

# Multi-objective dynamic economic emission dispatch of electric power generation integrated with game theory based demand response programs



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## ARTICLE INFO

### Article history:

Received 24 July 2014

Accepted 1 November 2014

### Keywords:

Dynamic economic emission dispatch

Game theory

Demand response

Multi-objective optimization

Load curtailment

## ABSTRACT

The dynamic economic emission dispatch (DEED) of electric power generation is a multi-objective mathematical optimization problem with two objective functions. The first objective is to minimize all the fuel costs of the generators in the power system, whilst the second objective seeks to minimize the emissions cost. Both objective functions are subject to constraints such as load demand constraint, ramp rate constraint, amongst other constraints. In this work, we integrate a game theory based demand response program into the DEED problem. The game theory based demand response program determines the optimal hourly incentive to be offered to customers who sign up for load curtailment. The game theory model has in built mechanisms to ensure that the incentive offered the customers is greater than the cost of interruption while simultaneously being beneficial to the utility. The combined DEED and game theoretic demand response model presented in this work, minimizes fuel and emissions costs and simultaneously determines the optimal incentive and load curtailment customers have to perform for maximal power system relief. The developed model is tested on two test systems with industrial customers and obtained results indicate the practical benefits of the proposed model.

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## 1. Introduction

The Dynamic Economic Dispatch (DED) problem is a mathematical problem that has recently received much attention in the literature. The aim of the DED problem is to determine the optimal output of thermal generators that will supply electric load and ensure that the generator limits and ramp rate limits are not exceeded or violated [1]. Global increasing environmental awareness has led to many researchers considering the DED problem with emission considerations. The emissions of dangerous and harmful pollutants like SO<sub>2</sub>, NO<sub>x</sub>, CO and CO<sub>2</sub> have led to widespread calls for electric utilities to fashion out ways of curtailing these pollutants [2]. Some of the proposed solutions include installation of pollutant cleaning, switching to low-emission fuels, replacement of the aged fuel burners with cleaner ones, and emission dispatching [3]. Emission dispatching is however the most preferred solution because of its low capital burden and ease of implementation. The most common method of incorporating emission dispatch into the DED mathematical problem is termed the Dynamic Economic Emissions Dispatch (DEED). It is a multi-objective optimization problem that involves simultaneously minimizing

emission and fuel costs under conventional ramp rate constraints, load constraints, etc.

There have been a lot of research works on DED and DEED in the literature using different solution algorithms [4]. Most of the solution algorithms can be categorized as either conventional mathematical techniques or soft computing techniques. Examples of conventional mathematical techniques used in the literature include mixed integer quadratic programming [5], quadratic constrained programming [5] and benders decomposition [6] to mention a few. Major advantages of conventional mathematical techniques are that optimal solutions can often be guaranteed, they have fast computational times and they have no domain specific parameters to define. The disadvantage of these methods is that they often cannot handle non convex cost functions and are in danger of providing locally optimal solutions. Examples of soft computing methods include artificial physical optimization algorithm [7], artificial bee colony optimization [8], gravitational search algorithm [9], harmony search algorithm [10], biogeography based optimization [11] and spiral optimization algorithm [12], to mention a few. The advantage of soft computing methods is that they can usually handle non smooth or non-convex cost functions; however they have the drawback of long computational times and the need for definition of a large number of domain specific parameters.

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The aim of this research is to present a practical framework for integrating DEED with demand response programs. Demand response programs are essentially designed to control and curtail a customer's demand for electrical energy [13]. It is obvious that DEED and DR programs are necessary programs for effective power system management. Whilst DEED is concerned with optimal generator output at the supply side, DR programs are concerned with optimal load curtailment at the demand side. It therefore follows logically that integrating both formulations together with their interdependent constraints would yield a more practical system with more realistic solutions at both the supply and demand spectrum of the power grid [14]. There are only very few works that consider the DEED and DR jointly. Most of the works on DEED deal with novel soft computing algorithms [15–18] or integrating DEED with renewable energy sources like wind and solar energy [19–22]. Addition of these renewable sources serves as extra sources of generated power and thus increases the supply of electric power. However, it is well known that these sources are intermittent and there is the problem of grid integration. Other works on DR in the literature have focused extensively on the consumer side without consideration to the supply side [23–25]. A few works have considered the DEED problem with different DR formulations. In [14,26] the authors presented a framework for integrating renewable economic dispatch with demand response in a micro grid. The renewable sources considered were solar and wind energy. Obtained results from their simulations show that their approach reduces costs at both supply and demand side. However, the scope of the work concerns optimal dispatch for only renewable energy sources in a micro grid and not in a conventional grid. In [27,28] where the authors considered DEED with DR in a large grid, DR is considered as spinning reserves. The authors did not consider the individual customers cost of interruptions and the incentives offered to consumers to ensure that the customers were adequately attracted to participate. There is therefore a need for a practical mathematical formulation for the integration of both DEED and DR in such a manner that it can be applied to either a micro grid or a conventional grid. The DR program as of a necessity must also be structured so that it is attractive to customers. This synergy would enable us get optimal results at both the supply side and at the demand side instead of considering both independently [29]. To this end, we introduce a combined DEED with a game theory based DR program. It is termed a DR-DEED model and can be solved by either conventional mathematical techniques or by soft computing techniques. The game theory based DR program is a voluntary incentivised demand response program and is structured in such a way that the incentive offered to customers has to equal or be greater than their cost of participating in DR [34] and the greater your load curtailed, the greater your incentive. Moreover to show the benefits of our proposed approach, two scenarios are considered. Both scenarios are chosen to show that DR-DEED formulation can be applied either in a micro grid or in a larger conventional grid. The major contributions of this paper are: (i) The extension of the game theory demand response model for multiple customers for multiple time intervals. (ii) The incorporation of the extended game theory demand response model into the multi-objective DEED optimization problem. (iii) The addition of practical constraints like budgetary and customer maximum load constraints. (iv) The effectiveness of the final proposed mathematical model is shown with two scenarios. The first test case involves six generators and five customers while the second involves ten generators and seven customers. (v) Obtained results indicate that the proposed mathematical model leads to optimal generator dispatches, optimal load reduction and incentives. The rest of this paper is organized as follows: Section 2 presents the DEED formulations, Section 3 introduces the game theory based demand response

program formulation. Section 4 details the combined game theory and DEED mathematical model. Section 5 focuses on numerical simulations using the developed mathematical model and presents obtained results. The paper is concluded in Section 6.

## 2. The dynamic economic emission dispatch model

The DEED problem determines the optimal power generation schedule over a time interval whilst simultaneously minimizing fuel and emission costs. The mathematical representation is presented below [30]:

$$\min \sum_{t=1}^T \sum_{i=1}^I C_i(P_{i,t}), \quad (1)$$

$$\min \sum_{t=1}^T \sum_{i=1}^I E_i(P_{i,t}), \quad (2)$$

with

$$C_i(P_{i,t}) = a_i + b_i P_{i,t} + c_i P_{i,t}^2, \quad (3)$$

$$E_i(P_{i,t}) = e_i + f_i P_{i,t} + g_i P_{i,t}^2, \quad (4)$$

subject to the following network constraints:

$$\sum_{i=1}^I P_{i,t} = D_t + loss_t, \quad (5)$$

$$P_{i,\min} \leq P_{i,t} \leq P_{i,\max}, \quad (6)$$

$$-DR_i \leq P_{i,t+1} - P_{i,t} \leq UR_i, \quad (7)$$

where

$$loss_t = \sum_{i=1}^I \sum_{k=1}^K P_{i,t} B_{i,k} P_{k,t} \quad (8)$$

$P_{i,t}$  is the power generated from generator  $i$  at time  $t$ .

$C_i$  is the fuel cost of generator  $i$ .

$E_i$  is the emissions cost for generator  $i$ .

$D_t$  is the total system demand at time  $t$ .

$loss_t$  is the total system losses at time  $t$ .

$P_{i,\min}$  and  $P_{i,\max}$  are the minimum and maximum capacity of generator  $i$  respectively.

$DR_i$  and  $UR_i$  are the maximum ramp down and up rates of generator  $i$  respectively.

$a_i, b_i$  and  $c_i$  are the fuel cost coefficients of generator  $i$  respectively.

$e_i, f_i$  and  $g_i$  are the emission cost coefficients of generator  $i$  respectively.

$B_{i,k}$  is the  $ik$ th element of the loss coefficient square matrix of size  $I$ .

$I$  and  $T$  are the number of generators and the dispatch interval respectively.

The following is a brief description of the constraints:

- The first constraint (5) is the power balance constraint and ensures that at any time  $t$ , the total power generated equals the demand and total power losses.
- The second constraint (6) is the generation limits constraint and ensures that the generator limits are not exceeded.
- The final constraint (7) is the generator ramp rate limits constraints and ensures that the generator ramp rate limits are not violated.

The fuel cost and emission cost (Eqs. (3) and (4)) are both assumed to be quadratic functions of the generators active power output [30]. Power balance constraint, generation limit constraints and generator ramp rate limits constraints and network losses are the only practical constraints considered. Other constraints will be incorporated in future studies. The reason for this is that the authors do not want to lose sight of the main motivation of this work which is not to delve too deeply into various DEED setups but rather to present a practical framework for integrating DEED with DR programs.

The multi-objective optimization can be transformed into a single objective function using a weighting factor  $w$  subject to the same constraints (5)–(8).

$$\min \left[ w \sum_{t=1}^T \sum_{i=1}^I C_i(P_{i,t}) + (1-w) \sum_{t=1}^T \sum_{i=1}^I E_i(P_{i,t}) \right]. \quad (9)$$

### 3. Game theory based demand response design

Demand response, as one DSM approach can simply be defined as a change in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized [31,32]. In cases where the utility is a monopoly, an advantage of demand response programs is an improvement in power system efficiency and reliability. An added advantage is a reduction in operating costs and emissions. In deregulated markets, the same advantages in monopolistic markets apply. Furthermore, there is the advantage of reduced wholesale market prices [33].

One of the core requirements for electric utilities in the design of voluntary demand response programs is that the customer incentive (monetary benefit) always has to be greater than the customer outage cost [34]. It is noteworthy to add that typically the monetary benefit can either be in the form of reduced negotiated electricity rates or monetary disbursements [35]. Furthermore the customer always has the option of specifying how much electric power they are voluntarily shedding. In [34–36], the authors introduce the concept of demand management contracts. The authors define a contract as: “an agreement between utility and customer wherein the customer agrees to willingly shed load and in return receive monetary compensation” [34,35]. Furthermore in [32], the authors explicitly detail three crucial requirements in the design of such contracts. They are given as follows:

- The ability to differentiate between the customers: Different customers mean different load sizes and hence imply different outage costs.
- The ability to estimate customer outage costs: Since different customers have different costs, the utility should be able to accurately estimate individual customer outage costs [36].
- The ability to incorporate locational attributes of the customers into the contract design.

#### 3.1. Game theory based demand response formulations

To illustrate the concept of game theory demand contract formulations as was shown in [34], we begin initially by assuming a single customer.

We define  $c(\theta, x)$  as the cost incurred by a customer of type  $\theta$  who decreases power consumption by  $x$  MW. The benefit function of the customer is given as:

$$V_1(\theta, x, y) = y - c(\theta, x) \quad (10)$$

where  $y$  is the value of monetary compensation the customer receives. It follows logically, that the customer would only participate if  $V_1 \geq 0$ .

Similarly, the benefit function of the utility is given as:

$$V_2(\theta, \lambda) = \lambda x - y \quad (11)$$

$\lambda$  is the cost of not supplying power to a particular location on the grid. Under certain conditions, it might be costly for the power utility to supply electric power to some load buses on the grid. The electric utility can easily calculate this cost of not supplying power. This calculated value has hitherto been defined as the “value of power interruptibility” ( $\lambda$ ) [34] and is typically calculated from optimal power flow (OPF).

The objective of the utility is to maximize its benefit function:

$$\max_{x,y} [\lambda x - y] \quad (12)$$

- $\theta$ : “customer type”, normalized in  $[0, 1]$ .
- $x$ : quantity of power reduced by a participating customer.
- $c(x, \theta)$ : cost of reducing  $x$  MW by customer of type  $\theta$ .
- $\lambda$ : “value of power interruptibility”.

#### 3.2. Customer cost function

As stated before,  $c(\theta, x)$  is the cost incurred by a customer of type  $\theta$  who decreases power consumption by  $x$  MW. In this work, it is assumed that the mathematical function is given as in [34]:

$$c(\theta, x) = K_1 x^2 + K_2 x - K_2 x \theta \quad (13)$$

where  $K_1$  and  $K_2$  are cost co-efficients.  $\theta$  is the customer type [36] and is used to categorize the different kinds of customers based on their desire/readiness to curb electric power.

$\theta$  is normalized in the interval  $0 \leq \theta \leq 1$ , thus  $\theta = 1$  for the most willing customer and  $\theta = 0$  for the least willing. We provide a summary of all the conditions that the cost function must satisfy:

- Assumed form  $c(\theta, x) = K_1 x^2 + K_2 x - K_2 x \theta$ .
- $K_2 x \theta$  term sorts customers by way of  $\theta$ .
- As  $\theta$  increases marginal cost decreases: The most willing customer ( $\theta = 1$ ) has the least marginal cost and thus has the highest marginal benefit, whilst the least willing customer ( $\theta = 0$ ) has the highest marginal cost and thus the lowest marginal benefit.
- $\frac{\partial c}{\partial x} = 2K_1 x + K_2 - K_2 \theta$ .
- Non-negative marginal cost.
- Increasing marginal cost (convex cost function).
- Zero curtailment: curbing zero power should cost ( $c(\theta, 0) = 0$ ).

The concept of contract formulations to more than one customer is given as in [34]:

Thus, if  $y_j$  is the amount of payment paid to customer  $j$ , the customer benefit is obtained from:

$$u_j = y_j - (K_1 x_j^2 + K_2 x_j - K_2 x_j \theta_j), \quad \text{for } j = 1, \dots, J. \quad (14)$$

The utility benefit is determined from:

$$u_0 = \sum_{j=1}^J \lambda_j x_j - y_j \quad (15)$$

The objective is thus to maximize the expected utility benefit

$$\max_{x,y} \sum_{j=1}^J [\lambda_j x_j - y_j] \quad (16)$$

s.t.

$$y_j - (K_1 x_j^2 + K_2 x_j - K_2 x_j \theta_j) \geq 0, \quad \text{for } j = 1, \dots, J. \quad (17)$$

$$y_j - (K_1 x_j^2 + K_2 x_j - K_2 x_j \theta_j) \geq y_{j-1} - (K_1 x_{j-1}^2 + K_2 x_{j-1} - K_2 x_{j-1} \theta_{j-1}),$$

for  $j = 2, \dots, J$ . (18)

The mathematical formulation presented above has two variables; the power curtailed ( $x$  MW) and the incentive paid ( $y$  \$). Furthermore, the two constraints are defined and described below:

The “*Individual rationality constraint*” (Constraint (17)) ensures that each customer benefit is greater than or exceeds zero.

The “*Incentive compatibility constraint*” (Constraint (18)) ensures that customers are appropriately compensated for their level of load curbed).

In this paper, we extend the demand management contract formulations (Eqs. (14)–(18)) to more than one time interval and incorporate it into the DEED problem. We also modify the *individual rationality constraint* and the *incentive compatibility constraint* and enforce it over the total optimization horizon (a day) instead of a single time interval (every hour). This we believe makes more practical and economic sense. Finally, we add maximum power targets and total budget as practical constraints into the model. The mathematical model is given as:

$$\max_{x,y} \sum_{t=1}^T \sum_{j=1}^J [\lambda_{j,t} x_{j,t} - y_{j,t}] \tag{19}$$

s.t.

$$\sum_{t=1}^T [y_{j,t} - (K_{1,j} x_{j,t}^2 + K_{2,j} x_{j,t} - K_{2,t} x_{j,t} \theta_j)] \geq 0, \quad \text{for } j = 1, \dots, J. \tag{20}$$

$$\sum_{t=1}^T [y_{j,t} - (K_{1,j} x_{j,t}^2 + K_{2,j} x_{j,t} - K_{2,t} x_{j,t} \theta_j)] \geq \sum_{t=1}^T [y_{j-1,t} - (K_{1,j-1} x_{j-1,t}^2 + K_{2,j-1} x_{j-1,t} - K_{2,j-1} x_{j-1,t} \theta_{j-1})],$$

for  $j = 2, \dots, J$ . (21)

$$\sum_{t=1}^T \sum_{j=1}^J y_{j,t} \leq UB \tag{22}$$

$$\sum_{t=1}^T x_{j,t} \leq CM_j, \quad \text{for } j = 1, \dots, J. \tag{23}$$

where  $UB$  is the utility's total budget and  $CM_j$  is the daily limit of interruptible power for customer  $j$ .

Constraint (20) ensures that the total daily incentive received by a customer exceeds or equals his daily cost of interruption.

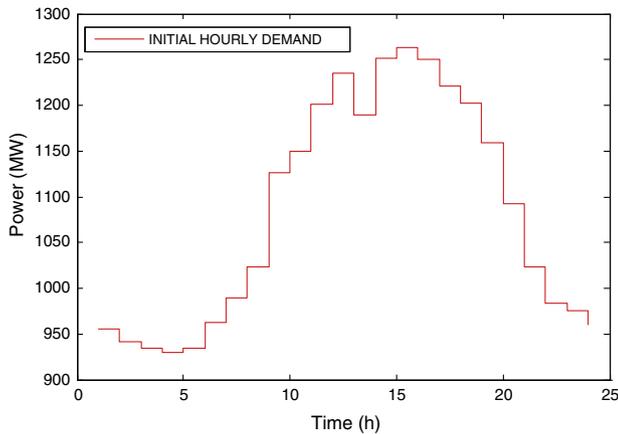


Fig. 1. Total initial hourly demand (Scenario 1).

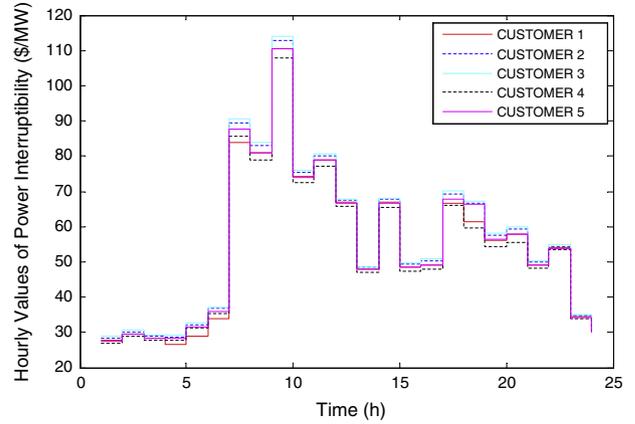


Fig. 2. Hourly values of power interruptibility ( $\lambda_{j,t}$ ) for different customers (Scenario 1).

Table 1

Customer cost function coefficients, customer type and daily customer power limit (Scenario 1).

| Customer ( $j$ ) | $K_{1,j}$ | $K_{2,j}$ | $\theta_j$ | $CM_j$ (MW) |
|------------------|-----------|-----------|------------|-------------|
| 1                | 1.847     | 11.64     | 0          | 200         |
| 2                | 1.378     | 11.63     | 0.1734     | 280         |
| 3                | 1.079     | 11.32     | 0.4828     | 410         |
| 4                | 0.9124    | 11.5      | 0.7208     | 500         |
| 5                | 0.8794    | 11.21     | 1          | 700         |

Constraint (21) ensures that the greater the customer power curtailed, the greater the customer benefit.

Constraint (22) ensures that the total incentive paid by the utility is less than or equal to the utility's budget.

Constraint (23) ensures that the total daily power curtailed by each customer is less than or equal to its daily limit of interruptible power.

In the next section, we present the combined DEED and game theory based demand response model.

#### 4. Combined DEED and game theory based mathematical model

Thus, the weighted single objective DR-DEED mathematical formulation from the utility perspective can be given as:

$$\min \left[ w_1 \left[ \sum_{t=1}^T \sum_{i=1}^I C_i(P_{i,t}) \right] + w_2 \left[ \sum_{t=1}^T \sum_{i=1}^I E_i(P_{i,t}) \right] - w_3 \left[ \sum_{t=1}^T \sum_{j=1}^J [\lambda_{j,t} x_{j,t} - y_{j,t}] \right] \right] \tag{24}$$

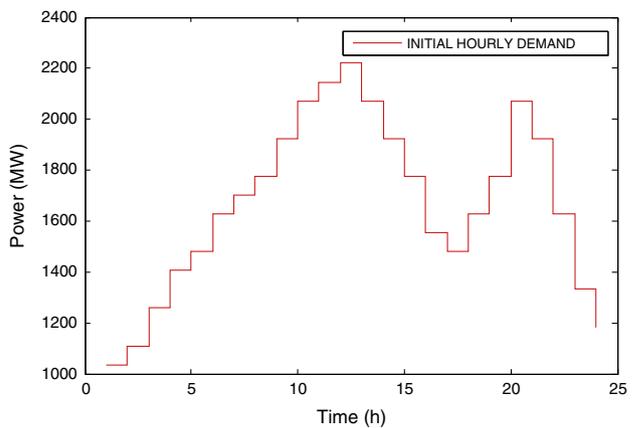


Fig. 3. Total initial hourly demand (Scenario 2).

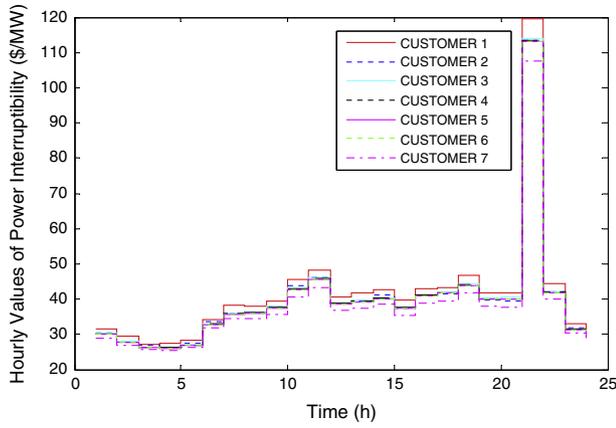


Fig. 4. Hourly values of power interruptibility ( $\lambda_{j,t}$ ) for different customers (Scenario 2).

Table 2  
Customer cost function coefficients, customer type and daily customer power limit (Scenario 2).

| Customer ( $j$ ) | $K_{1,j}$ | $K_{2,j}$ | $\theta_j$ | $CM_j$ (MW) |
|------------------|-----------|-----------|------------|-------------|
| 1                | 1.847     | 11.64     | 0          | 180         |
| 2                | 1.378     | 11.63     | 0.14       | 230         |
| 3                | 1.079     | 11.32     | 0.26       | 310         |
| 4                | 0.9124    | 11.5      | 0.37       | 390         |
| 5                | 0.8794    | 11.21     | 0.55       | 440         |
| 6                | 1.378     | 11.63     | 0.84       | 530         |
| 7                | 1.5231    | 11.5      | 1          | 600         |

subject to the following network constraints:

$$\sum_{i=1}^I P_{i,t} = D_t + loss_t - \sum_{j=1}^J x_{j,t}, \quad (25)$$

$$P_{i,\min} \leq P_{i,t} \leq P_{i,\max}, \quad (26)$$

$$-DR_i \leq P_{i,t+1} - P_{i,t} \leq UR_i, \quad (27)$$

$$\sum_{t=1}^T [y_{j,t} - (K_{1,j}x_{j,t}^2 + K_{2,j}x_{j,t} - K_{2,t}x_{j,t}\theta_j)] \geq 0, \quad \text{for } j = 1, \dots, J. \quad (28)$$

$$\begin{aligned} & \sum_{t=1}^T [y_{j,t} - (K_{1,j}x_{j,t}^2 + K_{2,j}x_{j,t} - K_{2,t}x_{j,t}\theta_j)] \\ & \geq \sum_{t=1}^T [y_{j-1,t} - (K_{1,j-1}x_{j-1,t}^2 + K_{2,j-1}x_{j-1,t} - K_{2,j-1}x_{j-1,t}\theta_{j-1})], \\ & \text{for } j = 2, \dots, J. \end{aligned} \quad (29)$$

$$\sum_{t=1}^T \sum_{j=1}^J y_{j,t} \leq UB \quad (30)$$

$$\sum_{t=1}^T x_{j,t} \leq CM_j, \quad \text{for } j = 1, \dots, J. \quad (31)$$

where  $w_1$ ,  $w_2$  and  $w_3$  are the weights and the following condition is required to be satisfied when choosing weights:

$$w_1 + w_2 + w_3 = 1 \quad (32)$$

$$loss_t = \sum_{i=1}^I \sum_{k=1}^K P_{i,t} B_{i,k} P_{k,t} \quad (33)$$

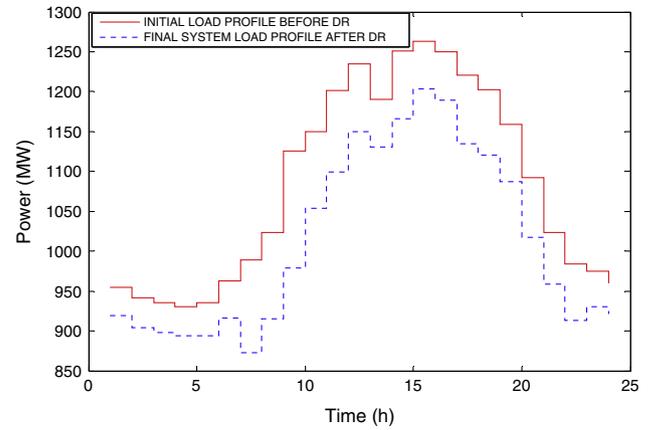


Fig. 5. Total load profile before and after demand response (Scenario 1).

The variables to be determined by the optimization model are  $x_{j,t}$ ,  $y_{j,t}$  and  $P_{i,t}$ .

### 5. Numerical simulations, obtained results and discussions

In this section, we present the parameters and results of the proposed optimization model given in Eqs. (24)–(33).

#### 5.1. Customer side data

A number of assumptions were made concerning the mathematical model. It is assumed that the customer makes known to the utility, its daily limit of interruptible power ( $CM_j$ ). This customer daily limit of interruptible power determines the customer willingness ( $\theta_j$ ). As stated earlier  $\theta_j$  is normalized to the interval  $0 \leq \theta \leq 1$ , thus Customer 1 who has the lowest  $CM_j$  has a  $\theta_j$  value of 0 and Customer 2 who has the highest  $CM_j$  has a  $\theta_j$  value of 1 and all other values of  $CM_j$  fall within the normalized interval. The customers are thus ranked in order of increasing willingness to curtail power. It is also assumed, the utility knows the different coefficients of the customers outage cost function ( $K_{1,j}$ ) and ( $K_{2,j}$ ). In this work Locational Marginal Prices (LMP) [33] are used as  $\lambda$  or “locational attribute” or “value of power interruptibility” as this gives the cost of NOT delivering power to a specific location or customer [36]. Since we need hourly  $\lambda$  values, hourly Locational Marginal Prices (LMP) are used from the Pennsylvania–New Jersey–Maryland (PJM) Market [37].

The goal is to obtain the optimal customer power to be curtailed ( $x_{j,t}$ ), optimal incentive to be paid to customers ( $y_{j,t}$ ) and power generated from all generators ( $P_{i,t}$ ). Two test scenarios are utilized in this work to investigate the effectiveness of the proposed mathematical formulations (Eqs. (23)–(31)). The reason for choosing these scenarios is because they show that optimizing DR-DEED jointly instead of independently and enables optimal results at both the supply side and at the demand side instead of considering both independently. The two scenarios also show that integration of both DEED and DR can be applied to either a micro grid or a conventional grid. The first scenario consists of six generator units and five aggregated customers. The maximum load is 1263 MW with a single load peak synonymous with industrial customers. The second scenario is a larger power system with greater demand, greater utility budget, more number of generators and customers. Scenario 2 consists of ten generator units and seven aggregated customers. The maximum load is 2220 MW with two peaks synonymous with residential customers. In both scenarios, it is assumed that the utility gives equal preference to each of the three objectives. Using a weighted sum approach we convert the three

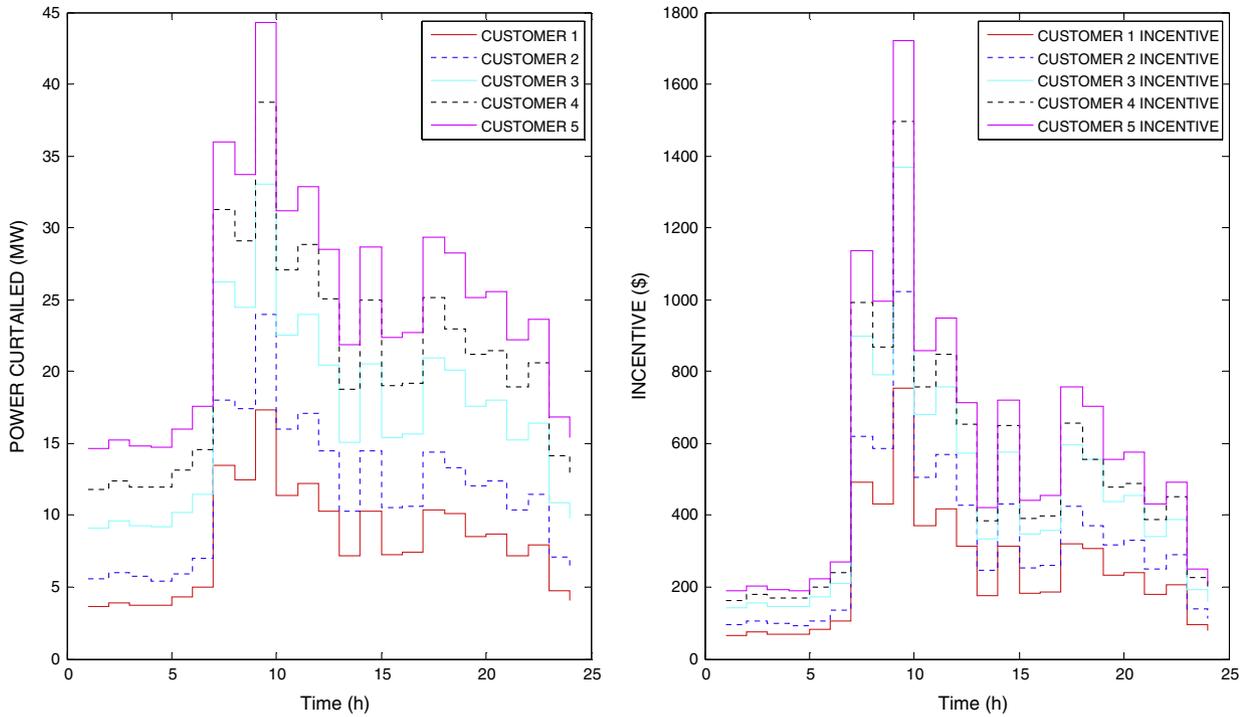


Fig. 6. Optimal power curtailed and optimal incentive for all customers (Scenario 1).

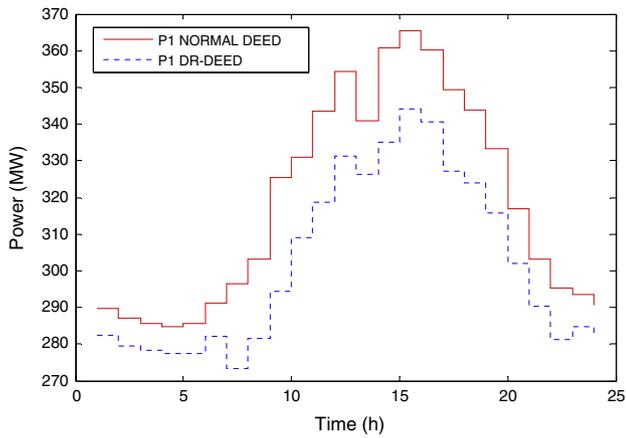


Fig. 7. Generation output of unit 1 (Scenario 1).

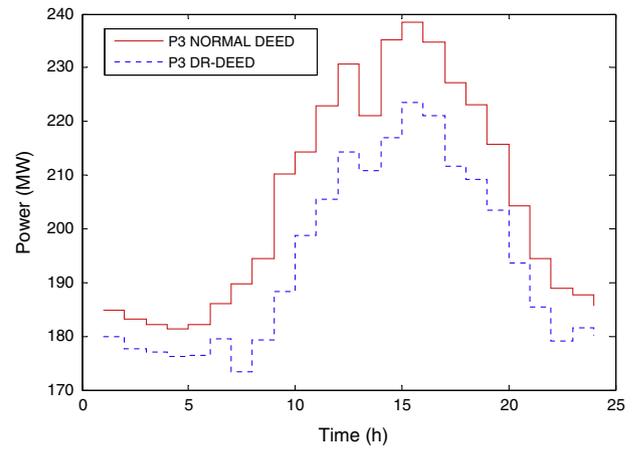


Fig. 9. Generation output of unit 3 (Scenario 1).

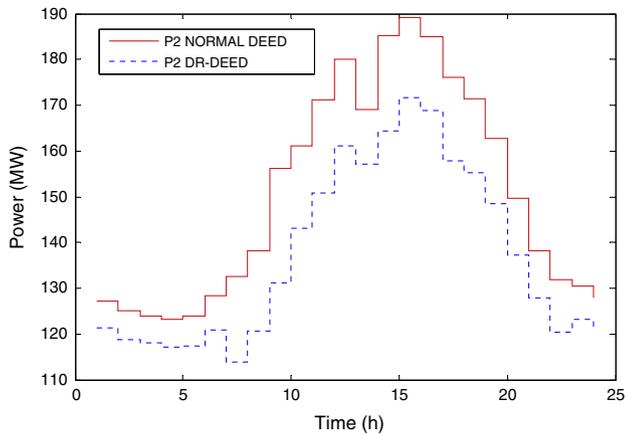


Fig. 8. Generation output of unit 2 (Scenario 1).

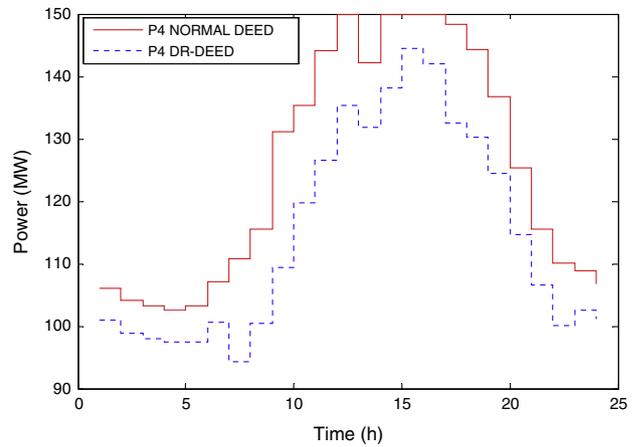


Fig. 10. Generation output of unit 4 (Scenario 1).

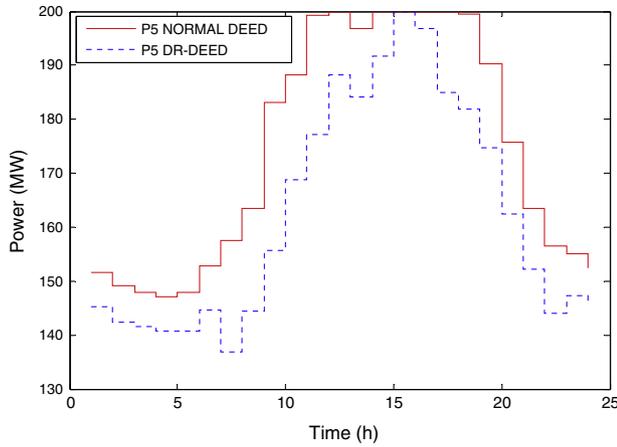


Fig. 11. Generation output of unit 5 (Scenario 1).

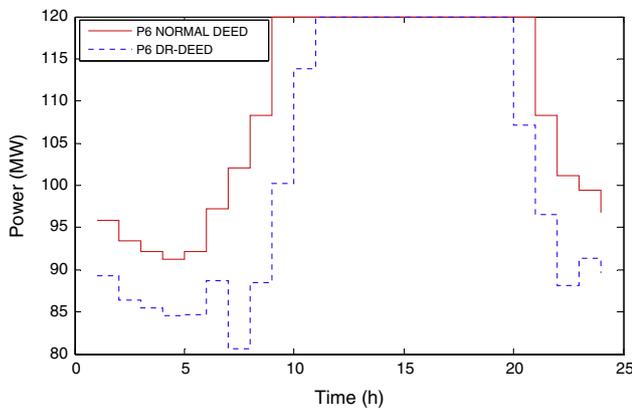


Fig. 12. Generation output of unit 6 (Scenario 1).

**Table 3**  
Final results from the combined DR-DEED program (Scenario 1).

|                | Total power saved (MW) | Total incentive received (\$) |
|----------------|------------------------|-------------------------------|
| Customer 1     | 195.18                 | 5775.42                       |
| Customer 2     | 276.01                 | 7791.49                       |
| Customer 3     | 405.23                 | 10774.40                      |
| Customer 4     | 495.07                 | 11995.48                      |
| Customer 5     | 581.53                 | 13663.22                      |
| <b>Utility</b> | <b>1953.02</b>         | <b>50000.00</b>               |

Bold text shows the total power saved and total incentive paid to customers by the utility.

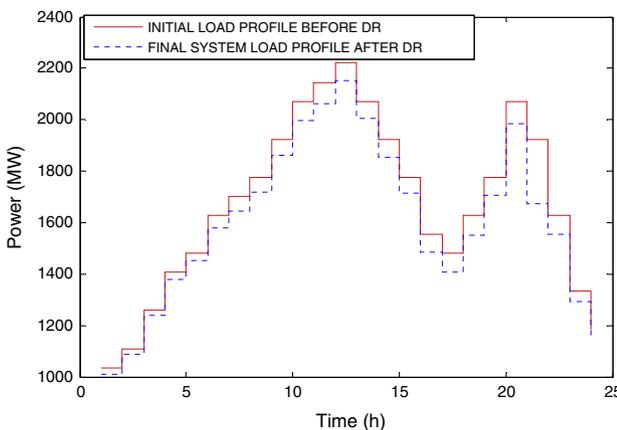


Fig. 13. Total load profile before and after demand response (Scenario 2).

objectives to a single objective and give equal weights to the three objectives, thus  $w_1 = w_2 = w_3 = \frac{1}{3}$ .

5.2. Scenario 1 (six generator units and five customers)

The fuel cost coefficients and the emission cost coefficients are obtained from [30] and shown in Table A1 in the appendix. Fig. 1 presents the total initial hourly demand, with one mid-day peak which is consistent with industrial customers, Fig. 2 gives the hourly values of power interruptibility ( $\lambda_{j,t}$ ) obtained from the PJM Market on the 30th of April 2014 and Table 1 gives the cost function coefficients, customer type and daily customer power limit. It is further assumed that the utility has a daily budget of \$50000. The transmission loss formula coefficients for the six unit test system are given by Eq. (34).

$$B = 10^{-4} \times \begin{bmatrix} 0.420 & 0.051 & 0.045 & 0.057 & 0.078 & 0.066 \\ 0.051 & 0.180 & 0.039 & 0.048 & 0.045 & 0.060 \\ 0.045 & 0.039 & 0.195 & 0.051 & 0.072 & 0.057 \\ 0.057 & 0.048 & 0.051 & 0.213 & 0.090 & 0.075 \\ 0.078 & 0.045 & 0.072 & 0.090 & 0.207 & 0.096 \\ 0.066 & 0.060 & 0.057 & 0.075 & 0.096 & 0.255 \end{bmatrix} \text{ per MW} \tag{34}$$

5.3. Scenario 2 (ten generator units and seven customers)

The fuel cost coefficients and the emission cost coefficients are obtained from [4] and shown in Table A.2. Fig. 3 presents the total initial hourly demand. Fig. 4 gives the hourly values of power interruptibility ( $\lambda_{j,t}$ ) obtained from the PJM Market on the 1st of May 2014 and Table 2 gives the cost function coefficients, customer type and daily customer power limit. It is further assumed that the utility has a daily budget of \$100000. The transmission loss formula coefficients for the ten unit test system are given by Eq. (35).

$$B = 10^{-5} \times \begin{bmatrix} 4.9 & 1.4 & 1.5 & 1.5 & 1.6 & 1.7 & 1.7 & 1.8 & 1.9 & 2.0 \\ 1.4 & 4.5 & 1.6 & 1.6 & 1.7 & 1.5 & 1.5 & 1.6 & 1.8 & 1.8 \\ 1.5 & 1.6 & 3.9 & 1.0 & 1.2 & 1.2 & 1.4 & 1.4 & 1.6 & 1.6 \\ 1.5 & 1.6 & 1.0 & 4.0 & 1.4 & 1.0 & 1.1 & 1.2 & 1.4 & 1.5 \\ 1.6 & 1.7 & 1.2 & 1.4 & 3.5 & 1.1 & 1.3 & 1.3 & 1.5 & 1.6 \\ 1.7 & 1.5 & 1.2 & 1.0 & 1.1 & 3.6 & 1.3 & 1.2 & 1.4 & 1.5 \\ 1.7 & 1.5 & 1.4 & 1.1 & 1.3 & 1.2 & 3.8 & 1.6 & 1.6 & 1.8 \\ 1.8 & 1.6 & 1.4 & 1.2 & 1.3 & 1.2 & 1.6 & 4.0 & 1.5 & 1.6 \\ 1.9 & 1.8 & 1.6 & 1.4 & 1.5 & 1.4 & 1.6 & 1.5 & 4.2 & 1.9 \\ 2.0 & 1.8 & 1.6 & 1.5 & 1.6 & 1.5 & 1.8 & 1.6 & 1.9 & 4.4 \end{bmatrix} \text{ per MW} \tag{35}$$

5.4. Solution methodology and results

The optimization model is built and solved using the CONOPT solver in the Advanced Interactive Multidimensional Modelling System (AIMMS) [38] on a computer with an Intel (R) Core™ processor and 4 GB of RAM. All scenarios are solved in less than 1 s. AIMMS is an Algebraic Modelling language (AML) used for solving optimization and scheduling type mathematical problems. A major advantage of using AIMMS is the similarity of the software’s syntax to the mathematical representation of optimization problems. The software supports the solution of a large number of optimization problem types and allows for an easy reproduction of their results. For Scenario 1, Fig. 5 shows the total load demand profile before and after demand response, Fig. 6 shows the optimal power curtailed and optimal determined incentive for all the five customers. Figs. 7–12 show the optimal power generated for all generators under normal DEED and after the DR program schedule has been

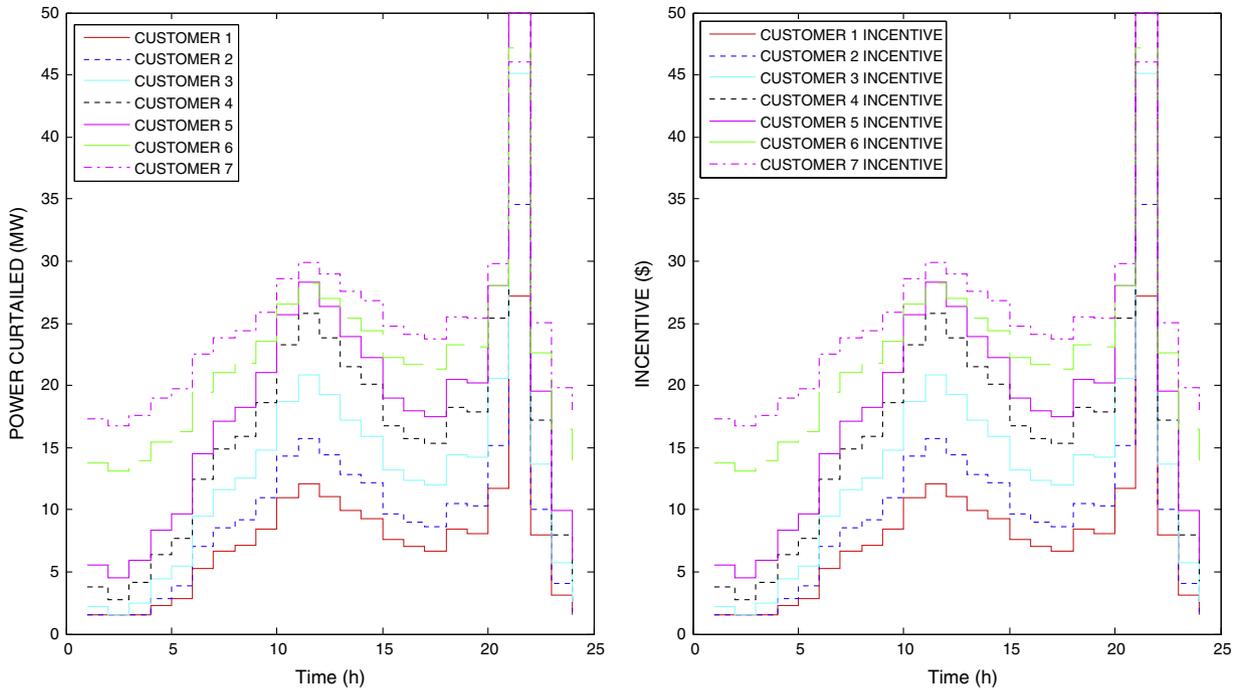


Fig. 14. Optimal power curtailed and optimal incentive for all customers (Scenario 2).

Table 4  
Final results from the combined DR-DEED program (Scenario 2).

|                | Total power saved (MW) | Total incentive received (\$) |
|----------------|------------------------|-------------------------------|
| Customer 1     | 180.00                 | 5849.30                       |
| Customer 2     | 230.00                 | 6901.25                       |
| Customer 3     | 310.00                 | 8992.28                       |
| Customer 4     | 390.00                 | 10762.01                      |
| Customer 5     | 440.00                 | 11338.26                      |
| Customer 6     | 530.00                 | 18720.82                      |
| Customer 7     | 590.57                 | 23448.62                      |
| <b>Utility</b> | <b>2670.57</b>         | <b>86012.54</b>               |

Bold text shows the total power saved and total incentive paid to customers by the utility.

Table 5  
Various weighting factor values.

|                | $w_1$         | $w_2$         | $w_3$         |
|----------------|---------------|---------------|---------------|
| Base Case (BC) | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| Case 2 (C2)    | 1             | 0             | 0             |
| Case 3 (C3)    | 0             | 1             | 0             |
| Case 4 (C4)    | 0             | 0             | 1             |

Table 6  
Optimal DR-DEED results with various weighting factor values (Test System 1).

|           | Cost (DR-DEED) (\$) | Emissions (DR-DEED) (lb) | Power generated (DR-DEED) (MW) | Loss (DR-DEED) (MW) |
|-----------|---------------------|--------------------------|--------------------------------|---------------------|
| <b>BC</b> | <b>291898.16</b>    | <b>24474.04</b>          | <b>24266.59</b>                | <b>265.61</b>       |
| C2        | 288430.67           | 31426.55                 | 24205.79                       | 308.20              |
| C3        | 299397.36           | 21106.33                 | 24176.39                       | 231.01              |
| C4        | 300815.08           | 21473.48                 | 24291.10                       | 234.85              |

Bold text shows the base case (BC) when equal preference is given to the three objectives.

Table 7  
Optimal DR-DEED results with various weighting factor values (Test System 2).

|           | Cost (DR-DEED) (\$) | Emissions (DR-DEED) (lb) | Power generated (DR-DEED) (MW) | Loss (DR-DEED) (MW) |
|-----------|---------------------|--------------------------|--------------------------------|---------------------|
| <b>BC</b> | <b>989439.22</b>    | <b>192743.96</b>         | <b>38574.55</b>                | <b>1137.11</b>      |
| C2        | 968899.75           | 356332.96                | 38682.66                       | 1254.66             |
| C3        | 994464.46           | 183068.82                | 38548.88                       | 1120.88             |
| C4        | 1006653.84          | 210026.09                | 39125.16                       | 1190.38             |

Bold text shows the base case (BC) when equal preference is given to the three objectives.

implemented. Table 3 presents the final parameters from the combined DR-DEED program for the case of Scenario 1.

For Scenario 2, Fig. 13 shows the total load demand profile before and after demand response, Fig. 14 shows the optimal power curtailed and optimal determined incentive for all the seven customers. Table 4 presents the final parameters from the combined DR-DEED program for Scenario 2.

5.5. Discussion of results

Considering Scenario 1, from Fig. 5 it is observed that the combined DR-DEED model brings about a reduction in the load profile. As shown in Fig. 6, each customer contributes to the eventual power reduction shown in Fig. 5. Another observation as shown in Fig. 6 is that the incentive received by customers increases as the customer willingness increases. Thus the most willing customer (with  $CM_j$  of 700 MW and  $\theta_j$  of 1) has a higher incentive than the least willing customer (with  $CM_j$  of 200 MW and  $\theta_j$  of 0). Figs. 7–12 simply show that the generators actually reduce power

**Table 8**  
Optimal DEED results with various weighting factor values (Test System 1).

|           | Cost (DEED) (\$) | Emissions (DEED) (lb) | Power generated (DEED) (MW) | Loss (DEED) (MW) |
|-----------|------------------|-----------------------|-----------------------------|------------------|
| $w = 0$   | 322786.57        | 25639.31              | 26233.14                    | 279.14           |
| $w = 0.5$ | <b>317046.37</b> | <b>28029.95</b>       | <b>26262.74</b>             | <b>308.74</b>    |
| $w = 1$   | 315021.43        | 35096.95              | 26308.30                    | 354.30           |

Bold text shows the base case (BC) when equal preference is given to the three objectives.

**Table 9**  
Optimal DEED results with various weighting factor values (Test System 2).

|           | Cost (DEED) (\$)  | Emissions (DEED) (lb) | Power generated (DEED) (MW) | Loss (DEED) (MW) |
|-----------|-------------------|-----------------------|-----------------------------|------------------|
| $w = 0$   | 1057670.13        | 248103.17             | 41438.53                    | 1330.53          |
| $w = 0.5$ | <b>1052722.84</b> | <b>249936.27</b>      | <b>41440.51</b>             | <b>1332.51</b>   |
| $w = 1$   | 1035411.67        | 380595.81             | 41540.71                    | 1432.71          |

Bold text shows the base case (BC) when equal preference is given to the three objectives.

**Table A.1**  
Data of the six-unit system.

| $i$ | $a_i$ (\$/h) | $b_i$ (\$/MW h) | $c_i$ (\$/MW <sup>2</sup> h) | $e_i$ (lb/h) | $f_i$ (lb/MW h) | $g_i$ (lb/MW <sup>2</sup> h) | $P_{i,min}$ (MW) | $P_{i,max}$ (MW) | $DR_i$ (MW/h) | $UR_i$ (MW/h) |
|-----|--------------|-----------------|------------------------------|--------------|-----------------|------------------------------|------------------|------------------|---------------|---------------|
| 1   | 240          | 7               | 0.007                        | 13.8593      | 0.32767         | 0.00419                      | 100              | 500              | 120           | 80            |
| 2   | 200          | 10              | 0.0095                       | 13.8593      | 0.32767         | 0.00419                      | 50               | 200              | 90            | 50            |
| 3   | 220          | 8.5             | 0.009                        | 40.2669      | -0.54551        | 0.00683                      | 80               | 300              | 100           | 65            |
| 4   | 200          | 11              | 0.009                        | 40.2669      | -0.54551        | 0.00683                      | 50               | 150              | 90            | 50            |
| 5   | 220          | 10.5            | 0.008                        | 42.8955      | -0.51116        | 0.00461                      | 50               | 200              | 90            | 50            |
| 6   | 190          | 12              | 0.0075                       | 42.8955      | -0.51116        | 0.00461                      | 50               | 120              | 90            | 50            |

**Table A.2**  
Data of the ten-unit system.

| $i$ | $a_i$ (\$/h) | $b_i$ (\$/MW h) | $c_i$ (\$/MW <sup>2</sup> h) | $e_i$ (lb/h) | $f_i$ (lb/MW h) | $g_i$ (lb/MW <sup>2</sup> h) | $P_{i,min}$ (MW) | $P_{i,max}$ (MW) | $DR_i$ (MW/h) | $UR_i$ (MW/h) |
|-----|--------------|-----------------|------------------------------|--------------|-----------------|------------------------------|------------------|------------------|---------------|---------------|
| 1   | 958.2        | 21.6            | 0.00043                      | 360.0012     | -3.9864         | 0.04702                      | 150              | 470              | 80            | 80            |
| 2   | 1313.6       | 21.05           | 0.00063                      | 350.0056     | -3.9524         | 0.04652                      | 135              | 460              | 80            | 80            |
| 3   | 604.97       | 20.81           | 0.00039                      | 330.0056     | -3.9023         | 0.04652                      | 73               | 340              | 80            | 80            |
| 4   | 471.6        | 23.9            | 0.0007                       | 330.0056     | -3.9023         | 0.04652                      | 60               | 300              | 50            | 50            |
| 5   | 480.29       | 21.62           | 0.00079                      | 13.8593      | 0.3277          | 0.0042                       | 73               | 243              | 50            | 50            |
| 6   | 601.75       | 17.87           | 0.00056                      | 13.8593      | 0.3277          | 0.0042                       | 57               | 160              | 50            | 50            |
| 7   | 502.7        | 16.51           | 0.00211                      | 40.2669      | -0.5455         | 0.0068                       | 20               | 130              | 30            | 30            |
| 8   | 639.4        | 23.23           | 0.0048                       | 40.2669      | -0.5455         | 0.0068                       | 47               | 120              | 30            | 30            |
| 9   | 455.6        | 19.58           | 0.10908                      | 42.8955      | -0.5112         | 0.0046                       | 20               | 80               | 30            | 30            |
| 10  | 692.4        | 22.54           | 0.00951                      | 42.8955      | -0.5112         | 0.0046                       | 55               | 55               | 30            | 30            |

output in light of the demand reduction by willing customers. This shows that demand response programs especially in the form of incentive payments are useful in altering customer load patterns and total system demand. This reduction of the customers' load patterns in turn reduces the probability of events like blackouts and brown outs thus improving the reliability or security of the power system. Table 3 details the total power saved and total incentive received by each customer over a 24 h period. As can be seen from Table 3, the higher the customer willingness, the greater the power curtailed and incentive received.

Obtained results from Scenario 2 which consists of more generators and customers than Scenario 1, corroborate the conclusions drawn from Scenario 1. The combined DR-DEED formulation reduces total demand over a 24 h period by 2670.57 MW (see Figs. 13 and 14) and an inspection of the incentive received by each customer (see Fig. 14 and Table 4) shows that the customers are compensated commensurate with the level of load they are actually willing to curbe (i.e. customer willingness). Furthermore, there is also a reduction in power generated by the generators due to the curtailed customer demand. The optimal customer power curtailed ( $X_{j,t}$ ), optimal incentive paid to customers ( $Y_{j,t}$ ) and power generated from all generators ( $P_{i,t}$ ) (the three variables obtained by the mathematical model) for both Scenario 1 and Scenario 2 are given in the appendix. (Tables A.3–A.8).

In the simulations done, it is assumed that the utility gives equal preference to the three objectives and gives equal weights to the three objectives, thus  $w_1 = w_2 = w_3 = \frac{1}{3}$ . It is important in multi-objective optimization problems with conflicting and competing objectives, to show how giving increased preference to one objective at the expense of the other influences the obtained results. We therefore present an analysis of optimization results using the base case when the utility gives equal preference to each objective ( $w_1 = w_2 = w_3 = \frac{1}{3}$ ), when the utility chooses to minimize cost alone ( $w_1 = 1, w_2 = w_3 = 0$ ), when the utility chooses to maximize emissions alone ( $w_2 = 1, w_1 = w_3 = 0$ ) and when the utility chooses to maximize its DR benefit alone ( $w_3 = 1, w_1 = w_2 = 0$ ). The four parameters evaluated are the total generator costs (\$), total emissions (lb), total generator power (MW) and total power losses (MW). Table 5 gives the various weight cases and Tables 6 and 7 give the various results for Scenario 1 and 2 respectively. They show that the best results are obtained when DR and DEED are considered jointly. Considering DR alone i.e. maximizing only the utility benefit (C4), produces suboptimal results. C2 always gives the lowest cost, but gives the highest emissions and the highest losses. C3 gives the lowest emissions but does not give the lowest cost. Depending on the most pressing objective of the utility, the model can be adjusted accordingly. However analyses of the results show that the results are best with cases BC and C3.

**Table A.3**  
Optimal customer power curtailed ( $x_{j,t}$ ) (Scenario 1).

| $t$            | $j$   |       |       |       |       |
|----------------|-------|-------|-------|-------|-------|
|                | 1     | 2     | 3     | 4     | 5     |
| $x_{j,t}$ (MW) |       |       |       |       |       |
| 1              | 3.64  | 5.61  | 9.10  | 11.76 | 14.67 |
| 2              | 3.93  | 5.99  | 9.58  | 12.36 | 15.25 |
| 3              | 3.74  | 5.73  | 9.24  | 11.97 | 14.80 |
| 4              | 3.72  | 5.38  | 9.20  | 11.99 | 14.75 |
| 5              | 4.29  | 5.90  | 10.18 | 13.14 | 15.96 |
| 6              | 5.02  | 7.01  | 11.46 | 14.57 | 17.55 |
| 7              | 13.49 | 18.00 | 26.27 | 31.27 | 35.98 |
| 8              | 12.43 | 17.41 | 24.47 | 29.12 | 33.67 |
| 9              | 17.31 | 23.97 | 33.02 | 38.76 | 44.25 |
| 10             | 11.36 | 16.00 | 22.55 | 27.10 | 31.22 |
| 11             | 12.22 | 17.11 | 23.95 | 28.80 | 32.87 |
| 12             | 10.25 | 14.51 | 20.46 | 25.05 | 28.51 |
| 13             | 7.14  | 10.32 | 15.09 | 18.80 | 21.86 |
| 14             | 10.27 | 14.52 | 20.54 | 25.01 | 28.65 |
| 15             | 7.30  | 10.52 | 15.42 | 19.04 | 22.37 |
| 16             | 7.40  | 10.66 | 15.66 | 19.19 | 22.75 |
| 17             | 10.40 | 14.44 | 20.95 | 25.12 | 29.36 |
| 18             | 10.15 | 13.29 | 20.14 | 22.98 | 28.26 |
| 19             | 8.52  | 12.08 | 17.59 | 21.18 | 25.11 |
| 20             | 8.71  | 12.39 | 17.98 | 21.45 | 25.60 |
| 21             | 7.18  | 10.40 | 15.23 | 18.94 | 22.18 |
| 22             | 7.92  | 11.43 | 16.44 | 20.59 | 23.66 |
| 23             | 4.73  | 7.11  | 10.91 | 14.12 | 16.82 |
| 24             | 4.07  | 6.21  | 9.78  | 12.75 | 15.44 |

**Table A.5**  
Optimal power generated by generators ( $P_{i,t}$ ) (Scenario 1).

| $t$            | $i$    |        |        |        |        |        |
|----------------|--------|--------|--------|--------|--------|--------|
|                | 1      | 2      | 3      | 4      | 5      | 6      |
| $P_{i,t}$ (MW) |        |        |        |        |        |        |
| 1              | 282.56 | 121.40 | 179.94 | 100.98 | 145.23 | 89.29  |
| 2              | 279.47 | 118.87 | 177.75 | 98.79  | 142.48 | 86.43  |
| 3              | 278.38 | 117.98 | 176.99 | 98.02  | 141.52 | 85.43  |
| 4              | 277.46 | 117.22 | 176.34 | 97.37  | 140.70 | 84.58  |
| 5              | 277.58 | 117.32 | 176.42 | 97.45  | 140.80 | 84.68  |
| 6              | 282.00 | 120.93 | 179.54 | 100.58 | 144.72 | 88.76  |
| 7              | 273.23 | 113.77 | 173.35 | 94.38  | 136.95 | 80.67  |
| 8              | 281.69 | 120.68 | 179.33 | 100.36 | 144.45 | 88.49  |
| 9              | 294.38 | 131.06 | 188.30 | 109.33 | 155.72 | 100.21 |
| 10             | 309.17 | 143.14 | 198.75 | 119.79 | 168.84 | 113.88 |
| 11             | 318.67 | 150.91 | 205.47 | 126.50 | 177.27 | 120.00 |
| 12             | 331.14 | 161.10 | 214.28 | 135.31 | 188.33 | 120.00 |
| 13             | 326.31 | 157.15 | 210.86 | 131.90 | 184.04 | 120.00 |
| 14             | 335.06 | 164.30 | 217.05 | 138.09 | 191.81 | 120.00 |
| 15             | 344.10 | 171.70 | 223.44 | 144.48 | 199.84 | 120.00 |
| 16             | 340.62 | 168.85 | 220.98 | 142.01 | 196.74 | 120.00 |
| 17             | 327.29 | 157.95 | 211.56 | 132.59 | 184.91 | 120.00 |
| 18             | 323.92 | 155.20 | 209.18 | 130.21 | 181.93 | 120.00 |
| 19             | 315.81 | 148.57 | 203.44 | 124.48 | 174.73 | 120.00 |
| 20             | 301.91 | 137.20 | 193.61 | 114.65 | 162.39 | 107.16 |
| 21             | 290.42 | 127.81 | 185.49 | 106.53 | 152.19 | 96.55  |
| 22             | 281.30 | 120.36 | 179.05 | 100.09 | 144.11 | 88.13  |
| 23             | 284.80 | 123.23 | 181.53 | 102.56 | 147.21 | 91.36  |
| 24             | 282.88 | 121.65 | 180.16 | 101.20 | 145.50 | 89.58  |

**Table A.4**  
Optimal customer incentive ( $y_{j,t}$ ) (Scenario 1).

| $t$            | $j$    |         |         |         |         |
|----------------|--------|---------|---------|---------|---------|
|                | 1      | 2       | 3       | 4       | 5       |
| $y_{j,t}$ (\$) |        |         |         |         |         |
| 1              | 66.77  | 97.38   | 142.73  | 163.93  | 189.25  |
| 2              | 74.23  | 107.15  | 155.22  | 178.97  | 204.42  |
| 3              | 69.36  | 100.37  | 146.19  | 169.21  | 192.72  |
| 4              | 68.97  | 91.72   | 145.25  | 169.64  | 191.29  |
| 5              | 83.85  | 104.57  | 171.44  | 199.84  | 224.04  |
| 6              | 104.85 | 135.13  | 208.72  | 240.43  | 270.90  |
| 7              | 493.22 | 619.34  | 898.62  | 992.56  | 1138.20 |
| 8              | 430.06 | 584.81  | 789.42  | 867.07  | 996.98  |
| 9              | 755.00 | 1022.53 | 1369.56 | 1495.06 | 1721.79 |
| 10             | 370.73 | 506.67  | 680.55  | 757.22  | 857.15  |
| 11             | 417.86 | 568.09  | 758.94  | 849.43  | 950.14  |
| 12             | 313.12 | 429.42  | 571.70  | 653.06  | 714.90  |
| 13             | 177.38 | 246.00  | 334.09  | 383.01  | 420.35  |
| 14             | 314.36 | 430.13  | 575.72  | 651.20  | 721.88  |
| 15             | 183.30 | 253.79  | 346.86  | 391.88  | 439.96  |
| 16             | 187.43 | 259.17  | 356.34  | 397.59  | 455.04  |
| 17             | 320.93 | 426.30  | 596.22  | 656.51  | 757.93  |
| 18             | 308.29 | 371.06  | 555.80  | 555.61  | 702.35  |
| 19             | 233.26 | 317.23  | 436.70  | 477.43  | 554.32  |
| 20             | 241.40 | 330.81  | 454.14  | 488.53  | 576.21  |
| 21             | 178.82 | 249.18  | 339.32  | 388.05  | 432.79  |
| 22             | 207.89 | 289.72  | 387.94  | 452.73  | 492.38  |
| 23             | 96.45  | 138.15  | 192.33  | 227.32  | 248.68  |
| 24             | 77.88  | 112.79  | 160.60  | 189.18  | 209.53  |

**Table A.6**  
Optimal customer power curtailed ( $x_{j,t}$ ) (Scenario 2).

| $t$            | $j$   |       |       |       |       |       |       |
|----------------|-------|-------|-------|-------|-------|-------|-------|
|                | 1     | 2     | 3     | 4     | 5     | 6     | 7     |
| $x_{j,t}$ (MW) |       |       |       |       |       |       |       |
| 1              | 1.50  | 1.50  | 2.17  | 3.78  | 5.57  | 13.73 | 17.27 |
| 2              | 1.50  | 1.50  | 1.50  | 2.75  | 4.52  | 13.06 | 16.71 |
| 3              | 1.50  | 1.56  | 2.47  | 4.18  | 5.96  | 13.98 | 17.62 |
| 4              | 2.28  | 2.87  | 4.39  | 6.40  | 8.31  | 15.48 | 18.95 |
| 5              | 2.88  | 3.83  | 5.45  | 7.68  | 9.62  | 16.31 | 19.73 |
| 6              | 5.24  | 7.04  | 9.49  | 12.47 | 14.54 | 19.46 | 22.49 |
| 7              | 6.71  | 8.49  | 11.60 | 14.88 | 17.09 | 21.08 | 23.80 |
| 8              | 7.17  | 9.18  | 12.51 | 15.94 | 18.21 | 21.79 | 24.37 |
| 9              | 8.47  | 10.92 | 14.80 | 18.62 | 20.99 | 23.57 | 25.91 |
| 10             | 10.93 | 14.31 | 18.69 | 23.31 | 25.69 | 26.57 | 28.57 |
| 11             | 12.08 | 15.76 | 20.84 | 25.80 | 28.28 | 28.22 | 29.93 |
| 12             | 11.07 | 14.44 | 19.22 | 23.88 | 26.39 | 27.01 | 28.99 |
| 13             | 9.97  | 12.85 | 17.21 | 21.49 | 23.89 | 25.42 | 27.57 |
| 14             | 9.29  | 12.17 | 15.91 | 20.07 | 22.27 | 24.38 | 26.78 |
| 15             | 7.61  | 9.64  | 13.20 | 16.75 | 18.99 | 22.29 | 24.73 |
| 16             | 7.07  | 8.96  | 12.37 | 15.76 | 17.99 | 21.65 | 24.09 |
| 17             | 6.71  | 8.65  | 11.99 | 15.30 | 17.51 | 21.35 | 23.78 |
| 18             | 8.42  | 10.51 | 14.43 | 18.20 | 20.51 | 23.26 | 25.50 |
| 19             | 8.09  | 10.34 | 14.18 | 17.90 | 20.22 | 23.08 | 25.40 |
| 20             | 11.74 | 15.19 | 20.55 | 25.38 | 28.05 | 28.07 | 29.76 |
| 21             | 27.22 | 34.54 | 45.11 | 50.00 | 50.00 | 47.12 | 46.04 |
| 22             | 7.95  | 10.04 | 13.62 | 17.22 | 19.52 | 22.63 | 25.04 |
| 23             | 3.12  | 4.03  | 5.75  | 7.98  | 9.92  | 16.51 | 19.80 |
| 24             | 1.50  | 1.69  | 2.56  | 4.24  | 5.95  | 13.97 | 17.75 |

To provide a comparison of the DR-DEED with normal DEED we vary the weights for DEED with both the six bus and ten bus systems (Scenario 1 and 2 respectively). Tables 8 and 9 give the total generator costs (\$), total emissions (lb), total generator power (MW) and total power losses (MW) for DEED in both example scenarios. It is observed that as  $w$  increases, the costs decrease and the emission and losses increase. This means that as the weighting factor is increased (the importance of minimizing emissions is decreased, while the importance of minimizing costs increases), emissions and losses actually increase and costs decrease. This is expected and consistent with results obtained

from the literature [2,4,16,30]. To see the benefits of DR-DEED over conventional DEED we compare Table 6 (DR-DEED) and Table 8 (normal DEED) for Scenario 1 and Table 7 (DR-DEED) and Table 9 (normal DEED) for Scenario 2. When the objective is to solely minimize cost (C2 in Table 6 and  $w = 1$  in Table 8), DR-DEED gives lower cost, emissions, losses and generated power. When the objective is to solely minimize emissions (C3 in Table 7 and  $w = 0$  in Table 9), again DR-DEED give lower costs, emissions, losses and generated power. It can be rightly concluded, that integrating both DR and DEED formulations

**Table A.7**  
Optimal customer incentive ( $y_{j,t}$ ) (Scenario 2).

| $t$            | $j$     |         |         |         |         |         |         |
|----------------|---------|---------|---------|---------|---------|---------|---------|
|                | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
| $y_{j,t}$ (\$) |         |         |         |         |         |         |         |
| 1              | 21.62   | 18.10   | 23.30   | 40.42   | 55.41   | 285.29  | 454.27  |
| 2              | 21.62   | 18.10   | 14.99   | 26.86   | 40.81   | 259.35  | 425.32  |
| 3              | 21.62   | 19.00   | 27.21   | 46.21   | 61.27   | 295.16  | 472.92  |
| 4              | 36.09   | 40.13   | 57.57   | 83.66   | 102.73  | 359.00  | 547.06  |
| 5              | 48.88   | 58.43   | 77.66   | 109.56  | 129.86  | 396.98  | 592.75  |
| 6              | 111.57  | 138.68  | 176.58  | 232.12  | 259.41  | 557.81  | 770.40  |
| 7              | 161.10  | 184.38  | 242.20  | 309.93  | 342.92  | 651.42  | 862.98  |
| 8              | 178.27  | 207.84  | 273.79  | 347.38  | 383.38  | 695.02  | 904.22  |
| 9              | 231.19  | 273.69  | 360.14  | 451.39  | 493.33  | 809.33  | 1022.21 |
| 10             | 348.09  | 425.15  | 533.69  | 664.71  | 710.20  | 1022.34 | 1242.93 |
| 11             | 409.99  | 500.05  | 643.01  | 794.31  | 846.14  | 1150.16 | 1364.10 |
| 12             | 354.95  | 431.90  | 559.84  | 693.52  | 745.32  | 1055.71 | 1280.48 |
| 13             | 299.89  | 355.93  | 463.54  | 577.08  | 622.43  | 937.72  | 1157.52 |
| 14             | 267.63  | 325.70  | 406.25  | 512.89  | 548.39  | 864.74  | 1092.48 |
| 15             | 195.45  | 224.39  | 298.55  | 377.24  | 413.00  | 726.37  | 931.21  |
| 16             | 174.58  | 200.23  | 268.74  | 340.85  | 375.22  | 686.30  | 883.64  |
| 17             | 161.08  | 189.48  | 255.49  | 324.57  | 358.01  | 667.79  | 861.27  |
| 18             | 228.89  | 257.50  | 345.74  | 433.97  | 473.30  | 788.89  | 990.38  |
| 19             | 215.09  | 250.88  | 335.72  | 422.07  | 461.44  | 776.72  | 982.72  |
| 20             | 391.43  | 469.64  | 627.97  | 771.50  | 833.36  | 1138.29 | 1348.83 |
| 21             | 1685.36 | 1989.37 | 2573.84 | 2643.25 | 2450.73 | 3147.16 | 3229.11 |
| 22             | 209.08  | 239.30  | 314.07  | 395.41  | 433.74  | 748.06  | 955.03  |
| 23             | 54.23   | 62.58   | 83.91   | 115.92  | 136.68  | 406.20  | 596.89  |
| 24             | 21.62   | 20.78   | 28.48   | 47.19   | 61.18   | 295.01  | 479.98  |

together with their interdependent constraints gave better results than considering either DR or DEED independently.

**6. Conclusion**

This paper presents a modification of the DEED formulation with a game theory based DR program. The three objectives in the optimization problem are to minimize the fuel and emissions costs and maximize the utility DR benefit subject to the conven-

tional DEED constraints and some extra constraints. The model determines the optimal generator output from the available generators and the game theory demand response program helps the utility determine the optimal customer load to curtail and the optimal incentive to be paid to customers who agree to curtail their load. The game theory model used in developing the DR model also included extra practical constraints like maximum power targets and total budget. Furthermore the *individual rationality constraint* and the *incentive compatibility constraint* were modified and optimized over a day instead of just an hour. From obtained results, it can be observed that the DR-DEED program helps to reduce total demand over a 24 h period by 1953.02 MW in the first scenario and reduces the total demand by 2670.57 MW in the second scenario. Results obtained from the model also show that willing customers can provide a cost efficient way to reduce demand in the power system. The mathematical framework proposed in this work assumes that the utility knows the customers cost function coefficients and that the customers follow the optimal schedule obtained by the model. This sometimes does not always happen. Thus, future work is on-going on how to incorporate a feedback scheme that would be robust against uncertainties and disturbances. Another promising research direction would be to incorporate a penalty function for customers who refuse to curtail the requested amount of power.

**Acknowledgements**

The authors gratefully acknowledge the editor and the anonymous reviewers for their constructive feedback and suggestions which helped in greatly improving the quality of this paper.

**Appendix A**

See Tables A.1–A.8.

**Table A.8**  
Optimal power generated by generators ( $P_{i,t}$ ) (Scenario 2).

| $t$            | $i$    |        |        |        |        |        |        |        |       |       |
|----------------|--------|--------|--------|--------|--------|--------|--------|--------|-------|-------|
|                | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9     | 10    |
| $P_{i,t}$ (MW) |        |        |        |        |        |        |        |        |       |       |
| 1              | 150.00 | 135.00 | 73.00  | 60.00  | 165.66 | 160.00 | 130.00 | 60.00  | 20.00 | 55.00 |
| 2              | 150.00 | 135.00 | 76.17  | 60.00  | 211.46 | 160.00 | 130.00 | 90.00  | 21.92 | 55.00 |
| 3              | 150.00 | 135.00 | 119.16 | 85.66  | 243.00 | 160.00 | 130.00 | 120.00 | 39.66 | 55.00 |
| 4              | 155.62 | 162.08 | 164.93 | 131.13 | 243.00 | 160.00 | 130.00 | 120.00 | 58.55 | 55.00 |
| 5              | 171.55 | 178.11 | 181.05 | 147.14 | 243.00 | 160.00 | 130.00 | 120.00 | 65.20 | 55.00 |
| 6              | 200.80 | 207.55 | 210.64 | 176.54 | 243.00 | 160.00 | 130.00 | 120.00 | 77.41 | 55.00 |
| 7              | 216.29 | 223.14 | 226.30 | 192.10 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 8              | 234.44 | 241.40 | 244.66 | 210.33 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 9              | 270.10 | 277.29 | 280.73 | 246.17 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 10             | 303.12 | 310.52 | 314.13 | 279.35 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 11             | 319.59 | 327.10 | 330.79 | 295.90 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 12             | 358.70 | 366.46 | 340.00 | 300.00 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 13             | 305.72 | 313.14 | 316.77 | 281.97 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 14             | 268.07 | 275.25 | 278.68 | 244.13 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 15             | 234.12 | 241.08 | 244.34 | 207.01 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 16             | 178.39 | 184.99 | 187.96 | 157.01 | 243.00 | 160.00 | 130.00 | 120.00 | 68.05 | 55.00 |
| 17             | 162.11 | 168.61 | 171.50 | 137.65 | 243.00 | 160.00 | 130.00 | 120.00 | 61.26 | 55.00 |
| 18             | 193.61 | 200.32 | 203.36 | 169.31 | 243.00 | 160.00 | 130.00 | 120.00 | 74.41 | 55.00 |
| 19             | 230.27 | 237.21 | 240.45 | 212.13 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 20             | 305.30 | 312.46 | 315.88 | 262.13 | 243.00 | 160.00 | 130.00 | 120.00 | 80.00 | 55.00 |
| 21             | 225.30 | 232.46 | 235.88 | 212.13 | 243.00 | 160.00 | 130.00 | 120.00 | 59.34 | 55.00 |
| 22             | 197.21 | 203.94 | 207.00 | 162.13 | 243.00 | 160.00 | 130.00 | 120.00 | 75.91 | 55.00 |
| 23             | 150.00 | 136.54 | 139.26 | 112.13 | 243.00 | 160.00 | 130.00 | 120.00 | 47.96 | 55.00 |
| 24             | 150.00 | 135.00 | 88.67  | 62.13  | 243.00 | 160.00 | 130.00 | 109.28 | 27.08 | 55.00 |

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