 2006), hybrid genetic algorithms (GA) (Li & Aggarwal, 2000; Li et al., 1997), differential evolution (DE) (Balamurugan & Subramanian, 2007), particle swarm optimization (PSO) (Gaing, 2004; Panigrahi, Chattopadhyay, & Chakrabarti, 2007), evolutionary programming with sequential quadratic programming (EP-SQR) (Attaviriyunapap et al., 2002), particle swarm optimization with sequential quadratic programming (PSO-SQR) (Victoire & Jeyakumar, 2005a, 2005b) and differential evolution with Shor’s r-algorithm (Yuan, Wang, Zhang, & Yuan, 2009). Many of these techniques have proven their effectiveness in solving the DED problems without any or fewer restrictions on the shape of the cost function curves. There is also an interesting research work to consider the applications of economic dispatch in hydro-thermal systems (see, for example, Salam, 2008; Zeng, Wu, Liu, & Yuan, 2005 and the references therein), and sometimes the unit commitment problem (Padhy, 2004) is combined together in the generation dispatch (Samudi, Das, Ojha, Sreeni, & Cherian, 2008).

In literature the difference between the two formulations OCDD and DED has never been questioned. Arising from the same problem of meeting electricity demand, both formulations have great similarities. Firstly, they are subject to similar sets of constraints. Secondly, they are offered with a view to be implemented repeatedly and periodically over a receding time horizon of, say, one day or one week. This periodicity assumption comes from the fact that the demand is periodic due to cyclic consumption behavior and seasonal changes. In the above existing studies on the DED and OCDD formulations, the main attention focuses on finding the optimal dispatch over a fixed time horizon and the periodic implementation of such an optimal dispatch solution has been though regarded as straightforward, and thus left to the practitioners.

This paper starts by showing that the two formulations are different, and both formulations suffer from the same technical deficiencies for periodic implementations as illustrated in Example 1. Then a new approach based on model predictive control (MPC) ideas to periodically implement OCDD solutions is presented. It is also

shown that the solutions in the iteration steps of this MPC approach are converging to the global optimal solution of an extended version of the DED problem.

The MPC method has emerged since the early 1970s when the OCDD problem was first formulated, and the MPC method has been successfully applied particularly in the process control industry. Theoretical properties such as stability and robustness of the MPC have been studied by many authors since the early work of Kleinman (1970). Here the readers are referred to the excellent reviews (De Nicolao, Magni, & Scattolini, 2000; Findeisen, Imsland, Allgöwer, & Foss, 2003; Mayne, Rawlings, Rao, & Sckaert, 2000; Qin & Badgwell, 2003; Rawlings, 2000), and to the references therein. MPC is a feedback control technique that uses an explicit model of the plant to predict the future response of the plant over a finite horizon. The feedback controller is constructed by solving a finite horizon optimal control problem at each sampling instant using the current state of the plant as the initial state for the optimization and applying only “the first part” of the optimal control (Mayne et al., 2000). Up to the present, MPC has become one of the most widely used multivariable control algorithms in various industries, including chemical engineering, food processing, automotive, aerospace applications (Qin & Badgwell, 2003) and recently in power systems (Otomega, Marinakis, Glavic, & Van Cutsem, 2007). This is due to its facility of handling constraints, being able to use simple models, and its closed-loop stability and inherent robustness in many applications (see, e.g., Camacho & Bordons, 2004; De Nicolao et al., 2000; Findeisen et al., 2003; Mayne et al., 2000; Qin & Badgwell, 2003; Rawlings, 2000). Moreover, MPC solves optimal control problems *on-line* for the current state of the plant which is a mathematical programming problem and is often simpler than determining the feedback solution by dynamic programming (Mayne et al., 2000). Other potential benefits of MPC in power system control problems have been demonstrated in Otomega et al. (2007), Zima and Andersson (2006), Atic, Rerkpreedapong, Hasanovic, and Feliachi (2003) and Hiskens and Gong (2006). To the best knowledge of the authors, the research on the application of MPC in the dynamic dispatch problem has never been done before, even though the MPC idea may have already been applied in practical implementations. For example, the generation scheduling of the main electricity utility in South Africa is exactly using a primitive MPC idea: the generation is dispatched by solving the DED problem over a 7-day horizon; the optimal solution is implemented only for the first day; at the end of the first day this DED problem is recalculated to find the optimal solution for the next 7 days; and such a computation repeats every day and the 7-day period recedes over and over. Since this scheduling idea is only a primitive MPC idea, it is not surprising that ramp rate constraints are sometimes violated, and they are left to the automatic generation control (AGC) to manage at the machine’s level. Therefore it is necessary to develop a systematic MPC approach to avoid the ramp rate violations during the periodic implementations of the optimal solution.

For this purpose, a critical review and comparison of OCDD and DED is given first. Then a ten-unit system is studied to show possible ramp rate violations during periodic implementations of OCDD and DED solutions. After that, an extended version of the DED formulation is given to overcome ramp rate violations. Now an algorithm originated from the moving optimization horizon idea in MPC approaches is introduced to provide a robust optimal solution against certain disturbances. This MPC approach, based upon the OCDD framework, with a moving horizon, is by its nature a closed-loop design from a control theoretical point of view. It is shown that the MPC solutions asymptotically approach the optimal solution of the extended version of the DED problem. The robustness of the MPC algorithm is also shown. The MPC approach therefore provides

a bridge between the OCDD and DED formulations, apart from the additional advantages of reduced dimensionality. The MPC approach is then illustrated on a six-unit system under load demand balance, ramp rate constraints and generation capacity constraints. Simulation results show the closed-loop nature and the robustness of the MPC. This MPC approach should facilitate a revived interest from the control system perspective, thus allowing a number of possible extensions in line with the newest developments of the MPC techniques. A draft version of the paper was presented in Xia, Zhang, and Elaiw (2009).

The layout of the paper is as follows. Section 2 introduces the OCDD and DED formulations and illustrate possible ramp rate violations in periodic implementations of their optimal solutions. In Section 3 an extended DED model is presented to avoid ramp rate violations during periodic implementations and to be a starting point of the MPC approach. Section 4 proposes the MPC algorithms to the dynamic dispatch problem. Section 5 illustrates how the MPC approach is applied on a six-unit system, as well as the advantages of MPC and comparisons with other methods. The last section is the conclusions and remarks.

## 2. Periodic implementations of OCDD and DED solutions

This section introduces the OCDD and DED formulations and identifies their possible violation of ramp rates during periodic implementations of the optimal solutions. Simple forms of OCDD and DED problems are considered which involve three types of constraints—equality, dynamic and inequality constraints. There are roughly three main types of constraints in the dynamic dispatch problem: the load demand balance in terms of equality constraints, ramp rates in terms of dynamic constraints and generation capacity in terms of inequality constraints. So the consideration of simple forms of the OCDD and the DED problems is without loss of generality, because it contains all three types of constraints. It is worthy to point out that the load demand balance, in a vertically integrated utility environment, is an obligation, thus a hard constraint; while in other cases, such as competitive markets, it is a soft constraint since the demand may not be necessarily met. The ramp rate and generation capacity limitations are equipment constraints, thus hard ones. Some other constraints such as spinning reserve, security constraints, etc., can be taken into consideration in exactly the same fashion in both formulations of the dynamic dispatch problem but they all boil down mathematically to the aforementioned three types of constraints. General constraints and objectives including non-smooth and/or non-convex functions will be left to future research.

To this end, some notations are introduced first.

Let  $N$  be a fixed positive integer,  $T$  a sampling period and  $NT$  the dispatch period. The dynamic dispatch problem can be considered over time intervals, or dispatch interval,  $[iT, (i+N)T)$  for any  $i \geq 0$ . Here the time from 0 to  $+\infty$  is divided into small intervals  $[0, T), [T, 2T), [2T, 3T), \dots$ , and  $[iT, (i+1)T)$  denotes the union of  $[iT, (i+1)T), [(i+1)T, (i+2)T), \dots, [(j-1)T, jT)$ . An example is a sampling period of a hour and a dispatch interval of 24 h as in Example 1 in this section.

For the sake of simplicity, it is assumed throughout the paper that  $[i, j)$  denotes the time interval  $[iT, jT)$ . When  $T$  equals 1 h, then the interval  $[i, j)$  actually denotes the  $i$ -th hour,  $(i+1)$ -th hour,  $\dots$ , and the  $(j-1)$ -th hour.

Assume that  $n$  is the number of committed units,  $P_i^k$  is the average power generated by unit  $i$  during the  $k$ -th time interval  $[k-1, k)$ ;  $C_i(P_i^k)$  and  $R_i(P_i^k)$  are the generation and ramping costs respectively for unit  $i$  to produce  $P_i^k$ ;  $D^k$  is the demand at time  $k$  (i.e., the  $k$ -th time interval); the variable  $u_i^k$  is the ramp rate of the unit  $i$  at time  $k$  and is also called control variable in this paper;  $UR_i$

and  $DR_i$  are the maximum ramp up/down rates for unit  $i$ ;  $P_i^{\min}$  and  $P_i^{\max}$  are the minimum and maximum capacity of unit  $i$  respectively; the notation  $(P_i^k : 1 \leq i \leq n, l \leq k \leq l+j)$  denotes the vector  $(P_1^l, P_2^l, \dots, P_n^l, P_1^{l+1}, P_2^{l+1}, \dots, P_n^{l+1}, \dots, P_1^{l+j}, P_2^{l+j}, \dots, P_n^{l+j})$ , and  $C(P_i^k : 1 \leq i \leq n, l \leq k \leq l+j)$  denotes the cost (objective) function  $C$  with variables  $\{P_i^k : 1 \leq i \leq n, l \leq k \leq l+j\}$ . Define  $D = (D^1, D^2, \dots, D^N)^T$ ,  $P^k = (P_1^k, P_2^k, \dots, P_n^k)^T$ ,  $u^k = (u_1^k, \dots, u_n^k)^T$ ,  $k \geq 0$ . The function  $[C_i(P_i^k) + R_i(P_i^k)]$  is assumed to be a quadratic function  $a_i(P_i^k)^2 + b_i P_i^k + c_i$  with known positive constants  $a_i$ ,  $b_i$  and  $c_i$ .

The demand  $D^k$  is assumed to be periodic with period  $N$ . This periodic assumption is made to reflect the cyclic consumption behavior and seasonal changes over the dispatch interval.

The following convention is also made:

$$\sum_{i=j}^k x_i = \begin{cases} 0 & \text{if } j > k, \\ x_j + x_{j+1} + \dots + x_k & \text{if } j \leq k. \end{cases}$$

### 2.1. OCDD formulation

The dynamics of the power system can be considered as a discrete-time control system (Ross & Kim, 1980; Travers & Kaye, 1998):

$$P_i^{k+1} = P_i^k + Tu_i^k, \quad k \geq 0, \quad i = 1, 2, \dots, n. \quad (1)$$

The OCDD problem over the dispatch interval  $[0, N)$  is formulated as follows: given a set of generators, load demand  $D^k$  and initial generation  $P_i^0$ , find a set of control actions  $u_i^k$  to minimize the total generation cost and to meet the load demand of a power system over this dispatch period:

**Problem OCDD.** Given  $n$ ,  $N$ ,  $DR_i$ ,  $UR_i$ ,  $P_i^{\min}$ ,  $P_i^{\max}$ ,  $1 \leq i \leq n$ ,  $P^0$ , and  $D$ , solve the following minimization problem:

min  $C(u_i^j : 1 \leq i \leq n, 0 \leq j \leq N-1)$

$$= \sum_{k=1}^N \sum_{i=1}^n \left[ C_i \left( P_i^k + \sum_{j=0}^{k-1} Tu_i^j \right) + R_i \left( P_i^k + \sum_{j=0}^{k-1} Tu_i^j \right) \right]$$

subject to  $\sum_{i=1}^n \left( P_i^0 + \sum_{j=0}^{k-1} Tu_i^j \right) = D^k$ ,

$$-DR_i \leq u_i^j \leq UR_i, \quad P_i^{\min} \leq P_i^0 + \sum_{j=0}^{k-1} Tu_i^j \leq P_i^{\max},$$

$$(1 \leq i \leq n, 0 \leq j \leq N-1, 1 \leq k \leq N). \quad (2)$$

In the constraints above, the first constraint  $\sum_{i=1}^n (P_i^0 + \sum_{j=0}^{k-1} Tu_i^j) = D^k$  is the demand constraint which can be a hard or soft constraint depending on the electricity market, while the second and third constraints are the ramp rate and generation capacity constraints respectively, and they are the hard constraints that the system must satisfy.

### 2.2. DED formulation

Normally the DED problem for the dispatch interval  $[0, N)$  can be formulated as follows:

**Problem DED.** Given  $n$ ,  $N$ ,  $DR_i$ ,  $UR_i$ ,  $P_i^{\min}$ ,  $P_i^{\max}$ ,  $1 \leq i \leq n$ , and  $D$ , solve the following minimization problem:

min  $C(P_i^k : 1 \leq i \leq n, 1 \leq k \leq N)$

$$= \sum_{k=1}^N \sum_{i=1}^n [C_i(P_i^k) + R_i(P_i^k)]$$

$$\begin{aligned} \text{subject to } & \sum_{i=1}^n P_i^k = D^k, \\ & -DR_i \cdot T \leq P_i^{j+1} - P_i^j \leq UR_i \cdot T, \\ & P_i^{\min} \leq P_i^k \leq P_i^{\max}, \\ & (1 \leq i \leq n, 1 \leq j \leq N-1, 1 \leq k \leq N). \end{aligned} \quad (3)$$

Note that the variables in the above DED problem are  $\{P_i^k : 1 \leq i \leq n, 1 \leq k \leq N\}$ .

### 2.3. Differences between OCDD and DED, and the periodic implementations of their optimal solutions

By noting the transformation (1), the OCDD and DED formulations are quite similar since both problems are actually to find the optimal  $(P^1, P^2, \dots, P^N)$ . However, there are fundamental differences between the two formulations:

- (1) The OCDD formulation produces an optimal solution for a given initial value  $P^0$ , and the optimal solution also depends on  $P^0$ , while the DED problem does not consider the initial generation  $P^0$ , and is totally independent of  $P^0$ .
- (2) The OCDD formulation has the ramp limit for  $u^0$ , that is, the differences between  $P_i^1$  and  $P_i^0$  must satisfy the ramp constraints; however, the DED formulation considers the ramp rate constraints only for  $P_i^2 - P_i^1, P_i^3 - P_i^2, \dots, P_i^N - P_i^{N-1}$  and has ignored the ramp limit for  $P_i^1 - P_i^0$ , where  $i = 1, \dots, n$ .

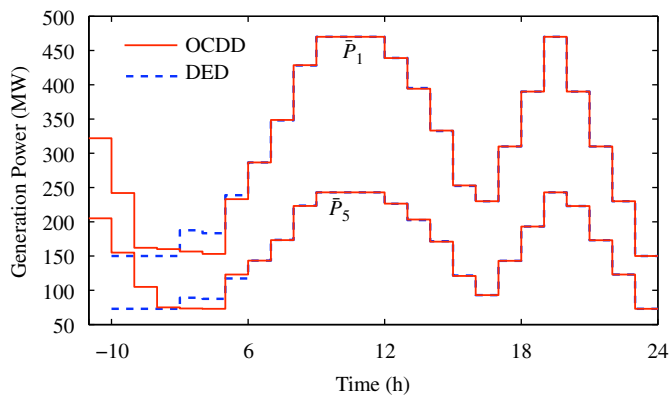


Fig. 1. The optimal trajectories of unit-1 and unit-5 of the ten-unit system for OCDD and DED.

Table 1  
Data of the ten-unit system.

Gen.	$P_i^{\min}$ (MW)	$P_i^{\max}$ (MW)	$a_i$ (\$/h)	$b_i$ (\$/MWh)	$c_i$ (\$/MW <sup>2</sup> h)	$UR_i$ (MW/h)	$DR_i$ (MW/h)	$P_i^0$ (MW)
1	150	470	958.20	21.60	0.00043	80	80	322
2	135	460	1313.6	21.05	0.00063	80	80	225
3	73	340	604.97	20.81	0.00039	80	80	80
4	60	300	471.60	23.90	0.0007	50	50	75
5	73	243	480.29	21.62	0.00079	50	50	205
6	57	160	601.75	17.87	0.00056	50	50	75
7	20	130	502.70	16.51	0.00211	30	30	50
8	47	120	639.40	23.23	0.0048	30	30	47
9	20	80	455.60	19.58	0.10908	30	30	50
10	55	55	692.40	22.54	0.00951	30	30	55

Table 2  
Load demand of the ten-unit system for 24 h.

Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Demand (MW)	1036	1110	1258	1406	1480	1628	1702	1776	1924	2072	2146	2220	2072	1924	1776	1554	1480	1628	1776	2072	1924	1628	1332	1184

The differences between the optimal solutions of OCDD and DED will be illustrated in Fig. 1 of Example 1.

Note that both the OCDD and DED problems are formulated over the dispatch interval  $[0, N)$  and do not consider the periodic implementations of the optimal solutions over the period  $[N, 2N), [2N, 3N), \dots$ . There is a simple way to periodically implement the optimal solutions: simply repeat the optimal solutions over other periods. However, the following Example 1 shows this simple repetition will possibly cause the ramp rate violations.

**Example 1.** This example illustrates the difference between the DED and OCDD approaches as well as the ramp rate violations of periodic implementations of DED and OCDD solutions on a ten-unit power system. This ten-unit system is a standard example in Attaviriyunupap et al. (2002), and all the data are taken from Attaviriyunupap et al. (2002) and are listed in Tables 1 and 2. The dispatch period is chosen to be a 24-h period with a 1 h sample period which is exactly the same as Attaviriyunupap et al. (2002). The sum of the generation and ramping costs is given by a quadratic function  $C_i(P_i) + R_i(P_i) = a_i + b_i P_i + c_i P_i^2$ . The initial  $P_i^0$  is chosen such that  $\sum_{i=1}^{10} P_i^0 = D^0$  and is given in Table 1. Here,  $P_i^0$  and  $D^0$  are the initial generation and load demand respectively during the interval  $[-1, 0)$ . Fig. 1 shows the optimal outputs of units 1 and 5 of the ten-unit system for both OCDD and DED problems. It can be observed that the solutions from the OCDD problem for the given initial  $P_i^0$  and that of the DED problem are different in the beginning of the dispatch period, but they do coincide from the 7-th time instant. Now the technical deficiencies that DED and OCDD suffer from during periodic implementations can be shown. Denote by  $(\bar{P}_i^k : 1 \leq i \leq 10, 1 \leq k \leq 24)$  the optimal solution computed by both the OCDD and DED models. Fig. 2 shows the optimal solution of unit 2 for the DED problem. The results show that the difference between  $\bar{P}_2^1$  and  $P_2^0$  does not satisfy the ramp rate constraint, since  $\bar{P}_2^1 - P_2^0 = -90 < -DR_2$ . Therefore, this optimal solution cannot be implemented at time instant  $t=1$  for the second generation unit. One may start at an initial condition  $P_i^0$  so that the ramp rate limits between  $\bar{P}_i^1$  and  $P_i^0$  can be satisfied. However, the optimal solution can only be implemented over the interval  $[0, 24]$ ; it cannot be implemented for the following 24 h by a simple repetition since  $\bar{P}_2^{24j+1} - \bar{P}_2^{24j} = -85 < -DR_2, j \geq 1$ . Similarly, Fig. 3 shows that the optimal solution of the OCDD for unit 3 cannot be implemented repeatedly every 24 h because  $\bar{P}_3^{24j+1} - \bar{P}_3^{24j} = -162 < -DR_3, j \geq 1$ .

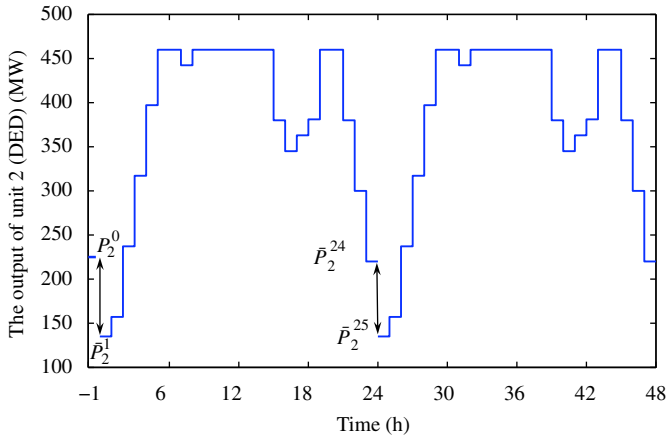


Fig. 2. The optimal trajectory of unit-2 of the ten-unit system for DED.

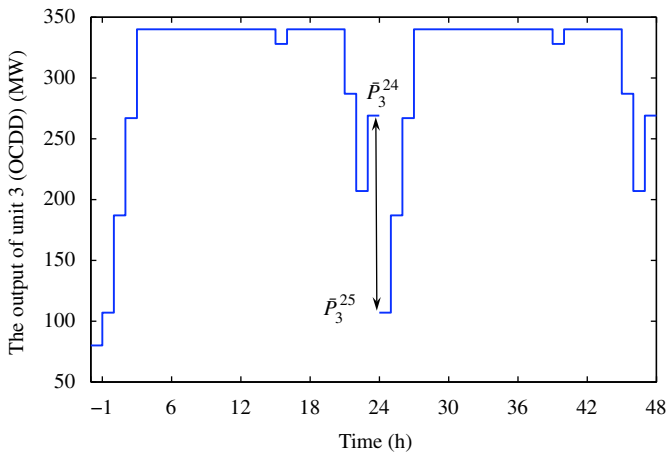


Fig. 3. The optimal trajectory of unit-3 of the ten-unit system for OCDD.

Although this same Example 1 is computed in Attaviryanupap et al. (2002), periodic implementation of the optimal solution is not considered in Attaviryanupap et al. (2002) and this results in the ramp rate violations mentioned above. In fact, the main purpose of Attaviryanupap et al. (2002) is to introduce a hybrid method using a combination of evolutionary programming and sequential quadratic programming so that a global optimal solution of a very general energy problem, in which the functions may be noncontinuous, nondifferentiable or non-smooth, can be found.

### 3. From OCDD and DED to the extended DED

Example 1 illustrates the possibility of ramp rate violations during the periodic implementations of OCDD and DED solutions. In order to avoid such a problem, the following extended DED (EDED) problem over the dispatch interval  $[0, N]$  is introduced and will be a starting point of the proposed MPC approach in this paper.

**Problem: Extended DED.** Given  $n, N, DR_i, UR_i, P_i^{\min}, P_i^{\max}, 1 \leq i \leq n$ , and  $D$ , solve the following minimization problem:

$$\begin{aligned} \min C(P_i^k : 1 \leq i \leq n, 1 \leq k \leq N) \\ = \sum_{k=1}^N \sum_{i=1}^n [C_i(P_i^k) + R_i(P_i^k)] \end{aligned}$$

subject to  $(P_i^k : 1 \leq i \leq n, 1 \leq k \leq N) \in \Omega_{EDED}$ , (4)

where the feasible domain  $\Omega_{EDED}$  is defined to be the set of  $(P_i^k : 1 \leq i \leq n, 1 \leq k \leq N)$  satisfying

$$\begin{aligned} \sum_{i=1}^n P_i^k &= D^k, \\ -DR_i \cdot T &\leq P_i^{j+1} - P_i^j \leq UR_i \cdot T, \\ -DR_i \cdot T &\leq P_i^1 - P_i^N \leq UR_i \cdot T, \\ P_i^{\min} &\leq P_i^k \leq P_i^{\max}, \\ (1 \leq i \leq n, 1 \leq j &\leq N-1, 1 \leq k \leq N). \end{aligned}$$

The only difference between the EDED problem (4) and the classical DED problem (3) is that the constraints

$$-DR_i \cdot T \leq P_i^1 - P_i^N \leq UR_i \cdot T \quad (1 \leq i \leq n) \quad (5)$$

do not appear in the classical problem (3). These constraints will mean that the difference between  $P^{N+1}$  and  $P^N$ , due to the difference between  $P^1$  and  $P^N$ , is bounded by some given constants, therefore the anticipated periodic and repeated implementation is practically feasible. The optimal solution of the EDED problem can be executed on each whole dispatch interval. Note that the EDED problem has the same cost (objective) function with the classical DED problem but more constraints, therefore, the cost determined by the optimal solution of the EDED is expectedly greater than or equal to that of the classical DED.

From a control theoretical point of view, both the OCDD and the DED formulations provide only open-loop optimal solutions to the generation dispatch problem, that is, the optimal solutions are predetermined before actual execution, and there is no measurement on the system states which is fed back to the optimization model. Therefore, a closed-loop control by the MPC method is introduced in the next section so that the measurement of states can be fed back to the optimization model, and the optimal solution is updated according to the feedback information at each time step.

## 4. MPC approach to DED

This section proposes an MPC approach based on the OCDD framework. The algorithms are presented first, followed by the results on convergence and robustness.

### 4.1. MPC algorithms

To introduce the MPC algorithms, a few steps of mathematical transformations are needed. Dummy variables are also introduced to avoid handling more mathematical notations.

The EDED problem (4) is defined over the time interval  $[0, N]$  with optimization variables  $P_i^1, P_i^2, \dots, P_i^N, i = 1, \dots, n$ . It is obvious that when the same dynamic economic dispatch problem is considered over the time interval  $[m, m+N]$ , then the optimization variables are changed into  $P_i^{m+1}, P_i^{m+2}, \dots, P_i^{m+N}, i = 1, \dots, n$ . By the transformation defined in (1), the set of variables  $\{P_i^{m+1}, P_i^{m+2}, \dots, P_i^{m+N}, 1 \leq i \leq n\}$  is transformed into  $\{P_i^{m+1}, u_i^{m+1}, \dots, u_i^{m+N-1}, 1 \leq i \leq n\}$ . This kind of transformation is convenient for the MPC formulation and the variables  $u_i^j$ , with  $m+1 \leq j \leq m+N-1, 1 \leq i \leq n$  are called control variables or system inputs in control theory.

In an MPC approach, a finite-horizon optimal control problem is repeatedly solved and the input is applied to the system based on the obtained optimal open-loop control. Consider a horizon with length  $N$ . Instead of solving the EDED problem with  $nN$  number of variables  $\{P_i^{m+1}, u_i^{m+1}, \dots, u_i^{m+N-1}, 1 \leq i \leq n\}$ , the MPC

algorithm solves the following problem which has only  $n(N-1)$  number of variables  $\{u_i^{m+1}, \dots, u_i^{m+N-1}, 1 \leq i \leq n\}$ :

**Problem** MPCED $_{\bar{P}^{m+1}}(u, [m, m+N])$ . Given  $n, N, DR_i, UR_i, P_i^{\min}, P_i^{\max}, 1 \leq i \leq n, D, \bar{P}^{m+1}$ , let

$$P_i^1 := P_i^{m+1}, \quad u_i^j := u_i^{m+j}, \quad D^k := D^{m+k},$$

$$1 \leq i \leq n, \quad 1 \leq j \leq N-1, \quad 1 \leq k \leq N, \quad (6)$$

and solve the following minimization problem:

$$\min C(u_i^j : 1 \leq i \leq n, j = 1, 2, \dots, N-1)$$

$$= \sum_{k=1}^N \sum_{i=1}^n \left[ C_i \left( P_i^1 + \sum_{j=1}^{k-1} Tu_i^j \right) + R_i \left( P_i^1 + \sum_{j=1}^{k-1} Tu_i^j \right) \right]$$

subject to  $(P_i^1, u_i^j : 1 \leq i \leq n, j = 1, 2, \dots, N-1)$

$$\in \Omega_D(P^1, u), \quad (7)$$

where the notation MPCED $_{\bar{P}^{m+1}}(u, [m, m+N])$  denotes the optimization problem is solved over the interval  $[m, m+N]$  with variables  $u_i^j$  and for known inputs  $P_i^{m+1}, 1 \leq i \leq n, j = m+1, \dots, m+N-1$ .

In order to make the above MPCED problem solvable, the following hypothesis is needed as in Han et al. (2001), Han and Gooi (2007) to make  $\Omega_D(P^1, u)$  nonempty. This hypothesis is easily fulfilled if the supplier has enough capacity to meet the demand and the demand does not change too much over adjacent sampling periods.

**Feasibility Hypothesis 1** (Han and Gooi, 2007; Han et al., 2001). After the change of variables in (6) over any dispatch interval  $[m, m+N]$  with  $m \geq 0$ , the set  $\Omega_D(P^1, u)$  is not empty.

Denote the optimal solution of (7) by  $(\bar{u}_i^j : 1 \leq i \leq n, 1 \leq j \leq N-1)$ , then use the inverse of (6) to change the dummy variables back by letting

$$\bar{u}_i^{j+m} := \bar{u}_i^j, \quad 1 \leq i \leq n, \quad 1 \leq j \leq N-1.$$

Now the optimal solution of MPCED $_{\bar{P}^{m+1}}(u, [m, m+N])$  is denoted by

$$\bar{u}|_m = (\bar{u}_i^j|_m : i = 1, 2, \dots, n; j = m+1, \dots, m+N-1),$$

where the notation  $\cdot|_m$  denotes the value obtained at the dispatch interval  $[m, m+N]$ . Denote  $\bar{u}^j|_m = (\bar{u}_1^j|_m, \bar{u}_2^j|_m, \dots, \bar{u}_n^j|_m)$ . The optimal solution  $\bar{u}|_m$  is applied only in the first sampling period  $[m, m+1]$ , that is,  $\bar{u}_i^{m+1}|_m$  is applied to the state  $\bar{P}_i^{m+1}$  to obtain  $\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m$ , and this  $\bar{P}_i^{m+2}$  is actually executed over the time period  $[m+1, m+2]$ , where  $i = 1, 2, \dots, n$ . Note that the optimal controller  $\bar{u}|_{m+1}$  of the dispatch interval  $[m+1, m+N+1]$  is indeed a function of the initial value  $\bar{P}^{m+2}$ , thus a closed-loop feedback is obtained. The above ideas can be strictly formulated into the following MPC algorithm.

**MPC Algorithm 1.** Initialization: Input the initial status  $\bar{P}^1 \triangleq P^1 = (P_1^1, P_2^1, \dots, P_n^1)$  and let  $m=0$ .

(1) Compute the open-loop optimal solution  $\bar{u}_i^j|_m$  of the problem MPCED $_{\bar{P}^{m+1}}(u, [m, m+N])$ , where  $i = 1, \dots, n, j = m+1, \dots, m+N-1$ .

(2) The (closed-loop) MPC controller  $\bar{u}_i^{m+1}|_m$  is applied to the plant in the sampling interval  $[m, m+1]$  (the remaining  $\{\bar{u}_i^j|_m : i = 1, 2, \dots, n; j = m+2, m+3, \dots, m+N-1\}$  are discarded) to obtain

the closed-loop MPC solution

$$\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m \quad (8)$$

over the period  $[m+1, m+2]$ .

(3) Let  $m:=m+1$  and go to step (1).

Generally, the above MPC algorithm never stops, and it updates the controller at each time interval  $[m, m+1]$  to include feedback information.

It is noticed that the MPC approach has the following advantages.

(1) Reduced dimension: The economic dispatch problem for a practical control problem can be of a size of over 100 generating units. For example, consider a system with 150 generating units, a dispatch interval of 24 h, and a sampling period of 1 h. Then  $T=1$  h,  $N=24$ , and the corresponding OCDD or DED problem must solve an optimization problem with  $150 \times 24=3600$  number of variables. However, in each iteration step of the MPC Algorithm 1, the algorithm starts with any  $P^{m+1}$  and solves an optimization problem with  $150 \times (24-1)=3450$  number of variables which reduces 150 dimensions in the optimization problem and makes the computation easier.

(2) Convergence and easy implementation: Theorem 1 will show that one can start the MPC Algorithm 1 with any  $P^1$  satisfying  $\sum_{i=1}^n P_i^1 = D^1$  and the optimal solution at each step will converge to the optimal solution of the EDED problem. This implies that the reduction of the number of variables in the MPC approach is both reasonable and feasible. This also means that the MPC approach can be initiated, or re-started after interruptions.

(3) Robustness and simplified model: In the beginning of each iteration after initialization on the dispatch interval  $[m, m+N]$ , the MPC algorithm measures the initial generation  $\bar{P}^{m+1}$ . Therefore, if there is any disturbance in  $\bar{P}^{m+1}$ , the MPC algorithm will detect the disturbed  $\bar{P}^{m+1}$  and the optimal MPC controller  $(\bar{u}_i^{m+1}|_m : i = 1, \dots, n)$  will make the compensation and correction automatically. This closed-loop nature of the MPC controller comes with an inherent property of robustness. Theorem 2 will show that the MPC algorithm is robust against small disturbances in  $\bar{P}^{m+1}$ .

The ramp rates of the dispatch problem are given by the relation in (1) (Ross & Kim, 1980; Travers & Kaye, 1998). However, (1) is actually a simplified model of the power changes of the generators. Due to the dynamical complexity of the generators, the change of the power from  $P_i^k$  to  $P_i^{k+1}$  can be a highly nonlinear process with uncertainties. It has been shown that MPC algorithms approximate many complex system behaviors by using simplified models to a satisfactory extent (see, e.g., Camacho & Bordons, 2004; De Nicolao et al., 2000; Findeisen et al., 2003; Mayne et al., 2000; Qin & Badgwell, 2003; Rawlings, 2000). The underlying reason can be understood as that the MPC Algorithm is robust against certain disturbances while the inaccuracy in system modeling can often be treated as disturbances to a simplified model. Then it follows from the robustness result in Theorem 2 that the MPC Algorithm 1 is able to use simplified model (1) to approximate the complex dynamics of the generators if this model inaccuracy does not exceed a predetermined bound.

Note that an input of an initial power  $P^1$  is needed in the MPC Algorithm 1. For this  $P^1$ , if there exist  $u^1, \dots, u^{N-1}$  such that  $(P^1, u^1, \dots, u^{N-1}) \in \Omega_D(P^1, u)$ , then  $\Omega_D(P^1, u)$  is nonempty, and the Feasibility Hypothesis 1 ensures the execution of MPC Algorithm 1. However, if  $P^1$  cannot ensure the nonemptiness of  $\Omega_D(P^1, u)$ , then the MPC Algorithm 1 cannot be executed. In this case, how could this algorithm be implemented? This question can be answered by considering the existing generation  $P^0$  at time  $t=0$ .

For the existing generation  $P^0$ , there are three cases, that is, the power output  $P^0$  may be less than, equal to, or greater than the

demand  $D^0$ . The third case can be dealt with exactly as the first case, therefore only the first two cases are discussed in the following. When the power output  $P^0$  equals the demand, that is,  $\sum_{i=1}^n P_i^0 = D^0$ , then the constraints  $P_i^{\min} \leq P_i^0 \leq P_i^{\max}, 1 \leq i \leq n$ , must be met automatically since the  $P_i^0$ 's are existing generation. Now let  $P^1 = P^0$  and all the  $u_i^j$ 's be zeros, then  $(P_i^1, u_i^j : 1 \leq i \leq n, 1 \leq j \leq N-1)$  is an element of  $\Omega_D(P^1, u)$ ,  $\Omega_D(P^1, u)$  is nonempty, and the MPC Algorithm 1 can be executed.

When the power output  $P^0$  is less than the demand, that is,  $\sum_{i=1}^n P_i^0 < D^0$ , there are two subcases. The first subcase is that the system does not have enough capacity to meet the demand, and has to switch on more generators by working out a unit commitment problem (Padhy, 2004). This applies if  $\sum_{i=1}^n P_i^0$  is much less than the demand  $D^0$ . In other words, the first subcase happens if  $|D^0 - \sum_{i=1}^n P_i^0| \geq M$  for a given  $M$ . Since the unit commitment problem is an independent problem out of the scope of this paper, the first subcase is resorted to a new unit commitment problem, and thus not considered in this paper. The second subcase is that the supplier still has enough capacity to meet the demand, and this happens if the difference between  $D^0$  and  $\sum_{i=1}^n P_i^0$  is small, or equivalently  $|D^0 - \sum_{i=1}^n P_i^0| < M$  for the given  $M$ . In the latter subcase, the supplier must sacrifice part of its running cost to meet the demand as soon as possible if the company is running in a competitive market. Since the ramp constraints limit the increase of the powers of the units, it may take some time to reach the desired demand. The solution of the following preliminary scheduling optimization problem shows how to schedule the powers of these committed units to meet the demand.

**Problem:  $k$ -th Preliminary Scheduling.**

$$\min \left( \sum_{i=1}^n P_i^k - D^k \right)^2 + \lambda_k \sum_{i=1}^n [C(P_i^k) + R(P_i^k)]$$

subject to  $-DR_i \cdot T \leq P_i^k - P_i^{k-1} \leq UR_i \cdot T,$

$$P_i^{\min} \leq P_i^k \leq P_i^{\max},$$

$$(1 \leq i \leq n), \tag{9}$$

where  $P^{k-1}$  is given,  $P^k$  is the variable in the problem, and  $\lambda_k \in [0, +\infty)$  is a weighting factor that represents the trade off between the increase of the powers of committed units and the corresponding running cost.

The following algorithm will be applied for the MPC approach to EDED problem.

**MPC Algorithm 2.** Initialization: Input a number  $M$ , and a  $P^0$  which is the existing power output of the units at the dispatch interval  $[-1, 0)$ .

(1) If  $\sum_{i=1}^n P_i^0 = D^0$ , then let  $P^1 = P^0$  and execute MPC Algorithm 1 with this  $P^1$ .

(2) If  $|D^0 - \sum_{i=1}^n P_i^0| < M$ , let  $k=1$  and do the following steps.

(2.1) Solve the  $k$ -th Preliminary Scheduling Problem with the initial value  $P^{k-1}$  and a weighting factor  $\lambda_k$ , and denote the obtained optimal solution by  $\bar{P}^k := (\bar{P}_1^k, \bar{P}_2^k, \dots, \bar{P}_n^k)$ .

(2.2) Execute  $\bar{P}^k$  as the actual power outputs of the  $n$  units, that is, let the power output of the  $i$ -th unit be  $\bar{P}_i^k$  at the sampling time period  $[k-1, k)$ , where  $i = 1, 2, \dots, n$ . If  $\sum_{i=1}^n \bar{P}_i^k < D^k$ , let  $k := k+1$  and go to step (2.1); otherwise let  $P^1 := \bar{P}^k$  and perform a change of time coordinate so that the time interval  $[i, j)$  is changed into  $[i-k+1, j-k+1)$ , now execute MPC Algorithm 1.

(3) If  $|D^0 - \sum_{i=1}^n P_i^0| \geq M$ , do a unit commitment problem by other algorithms to obtain a new  $P^0$  and go back to step (1).

**4.2. Convergence and robustness**

After having the above MPC algorithms, some basic questions arise: Do these algorithms converge? What are the physical meanings of the optimal solution of an MPC algorithm? Are the MPC algorithms robust when disturbances exist? These questions are answered in the following theorems.

**Theorem 1.** Suppose Feasibility Hypothesis 1 holds,  $P^*$  is the globally optimal solution of the EDED problem (4),  $\tilde{P}^*$  is the globally optimal solution of the DED problem (3), then

- (i) MPC Algorithm 1 converges to  $P^*$  if the initial power output  $P^1$  at time  $t=1$  satisfies  $\sum_{i=1}^n P_i^1 = D^1$ .
- (ii) MPC Algorithm 2 converges to  $P^*$ .
- (iii) the value of the objective function  $C(P_i^k : 1 \leq i \leq n, 1 \leq k \leq N) = \sum_{k=1}^N \sum_{i=1}^n [C(P_i^k) + R(P_i^k)]$  at the point  $P^*$  is greater than or equal to that at the point  $\tilde{P}^*$ .

This theorem tells that the solutions of the MPC Algorithms 1 and 2 converge to the optimal solution of the EDED problem (4), and this solution may be worse than the solution of the classical DED problem (3) due to the fact that classical DED has neglected some ramp limits. The proof of the theorem is quite lengthy and omitted here.

Now consider the robustness of the MPC algorithms. Note that MPC Algorithm 1 is the main algorithm in this MPC approach. Therefore, in order to discuss the robustness of the MPC approach, it suffices to discuss the robustness of MPC Algorithm 1. The uncertainties in energy demand, price, and reserve demand for the DED problem in a deregulated market are discussed by fuzzy optimization in Attaviriyapap et al. (2004). However, no theoretical result is given. For the sake of simplicity, suppose that disturbance happens only in the execution of the controller. That is, the disturbance happens only in step (2) of MPC Algorithm 1 so that when the control  $\bar{u}_i^{m+1}|_m$  is applied to the plant in the sampling interval  $[m, m+1)$ , the system actually executes  $\bar{P}_i^{m+2} = F(\bar{P}_i^{m+1}, \bar{u}_i^{m+1}|_m, w_i^{m+1})$  over the period  $[m+1, m+2)$ , where  $F$  is a function,  $w_i^{m+1}$  is a disturbance vector satisfying  $\|w_i^{m+1}\| < e$  and  $e$  is a positive constant. Although  $F$  is written in a general form to include general disturbances in nonlinear MPC (Findeisen & Allgöwer, 2002), it is often written in the addition form Garcia, Prett, and Morari (1989) as  $F(\bar{P}_i^{m+1}, \bar{u}_i^{m+1}|_m, w_i^{m+1}) = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m + Tw_i^{m+1}$ , therefore

$$\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m + Tw_i^{m+1}. \tag{10}$$

Whenever the robust version of MPC Algorithm 1 is mentioned, it always means that (8) is replaced by (10) during its execution.

**Theorem 2.** Suppose Feasibility Hypothesis 1 holds,  $P^*$  is the globally optimal solution of the EDED problem (4),  $\tilde{P}^*$  is the globally optimal solution of the DED problem (3),  $\Omega_{EDED}$  is the feasible domain of problem (4), the norm of the gradient of the cost function of problem (4) has the upper bound  $L$  on  $\Omega_{EDED}$ ,  $\varepsilon$  is a small enough positive constant,  $c$  is a positive constant which is less than  $\varepsilon$ , (10) is executed in step (2) of MPC Algorithm 1 instead of (8), the constant disturbance  $w_i^k$  satisfies  $\|w_i^k\| < e$ , and  $e$  is small enough so that  $e < \min\{c/L, (\varepsilon - c)/L\}$ , then there exists an integer  $N_0$  such that for any  $k > N_0$ , the optimal MPC solution  $\bar{P}^{k+1}$  of the  $k$ -th loop in MPC Algorithm 1 belongs to the domain  $\bar{\Omega} := \{P : \|P - P^*\| < c\}$ .

The lengthy proof of this theorem is omitted.

**Remark 1.** Theorem 2 shows that the MPC algorithm is robust against certain disturbances in the execution of the optimal controller. It may happen that there is disturbance or uncertainty in the forecasted demand, that is, the demand  $D^k$  is disturbed so

that the actual demand is  $\widetilde{D}^k$ . Denote the resulting feasible domain by  $\Omega_{\widetilde{D}}(P^1, u)$  which is often different from  $\Omega_D(P^1, u)$  in (7) of Problem MPCED $_{pm+1}(u, [m, m+N])$ . Let  $\widetilde{u}_i^{m+1}|_m$  be the disturbed optimal solution of (7) over  $\Omega_{\widetilde{D}}(P^1, u)$ . Then by steps (1) and (2) of the MPC Algorithm 1, the obtained  $\widetilde{u}_i^{m+1}|_m$  may be different from the optimal solution  $\bar{u}_i^{m+1}|_m$  which is obtained without demand disturbance. However, by the continuous dependence of the solution on the feasible domain, the difference  $\|\widetilde{u}_i^{m+1}|_m - \bar{u}_i^{m+1}|_m\|$  is sufficiently small when  $\|\widetilde{D}^k - D^k\|$  is small enough. Therefore, (8) in step (2) of the MPC Algorithm 1 can be written as

$$\bar{P}_i^{m+2} = \bar{P}_i^{m+1} + T\widetilde{u}_i^{m+1}|_m = \bar{P}_i^{m+1} + T\bar{u}_i^{m+1}|_m + Tw_i^{m+1}, \quad (11)$$

where  $w_i^{m+1} := \widetilde{u}_i^{m+1}|_m - \bar{u}_i^{m+1}|_m$ .

Hence the demand disturbance or uncertainties can also be written in the form of (10) and the result of Theorem 2 is applicable when  $\|\widetilde{D}^k - D^k\|$  is small enough.

### 5. Simulation results

This section presents an example to show the convergence and robustness properties of the proposed MPC algorithm. The optimization problem in the example is solved by sequential quadratic programming under load balance constraints, ramp rate constraints and generation capacity constraints. All computations are carried out by MATLAB program. In particular, the optimal control sequence is computed by the `fmincon` code of the MATLAB Optimization Toolbox.

**Example 2.** This example presents an application of MPC to the DED problem consisting of six units. This is a standard example in Gaing (2004), and all the system technical data are exactly the same as Gaing (2004) and are listed in Tables 3 and 4. The dispatch interval and the sampling period in Gaing (2004) are one day and 1 h respectively, and these will also be kept here in this example. The generation cost and ramping cost curves are given as quadratic functions  $C_i(P_i) + R_i(P_i) = a_i + b_i P_i + c_i P_i^2$ . The initial generations  $P_i^0$  have been chosen such that  $\sum_{i=1}^{10} P_i^0 < D^0$ . MPC Algorithm 2 is implemented over 48 h, where the  $k$ -th preliminary scheduling problem is run with  $\lambda_1 = 0.1, \lambda_2 = 0.01, \lambda_3 = 0.001, \lambda_4 = 0$ . The advantages of the MPC algorithm and its comparison with DED and OCDD are listed below.

(a) Reduced dimensions: Since there are 6 units, the dispatch period is a 24-h period, and the sampling period is 1 h; the

**Table 3**  
Data of the six-unit system.

Gen.	$p_i^{\min}$ (MW)	$p_i^{\max}$ (MW)	$a_i$ (\$/h)	$b_i$ (\$/MWh)	$c_i$ (\$/MW <sup>2</sup> h)	$UR_i$ (MW/h)	$DR_i$ (MW/h)	$P_i^0$ (MW)
1	100	500	240	7.0	0.0070	80	120	240
2	50	200	200	10.0	0.0095	50	90	50
3	80	300	220	8.5	0.0090	65	100	130
4	50	150	200	11.0	0.0090	50	90	80
5	50	200	220	10.5	0.0080	50	90	50
6	50	120	190	12.0	0.0075	50	90	50

**Table 4**  
Load demand of the six-unit system for 24 h.

Time (h)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Demand (MW)	955	942	935	930	935	963	989	1023	1126	1150	1201	1235	1190	1251	1263	1250	1221	1202	1159	1092	1023	984	975	960

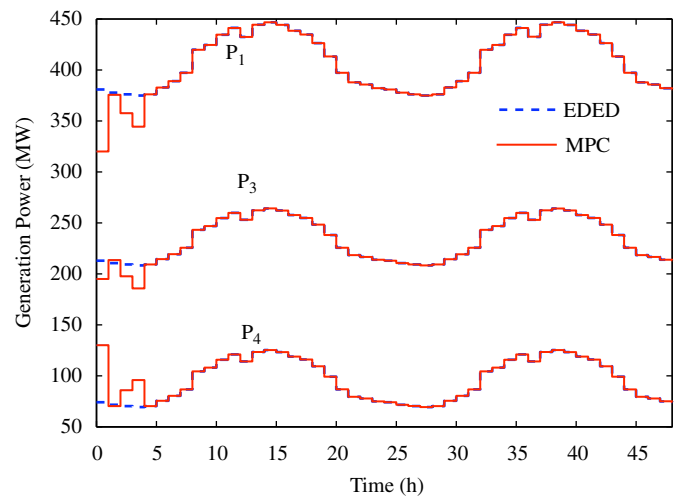
number of variables in the DED or OCDD model is  $6 \times 24 = 144$ , while the number of variables in the MPC approach is  $6 \times 23 = 138$ . Thus the optimization problem in the MPC approach has 6 less variables over one dispatch period.

(b) Convergence and easy implementation: Fig. 4 shows that the MPC closed-loop solutions asymptotically approach the optimal solutions of the EDED problem. Because of this convergence, restarting the MPC algorithm from any time will give rise to the same convergence, which further implies that the MPC algorithm can be executed at any sampling time point.

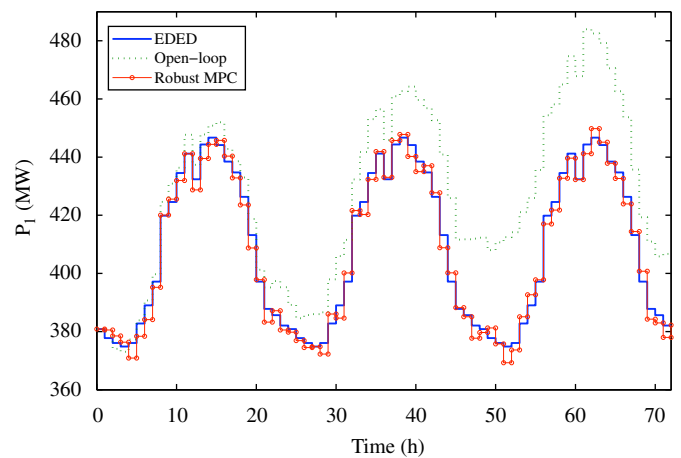
(c) Robustness and simplified model: To show the robustness of the MPC algorithm, (10) is executed and the disturbance  $w_i^m$  is generated by

$$w_i^m = -\varepsilon_i + 2\varepsilon_i r(m),$$

where the parameters  $r(m)$ 's are uniformly distributed random numbers on  $[0,1]$  and  $\varepsilon_i$ 's are given error bounds. Note that the



**Fig. 4.** Convergence of the closed-loop MPC solutions to those of EDED for the six-unit system.



**Fig. 5.** The generation output of unit-1 of the six-unit system under EDED, open-loop controller and robust MPC.



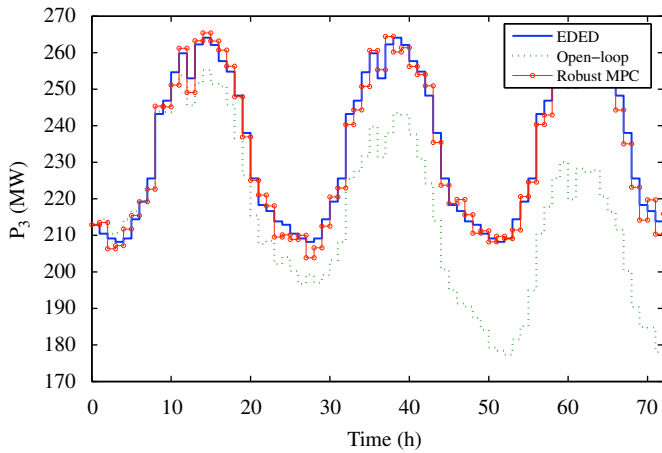


Fig. 6. The generation output of unit-2 of the six-unit system under EDED, open-loop controller and robust MPC.

Table 5  
Relative error for  $\varepsilon = (5,5,4,4,5,3)$ .

$i =$	1	2	3	4	5	6
$E_{\max}^i$ (%)	1.33	4.23	1.92	5.77	4.58	6.00
$E_{\min}^i$ (%)	1.12	2.92	1.51	3.19	2.90	3.59
$\varepsilon_{\max}^i$ (%)	1.39	4.88	2.00	6.80	4.93	10.30

sampling period  $T$  is 1 h, thus,  $w_i^m$  introduces an evenly distributed error from  $-\varepsilon_i$  to  $\varepsilon_i$  on  $\bar{P}_i^{m+1}$ . Denote  $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_6)$ . The robustness of the MPC algorithm is illustrated in the following different ranges of disturbances.

(c.1) Choose  $\varepsilon = (5,5,4,4,5,3)$ . To show the effectiveness of the MPC, the following cases are shown in Figs. 5 and 6;

- (i) the optimal solutions of the EDED problem ( $\bar{P}_i^k : 1 \leq i \leq n, 1 \leq k \leq 72$ );
- (ii) the (closed-loop) MPC solutions with disturbances; and
- (iii) the solutions of the disturbed system

$$P_i^{k+1} = P_i^k + T\bar{u}_i^k + Tw_i^k$$

with the open-loop controller ( $\bar{u}_i^k : 1 \leq i \leq 6, 1 \leq k \leq 72$ ) obtained by OCDD problem.

In cases (ii) and (iii) the initial  $P_i^1$  is chosen as the optimal solution of the EDED problem at  $t = 1$ , i.e.,  $P_i^1 = \bar{P}_i^1$ . From Figs. 5 and 6 it is obvious that the MPC algorithm can keep the disturbed system around the optimal solution of EDED which illustrates the robustness. The two figures illustrate also the feasibility to use a simplified model such as (1) to represent the real complex system.

For the 72 h' period, the impact of the random error  $w_i^m$  to the optimal solution in each of the MPC iteration loops can be roughly estimated by the maximum value and minimum value of the set  $\{\varepsilon_i/P_i^{m*} \times 100\% : m = 1, 2, \dots, 72\}$ , where  $P_i^{m*}$  denotes the optimal EDED solution and  $i = 1, \dots, 6$ . The maximum value and minimum value correspond, respectively, to the maximum and minimum error/uncertainty/disturbance introduced in the actuator implementation. For simplicity, denote them by  $E_{\max}^i$  and  $E_{\min}^i$  respectively. The performance of the optimal solution in each MPC iteration loop is the best evaluated by the maximum relative error of this solution to the EDED solution, that is, by  $\varepsilon_{\max}^i := \max\{|\bar{P}_i^m - P_i^{m*}|/P_i^{m*} \times 100\% : m = 1, 2, \dots, 72\}$ . Table 5 lists these errors, and it can be best understood by consider an example. For instance, the second column of the table states that for the error bound  $\varepsilon_1 = 5$ , it introduces a randomly distributed uncertainty or error within  $[-5, 5]$  on the generation of the first generator at

Table 6  
Relative error for  $\varepsilon = (25,6,12,5,6,4)$ .

$i =$	1	2	3	4	5	6
$E_{\max}^i$ (%)	6.67	5.07	5.76	7.21	5.49	8.00
$E_{\min}^i$ (%)	5.60	3.50	4.54	3.99	3.49	4.79
$\varepsilon_{\max}^i$ (%)	6.92	5.63	6.12	7.78	6.45	13.60

Table 7  
Relative error for  $\varepsilon = (25,25,20,20,25,15)$ .

$i =$	1	2	3	4	5	6
$E_{\max}^i$ (%)	6.67	21.13	9.60	28.85	22.88	30.00
$E_{\min}^i$ (%)	5.60	14.60	7.57	15.97	14.52	17.94
$\varepsilon_{\max}^i$ (%)	8.21	29.73	11.92	38.85	34.26	36.81

each iteration loop, and this uncertainty occupies at least 1.12% and at most 1.33% of the generation in the corresponding EDED solution, while the maximum relative error of each MPC solution within the 72 h is 1.39%. The last column indicates that  $\varepsilon_6 = 3$  introduces up to 6% of the implementation inaccuracy compared to the EDED solution, while the MPC solution at each loop will have at most an error of 10.30%.

(c.2). Choose  $\varepsilon = (25,6,12,5,6,4)$ . For simplicity and also to avoid repetition, the figures for the comparison of the MPC solution and the open loop solution are not provided. Instead, a table indicating the relative error of the MPC solution to the EDED solution is provided to illustrate the robustness. That is, the relative errors of this disturbance and the MPC solution compared to the EDED solution are given in Table 6. This table shows, for instance, with a maximum uncertainty of 7.21% in the optimal solution implementation for the 4-th generator, the MPC solution has at most an error of 7.78%. The worse case is that when the generation of the 6-th generator has the maximum uncertainty of 8%, the MPC solution has an error up to 13.60%. Note that this 13.60% error is under the worse case, and usually the error is less than this, and sometimes there is even no error. These MPC solution errors depend on the particular inaccuracies during the solution implementations.

(c.3) Choose  $\varepsilon = (25,25,20,20,25,15)$ . Then the relative errors of this disturbance and the MPC solution compared to the EDED solution are given in Table 7. This table shows that even under a very great implementation inaccuracy, the MPC solution does not change too far compared with the implementation inaccuracy. For example, when the 6-th generator has an implementation inaccuracy of 30%, the MPC solution has at most an error of 36.81%. Once again, this example illustrates the robustness of this MPC algorithm.

(d) Comparison with DED and OCDD under disturbances: For this example, the DED model optimizes the dispatch over the time interval  $[0,24]$  and often ignores any problem which might happen during the periodic implementation of the obtained optimal solution. Therefore a simple repetition of the optimal DED solution over other periods such as  $[24,48]$ ,  $[48,72]$ , etc., may lead to ramp rate violations as Example 1 shows. A fair comparison of DED with MPC should be in the case that the DED method has also a moving optimization horizon such as  $[0,24]$ ,  $[1,25]$ ,  $[2,26]$ , etc. Unfortunately, although reference (Gaing, 2004) studies the same Example 2 under extra spinning reserve constraints by the particle swarm optimization method, it optimizes the dispatch for a dispatch period of 24 h only and does not consider the periodic implementation of the optimal solutions. Other

references on DED such as Wood (1982), Han and Gooi (2007), Irisarri et al. (1998), Somuah and Khunaizi (1990), Li et al. (1997), Attaviryanupap et al. (2002), Jabr et al. (2000), Granelli et al. (1989), Hindi and Ghani (1991), Panigrahi et al. (2006), Li and Aggarwal (2000), Balamurugan and Subramanian (2007), Gaing (2004), Panigrahi et al. (2007), Victoire and Jeyakumar (2005a, 2005b), Yuan et al. (2009) also focus on the solution of the DED problem under various complex constraints or objectives over a fixed time period, say, 24 h, and do not consider the periodic implementation of the obtained optimal solutions. The same is true for reference (Travers & Kaye, 1998). It proposes a constructive dynamic programming approach to the OCDD model, however its attention is also on the optimization over a fixed time period and ignores the periodic implementation problem.

As for the comparison of OCDD and MPC, again one cannot simply repeat the OCDD solution over the time period [24,48], [48,72], etc., as Example 1 has shown the possible ramp rate violation of OCDD by this kind of simple repetition. Note that the OCDD solution over the period [24,48] depends on its initial generation at time 24, therefore it is reasonable to recalculate the OCDD problem over the period [24,48] by using the initial generation  $\bar{P}^{24}$  which is the generation output at the 24-th hour. For the generation at [48,72], the generation  $\bar{P}^{48}$  is used as its initial generation. Now the obtained optimal OCDD solution is the one named “open-loop” solution in Figs. 5 and 6. From these two figures it is clear that the OCDD solution does not vary too much from the EDED solution for the first 24 h; however, the accumulated deviations from the EDED solution becomes larger as time elapses, and the generation capacity constraints of unit 1 have been violated. Therefore one can conclude that the MPC solution is better than the OCDD solution under disturbances if the OCDD problem is recalculated only for the time period [24,48], [48,72], etc.

The MPC algorithm developed in this paper does not contradict with any existing DED or OCDD methods. These existing DED and OCDD methods provide various optimization solution methods to find the optimal dispatch over a fixed time horizon; while the MPC Algorithm 1 provides a periodic implementation framework and does not specify any special optimization method to solve the dispatch problem MPC-EDED over a fixed time period. Furthermore, the MPC approach in MPC Algorithm 1 is in fact a very general philosophy: calculating an optimization problem over a fixed period, implementing the solution only at the beginning part of this fixed period, recalculating the optimization problem over a new time horizon, and repeating these steps. Following this idea, it is possible to incorporate these existing solution methods for DED and OCDD into this MPC framework. That is, by adding constraints like (5) to avoid ramp rate violations in existing DED and OCDD models, then it is possible to apply the above-mentioned optimization methods at each loop of the MPC Algorithm 1 and thus the obtained results will not violate any ramp rate constraint and may also be robust against disturbances. Possible difficulties could be the theoretical proofs of the convergence and robustness of the MPC algorithm since the DED or OCDD model under consideration will be more complex. This challenging work is still under research.

## 6. Conclusions

The main purpose of this paper is to propose an MPC approach to the periodic implementation of optimal solutions of dynamic dispatch problem. The convergence of the MPC solutions to the optimal solutions of the EDED and certain robustness of the MPC algorithm are shown, and these results guarantee the existing MPC practises in DED. The differences between the OCDD and

DED approaches are discussed and also illustrated on a ten-unit system. The convergence and robustness of the MPC algorithms are demonstrated through the application of MPC to a standard dynamic dispatch problem with six units.

A number of generalizations can be drawn as the following suggests:

- (1) The constraints considered in this paper are linear. In future applications of the MPC method the constraints can be non-linear if they define a convex feasible domain. Future work will also include various other constraints such as security constraints, spinning reserve, etc.
- (2) The objective functions considered in this paper are supposed to be quadratic functions. Under deregulated markets, the objective functions will not be quadratic. However, if they are convex and differentiable, then the MPC approach developed in this paper is still valid. For non-convex or non-smooth objective functions, further research is needed.
- (3) The optimization problems in the simulations are solved by a simple MATLAB function `fmincon`. The MPC algorithms can be solved by any other optimization routine, provided the later is effective and efficient. Therefore, advanced algorithms can also be applied in the MPC approach to energy optimization problems.
- (4) A simple model characterizing the increase of power output between adjacent time intervals is adopted, and this is only a simplified model for the complex system behavior (see equation (1)). The MPC method can approach the complex system by using this simple model. Note that there are many complex mathematical models in energy optimization; for example, the optimal control of geysers (Zhang & Xia, 2007). The complexity of these models often makes many numerical optimization algorithms invalid. Now one can simplify these models and try the MPC approach. Moreover, the MPC approach can handle more sophisticated models and this allows for the consideration of more in-depth modeling of the ramping mechanisms of the generator's power and reserve.
- (5) This paper provides only initial applications of the MPC ideas in energy optimization, with theoretical results of convergence and robustness. Note that the robustness results reveal inherent feedback properties of the MPC solutions. Robust MPC is an on-going research topic on its own right (Bemporad & Morari, 1999). However, this paper shows that advanced results from robust MPC, continuous time MPC, and nonlinear MPC (Findeisen et al., 2003) are expected to have effective applications in more energy optimization problems.

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