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Application of model predictive control to optimal dynamic dispatch of generation with emission limitations

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ABSTRACT

Reducing emission from fossil-fueled electric power generating plants has received considerable attention in recent years in both regulated and deregulated power markets. Under regulated power systems, utilities solve the dynamic economic dispatch problem to determine the optimal scheduling of the committed unit's output at minimum fuel cost while satisfying a set of constraints. In this paper, we introduce the following problems where the emission effects are included in the mathematical model: (1) dynamic economic emission dispatch and (2) emission constrained dynamic economic dispatch. Under deregulated markets, the generation company can determine the optimal amounts of energy to be sold in the market by running profit-based dynamic economic dispatch problem to maximize its own profit. To take into account the emission limitations we introduced two problems: (1) profit-based dynamic economic emission dispatch problem and (2) profit-based emission constrained dynamic economic dispatch problem. In this paper we applied the model predictive control (MPC) approach proposed recently to the dynamic dispatch problems in both regulated and deregulated systems. The convergence and robustness of the MPC algorithms are demonstrated through the application of MPC to these problems with a six-unit system.

1. Introduction

The problem of optimal dynamic dispatch (ODD) of electric power generation has received considerable attention in both regulated and deregulated power markets. In the vertically integrated monopolistic and regulated environment, the problem is known as dynamic economic dispatch (DED) problem which is formulated to determine the optimal scheduling of the committed generating unit's output so as to meet the load demand over a dispatch period at minimum operating cost while satisfying ramp rate constraints and other constraints (see e.g. [3–15] and the review paper [22]). In this environment, utilities are obliged to serve all customers and meeting all demands. Recently many countries have gone through deregulation and restructuring of the electrical power systems with the aim of improving economic efficiency. Under deregulation, the DED has evolved from a minimum-cost policy to a maximum-profit policy, giving rise to the new profit-based dynamic economic dispatch (PBDED) problem [16]. The objective function of the PBDED is formulated to maximize the generation company's (GENCO's) own profit from selling energy into the market. Therefore, the GENCO can choose to sell energy less than the predicted values if a higher profit is realized. The PBDED problem can also be used to create the decision criteria for the GENCO.

The emission of gaseous pollutants including SO_2, NO_x, CO and CO_2 from fossil fuel fired thermal plants affects the human health directly or indirectly. Therefore, electric utilities or GENCOs are requested to reduce emission from their plants. As a result of public awareness of environmental protection, diverse compliance strategies have emerged. These strategies include installation of pollutant cleaning, switching to low emission fuels, replacement of the aged fuel burners with cleaner ones, as well as emission dispatching [23]. The last strategy is usually preferred to the existing systems because it is easy to implement and requires less additional cost. In the literature an overwhelming number of reported works incorporate the emission with the static economic dispatch (SED) problem (see e.g. [23]), but only few works address the DED problem with emission limitations [17–21]. In contrast to DED, the SED does not have the look-ahead capability and it does not incorporate the ramp rate constraint which is important to maintain the life of the generators [5].

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The emission can be taken into the DED formulation in several ways. One approach is similar to the DED problem with the objective to be minimized being emission instead of fuel cost, and the problem is referred to as pure dynamic emission dispatch (PDED) [19]. The second approach combines both DED and PDED in one problem to minimize both fuel cost and emission simultaneously under load demand constraint, ramp rate constraint and other constraints, resulting in a multi-objective optimization problem. This problem is referred to as dynamic economic emission dispatch (DEED) [17–19,21]. Another approach is to minimize the fuel cost while treating the emission as a constraint with a pre-specified limit [20], and the problem is referred to as emission constrained dynamic economic dispatch (ECDED) [20].

To the best of the knowledge of the authors the emission has never been considered in the PBDED formulation which represents one of the new contributions of the present paper. The emission can be taken into the PBDED problem by formulating profit-based dynamic economic emission dispatch (PBDEED) problem with the objective of maximizing the profit and minimizing emission simultaneously under ramp rate constraint and other constraints. The emission can also be taken into the PBDED problem by formulating profit-based emission constrained dynamic economic dispatch (PBECDED) problem with the objective to maximize the profit under emission constraint and other constraints.

A great majority of works have been devoted to solve the ODD problem using various optimization techniques and procedures incorporating extended and complex objective functions or constraints (see the review paper [22]). The early research activities were either mathematical programming based or heuristically based, such as the lambda iterative method [6], gradient projection method [7], Lagrange relaxation [8], linear programming [9], dynamic programming [1,2] and interior point method [10,5]. More recent works have centered around artificial intelligence (AI) methods, on par with the development of AI optimization theories, such as simulated annealing [11], hybrid genetic algorithms [12], differential evolution [13], particle swarm optimization [4,14], evolutionary programming with sequential quadratic programming [3], particle swarm optimization with sequential quadratic programming [15]. Many of these techniques have proven their effectiveness in solving the DED problems without any or fewer restrictions on the shape of the cost function curves.

All the above mentioned ODD formulations and their optimization algorithms suffer from the deficiency of not allowing to compensate for inaccuracies originating from modeling uncertainties, external disturbances, and unexpected reactions of some of the power system components. In the terminology of control theory, these formulations are in fact open-loop systems and there is no way to feedback the inaccuracy information to the systems so that the solutions can be compensated. In other words, these formulations are not closed-loop systems. A possible solution to this problem is to apply the model predictive control (MPC) method. This method obtains a feedback control by solving a finite horizon optimal control problem at each sampling instant using the current state of the plant as the initial state for the optimization and applying only "the first part" of the optimal control [24]. MPC method has emerged and been successfully applied particularly in the process control industry since 1970s. Theoretical properties such as stability and robustness of the MPC have been studied by many authors (see the review paper [24]). Up to present, MPC has become one of the most widely used multivariable control algorithms in various industries including chemical engineering, food processing, automotive, aerospace applications [25], and recently in power systems [26–28]. This is due to its facility of handling constraints, being able to use simple models, and its closed-loop stability and inherent robustness. Moreover, MPC solves optimal control problem *on-line* for the current state of the plant which is a mathematical programming problem and is much more simpler than determining the feedback solution by dynamic programming [24].

MPC has been proposed for the periodic implementations of the optimal solutions of the DED problem and the optimal dynamic resource allocation problem in [27,26], respectively. In these papers, the convergence and robustness of the MPC algorithm are proved. In [27], emission is not included in the DED formulation. In the present paper we first introduce the DEED, ECDED, PBDEED and PBECDED problems and then apply the MPC approach proposed in [27,26] to these problems.

The remainder of this paper is organized as follows: in Section 2, we introduce the ODD problems under regulated markets. The ODD problems under deregulated markets are outlined in Section 3. In Section 4, we outline the MPC approach for the DED problem and summarize the main results obtained in [27]. The simulation results for the application of MPC to the DEED, ECDED, PBDEED and PBECDED problems are given in Section 5. The last section is the conclusions.

Throughout the paper, the following notations and definitions will be used. For a sampling period T, the dynamic dispatch problem is considered over dispatch intervals, $[iT, (i+N)T))$ where the optimization is considered, for $i \geq 0$, N is a fixed positive integer, and NT is the dispatch period. For simplicity, we make the convention throughout the paper that $[i, j)$ denotes the time interval $[iT, jT)$. Assume that n is the number of committed units, P_i^t is the generation of unit i during the tth time interval $[t-1, t)$; $C_i(P_i^t)$ and $E_i(P_i^t)$ are the generation cost and the amount of emission respectively for unit i to produce P_i^t; D^t, SP^t are the demand and energy price at time t (i.e., the tth time interval); the control variable u_i^t is the ramp rate of the unit i at time t; UR_i and DR_i are the maximum ramp up/down rates for unit i; $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum capacity of unit i respectively. For any $m \geq 0$, $k \geq 1$ define $\mathbf{P}^m = (P_1^{1+m}, P_2^{1+m}, \ldots, P_n^{1+m}, P_1^{2+m}, P_2^{2+m}, \ldots, P_n^{2+m}, \ldots, P_1^{N+m}, P_2^{N+m}, \ldots, P_n^{N+m})$, and $P^k = (P_1^k, P_2^k, \ldots, P_n^k)'$. Define $\mathbf{U} = (u_1^1, u_2^1, \ldots, u_n^1, u_1^2, u_2^2, \ldots, u_n^2, \ldots, u_1^{N-1}, u_2^{N-1}, \ldots, u_n^{N-1})$ and $D = (D^1, D^2, \ldots, D^N)'$. The total fuel cost and emission from all units and over the dispatch period $[m, m+N)$ are denoted by $C(\mathbf{P}^m)$ and $E(\mathbf{P}^m)$, respectively. The demand D^t and the energy price SP^t are assumed to be periodic with period N. This periodic assumption is made to reflect the cyclic consumption behavior and seasonal changes over the dispatch interval.

2. Optimal dynamic dispatch under regulated markets

In this section we introduce the ODD problems under regulated markets taking into account the emission limitations.

2.1. Dynamic economic emission dispatch

It is well known that the fuel cost and the amount of emission conflict with each other. Minimization of fuel cost maximizes the amount of emission and vice versa. Therefore it is necessary to find out an operating point that strikes a balance between fuel cost and emission. This can be done by formulating DEED problem which is a multi-objective optimization problem with two conflicting objectives, the fuel cost and emission. The total fuel cost and pollutants emission over the dispatch period $[0, N]$ are given, respectively by:

$$C(\mathbf{P}^0) = \sum_{t=1}^{N} \sum_{i=1}^{n} C_i(P_i^t), \tag{1}$$

$$E(\mathbf{P}^0) = \sum_{t=1}^{N} \sum_{i=1}^{n} E_i(P_i^t). \tag{2}$$

The objective of the DEED problem is to determine the generation levels for the committed units which simultaneously minimize the total fuel cost and pollutants emission over the dispatch period $[0, N]$, while satisfying a set of constraints. The DEED can be mathematically stated as follows:

$$\begin{aligned}&\min_{\mathbf{P}^0}(C(\mathbf{P}^0), E(\mathbf{P}^0))\\&\text{subject to } P_i^t \in \Omega_{DEED}(\mathbf{P}^0), \quad i=1,2,\ldots,n, \quad t=1,2,\ldots,N.\end{aligned} \tag{3}$$

where the feasible domain Ω_{DEED} is defined to be the set of $(P_i^t : i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, N)$ satisfying the following constraints:

(i) Power balance constraint

$$\sum_{i=1}^{n} P_i^t = D^t, \quad t = 1, 2, \ldots, N \tag{4}$$

(ii) Generation limits

$$P_i^{\min} \le P_i^t \le P_i^{\max}, \quad i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, N \tag{5}$$

(iii) Generating unit ramp rate limits

$$\begin{aligned}&-DR_i \cdot T \le P_i^{t+1} - P_i^t \le UR_i \cdot T, \quad i = 1, 2, \ldots, n,\\&\quad t = 1, 2, \ldots, N-1,\end{aligned} \tag{6}$$

$$-DR_i \cdot T \le P_i^1 - P_i^N \le UR_i \cdot T, \quad i = 1, 2, \ldots, n. \tag{7}$$

In this paper we assume, for simplicity, that the cost and emission functions are quadratic functions as:

$$C_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2, \tag{8}$$

$$E_i(P_i^t) = \alpha_i + \beta_i P_i^t + \gamma_i (P_i^t)^2, \tag{9}$$

where a_i, b_i and c_i are the fuel cost coefficients of generator i and they are constants. The parameters α_i, β_i and γ_i are the coefficient of ith generator emission characteristics [20].

This multi-objective optimization problem can be converted into a single objective optimization as:

$$\min_{\mathbf{P}^0} H(\mathbf{P}^0) = \alpha \sum_{t=1}^{N}\sum_{i=1}^{n} C_i(P_i^t) + (1-\alpha)\sum_{t=1}^{N}\sum_{i=1}^{n} E_i(P_i^t) \tag{10}$$

$$\text{subject to} \quad P_i^t \in \Omega_{DEED}(\mathbf{P}^0), \quad i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, N \tag{11}$$

where $\alpha \in [0, 1]$ is a weighting factor. It will be noted that, when $\alpha = 1$, the problem (10) and (11) determines the optimal amount of the generated power by minimizing the cost regardless of emission and the DEED problem leads to the DED problem [27]. If $\alpha = 0$ then, the DEED problem determine the optimal amount of the generated power by minimizing the emission regardless of cost and the DEED problem leads to the pure dynamic emission dispatch (PDED) [19]. This optimization problem (10) and (11) can be solved by e.g., quadratic programming since H is a quadratic function.

The constraints (4)–(6) are usually used in the conventional DEED problem [17–19]. Since the demand and constraints are periodic, one may obtain the solution of the conventional DEED problem (3)–(6) over e.g. 24 h ($N = 24$ and $T = 1$) then this solution is implemented not only for the first day, but also for all the other week days. Sometimes such an optimal solution is not able to be practically implemented, or in other words, the solution is not practically feasible. The ramp rate constraint may be violated when the generators are moved from the 24th hour of a day to the first hour of the next day. This problem can be resolved by including the ramp limit on the difference between P_i^{24} and $P_i^{25} = P_i^1$. This can be achieved by adding the constraint (7) to the conventional DEED problem [27].

Here we have considered a simple form of DEED problem involving three types of constraints, equality, dynamic and inequality constraints. There are roughly three main types of constraints in the DEED problem: the load demand balance in terms of equality constraints, ramp rates in terms of dynamic constraints and generation capacity in terms of inequality constraints. So the consideration of simple form of the DEED problem is without loss of generality, because it contains all three types of constraints. Some other constraints such as spinning reserve, security constraints, etc., can be taken into consideration in exactly the same fashion in both formulations of the dynamic dispatch problem but they all boil down mathematically to the afore-mentioned three types of constraints. The application of MPC to the DEED with transmission line losses, general constraints and objectives including non-smooth and/or non-convex functions will be left to our future research.

We note that the above DEED problem can be solved over the dispatch period $[m, m+N]$ for any $m \ge 0$ and it can be formulated as:

$$\begin{aligned}&\min_{\mathbf{P}^m} H(\mathbf{P}^m)\\&\text{subject to} \quad P_i^t \in \Omega_{DEED}(\mathbf{P}^m), \quad i = 1, 2, \ldots, n, \ t = m+1, m+2, \ldots, m+N.\end{aligned}$$

Since the demand is periodic and $P_i^{\min}$, $P_i^{\max}$, UR_i, DR_i, α, and T, do not change over time, then $P^{m+1} = P^{m+N+1}$, and Ω_{DEED} satisfies

$$\begin{aligned}\Omega_{DEED}(\mathbf{P}^{m+1}) &= \Omega_{DEED}(P^{m+2}, \ldots, P^{m+N}, P^{m+N+1})\\&= \Omega_{DEED}(P^{m+2}, \ldots, P^{m+N}, P^{m+1}) = \Omega_{DEED}(\mathbf{P}^m)\end{aligned}$$

then Ω_{DEED} is shift-invariant (see [26]). The shift-invariant property of Ω_{DEED} is needed for the application of the MPC approach to the DEED problem.

2.2. Emission constrained dynamic economic dispatch

The main objective of this problem is to minimize the total fuel cost under power balance constraint, ramp rate constraint, and generation capacity constraints. In addition the total emission from all units and over the dispatch period need to be below the allowable emission limit $E^{\max}$. The ECDED problem can be formulated as:

$$\begin{aligned}&\min_{\mathbf{P}^0} C(\mathbf{P}^0) = \sum_{t=1}^{N}\sum_{i=1}^{n} C_i(P_i^t)\\&\text{subject to} \quad P_i^t \in \Omega_{ECDED}(\mathbf{P}^0), \quad i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, N\end{aligned}$$

where the feasible domain Ω_{ECDED} is defined to be the set of $(P_i^t : i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, N)$ satisfying the constraints (4)–(7) and

$$\sum_{t=1}^{N}\sum_{i=1}^{n} E_i(P_i^t) \le E^{\max}. \tag{12}$$

3. Optimal dynamic dispatch under deregulated markets

After deregulation of the electrical power systems, competitive electricity markets have been created. Deregulation means consumers will have their choice of electricity generation suppliers. In this environment, suppliers and sometimes customers can participate in the energy market and all the transactions are made based on the price rather than cost. The Independent System Operator (ISO) has been created to coordinate, control and monitor the operations of the electrical power systems. In some markets the ODD problem is the responsibility of the GENCO for scheduling and operating its power plants. In this case the GENCO solve the PBDED problem not for minimizing the production cost as in the regulated system but for maximizing its own profit. Therefore, GENCO can produce power less than the forecasted demand if this will maximize its own profit. The system-wide balance of supply and demand is managed by ISO. In a day-ahead energy market, suppliers submit monotonically increasing bid curves (a pair of quantity and price) to the ISO. The ISO aggregates the bid curves into an aggregated supply curve for each hour to determine the market clearing price for that hour on the basis of the forecast load.

Due to increasing concern over the environmental considerations, GENCOs are requested to minimize the level of pollution from their plants. The emission limitations has recently been included in the profit-based unit commitment problem in [30], but not in the PBDED problem. Therefore, the objective of this section is to take into account the emission limitations in the PBDED by formulating the PBDEED and PBECDED problems.

3.1. Profit-based dynamic economic emission dispatch

Now we introduce the PBDEED formulation with the aim to produce electricity with minimum operating cost and sell it with maximum profits and environmental protection by limiting the emission of greenhouse gases into the atmosphere.

The total profit over the dispatch period $[0, N]$ is given by:

$$PF(\mathbf{P}^0) = \underbrace{\sum_{t=1}^{N}\sum_{i=1}^{n} SP^t \cdot P_i^t}_{\text{revenue}} - \underbrace{\sum_{t=1}^{N}\sum_{i=1}^{n} C_i(P_i^t)}_{\text{cost}}$$

where SP^t is the forecasted energy price at time t. Let us define a function $G(\boldsymbol{.}) = -PF(\boldsymbol{.})$ which measures the profit attained by the conversion of the energy available in fossil fuels into electric energy.

The objective of the PBDEED is to simultaneously minimize the emission and maximize the profit and satisfying a set of constraints. The PBDEED can be mathematically stated as follows:

$$\min_{\mathbf{P}^0}(G(\mathbf{P}^0), E(\mathbf{P}^0))$$
$$\text{subject to} \quad P_i^t \in \Omega_{PBDEED}(\mathbf{P}^0), \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N,$$

where the feasible domain Ω_{PBDEED} is defined to be the set of $(P_i^t : i = 1, 2, \ldots, n, t = 1, 2, \ldots, N)$ satisfying the following constraints:

$$\sum_{i=1}^{n} P_i^t \le D^t, \quad t = 1, 2, \ldots, N \tag{13}$$

$$P_i^{\min} \le P_i^t \le P_i^{\max}, \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N, \tag{14}$$

$$-DR_i \cdot T \le P_i^{t+1} - P_i^t \le UR_i \cdot T, \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N-1, \tag{15}$$

$$-DR_i \cdot T \le P_i^1 - P_i^N \le UR_i \cdot T, \quad i = 1, 2, \ldots, n. \tag{16}$$

This multi-objective optimization problem can be converted into a single objective optimization as follows:

$$\min_{\mathbf{P}^0} H(\mathbf{P}^0) = \alpha \left[\sum_{t=1}^{N}\sum_{i=1}^{n} C_i(P_i^t) - SP^t \cdot P_i^t \right] + (1-\alpha)\sum_{t=1}^{N}\sum_{i=1}^{n} E_i(P_i^t), \tag{17}$$

$$\text{subject to} \quad P_i^t \in \Omega_{PBDEED}(\mathbf{P}^0), \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N, \tag{18}$$

where $\alpha \in [0, 1]$ is a weighting factor. It will be noted that, when $\alpha = 1$, the problem (17) and (18) determines the optimal amount of the generated power by maximizing the profit regardless of emission and the PBDEED problem leads to the PBDED problem [16]. If $\alpha = 0$ then, the PBDEED problem determine the optimal amount of the generated power by minimizing the emission regardless of profit [30]. Of course the later case is not useful for GENCOs.

Constraint (13) means that under the deregulated environment, GENCO is not obliged to meet all demand, but may sell its energy at less than the system's forecasted demand equilibrium.

Now we can make a comparison between the DEED and PBDEED.

1. The DEED objective is to simultaneously minimize the emission and the generation cost while the PBDEED objective is to simultaneously minimize emission and maximize the GENCO's profit on the basis of the forecast energy prices.
2. In DEED, demand forecast advises the power system operator of the amount of power to be generated. But in PBDEED, bilateral and forward contracts will make a part of the total demand and the remaining part will be forecast as in DEED.
3. In DEED, the total generation must equal the total demand. In contrast, the GENCO has the option to consider a PBDEED schedule that produces less than the predicted power demand, with its aim to maximize its own profit.

3.2. Profit-based emission constrained dynamic economic dispatch

The main objective of this problem is to maximize the profit under constraints (13)–(16). In addition the total emission from all units and over the dispatch period need to be below the allowable emission limit $E^{\max}$. The PBECDED problem can be formulated as:

$$\min_{\mathbf{P}^0} G(\mathbf{P}^0) = \sum_{t=1}^{N}\sum_{i=1}^{n} C_i(P_i^t) - SP^t \cdot P_i^t$$
$$\text{subject to} \quad P_i^t \in \Omega_{PBECDED}(\mathbf{P}^0), \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N,$$

where the feasible domain $\Omega_{PBECDED}$ is defined to be the set of $(P_i^t : i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, N)$ satisfying the constraints (13)–(16) and (12).

From a control theoretical point of view, the ODD formulations introduced in Sections 2 and 3 provide only open-loop optimal solutions to the generation dispatch problem, that is, the optimal solutions are predetermined before actual execution, and there is no measurement on the system states which is fed back to the optimization model. Therefore, we want to introduce a closed-loop control by the MPC method in the next section so that the measurement of states can be fed back to the optimization model, and the optimal solution is updated according to the feedback information at each time step.

4. MPC approach to DED

In this section, we first outline the MPC approach proposed in [27] for the DED problem and then show that the MPC can be applied to the others ODD problems formulated in Sections 2 and 3. The DED formulation is obtained by letting $\alpha = 1$ in the DEED problem, i.e.

$$\min_{\mathbf{P}^0} C(\mathbf{P}^0)$$
$$\text{subject to} \quad P_i^t \in \Omega_{DED}(\mathbf{P}^0), \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N.$$

where $C(\mathbf{P}^0)$ is given by (1) and $\Omega_{DED} = \Omega_{DEED}$.

We introduce the control variables u_i^t as [1,2]:

$$u_i^t = \frac{P_i^{t+1} - P_i^t}{T}, \quad i = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, N-1, \tag{19}$$

where u_i^t is the ramping action of unit i at time t. This equation actually defines coordinate transformation between the variables $\{P_i^t : i = 1, 2, \ldots, n,\ t = 1, 2, \ldots, N\}$ and the variables $\{u_i^t : i = 1, 2, \ldots, n,\ t = 1, 2, \ldots, N-1\}$. It is obvious that the inverse coordinate transformation is given by

$$P_i^t = P_i^1 + \sum_{j=1}^{t-1} T u_i^j, \quad t = 2, 3, \ldots, N. \tag{20}$$

The optimal solution of the DED problem is implemented repeatedly at instants which equal to multiples of N. To introduce the MPC approach, let us consider the DED problem starting at an arbitrary instant $t = m$ and over a dispatch interval $[m, m+N)$. Then the optimization variables are changed into $\left\{P_i^{m+1}, P_i^{m+2}, \ldots, P_i^{m+N}, i = 1, 2, \ldots, n\right\}$. By the transformation defined in (20), the optimization variables $\left\{P_i^{m+1}, P_i^{m+2}, \ldots, P_i^{m+N}, i = 1, 2, \ldots, n\right\}$ are transformed into $\left\{P_i^{m+1}, u_i^{m+1}, \ldots, u_i^{m+N-1}, i = 1, 2, \ldots, n\right\}$.

In an MPC approach, a finite-horizon optimal control problem is repeatedly solved and the input is applied to the system based on the obtained optimal open-loop control. In our case, the horizon is chosen to be N. Instead of solving the DED problem with nN number of variables $\left\{P_i^{m+1}, u_i^{m+1}, \ldots, u_i^{m+N-1}, i = 1, 2, \ldots, n\right\}$, the MPC algorithm solves the following problem which has only $n(N-1)$ number of variables $\left\{u_i^{m+1}, \ldots, u_i^{m+N-1}, i = 1, 2, \ldots, n\right\}$:

Problem $\text{MPCDED}_{pm+1}(u, [m, m+N))$ given $n, N, DR_i, UR_i, P_i^{\min}, P_i^{\max}, i = 1, 2, \ldots, n, D, P^{m+1}$, let

$$P_i^1 := P_i^{m+1}, u_i^j := u_i^{m+j}, D^t := D^{m+t}, \quad i = 1, 2, \ldots, n,$$
$$t = 1, 2, \ldots, N, \ j = 1, 2, \ldots, N-1, \tag{21}$$

and solve the following minimization problem

$$\min_{\mathbf{U}} \sum_{t=1}^{N} \sum_{i=1}^{n} C_i \left(P_i^1 + \sum_{j=1}^{t-1} T u_i^j \right)$$
$$\text{subject to} \quad u_i^j \in \Omega_D(P^1, U), \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, N-1$$

where the feasible domain $\Omega_D(P^1, U)$ is defined to be the set of $\{P_i^1, u_i^t : i = 1, 2, \ldots, n, t = 1, 2, \ldots, N-1\}$ satisfying

$$\sum_{i=1}^{n} \left(P_i^1 + \sum_{j=1}^{t-1} T u_i^j \right) = D^t, \quad t = 1, 2, \ldots, N,$$
$$P_i^{\min} \le P_i^1 + \sum_{j=1}^{t-1} T u_i^j \le P_i^{\max}, \quad i = 1, 2, \ldots, n,\ t = 1, 2, \ldots, N,$$
$$-DR_i \le u_i^j \le UR_i, \quad i = 1, 2, \ldots, n,\ t = 1, 2, \ldots, N-1.$$

The notation $\text{MPCDED}_{pm+1}(u, [m, m+N))$ denotes the optimization problem is solved over the interval $[m, m+N)$ with variables u_i^j and for known inputs $P_i^{m+1}, i = 1, 2, \ldots, n, j = m+1, \ldots, m+N-1$.

In order to make the MPCDED problem solvable, the following hypothesis is needed as in [5,27].

Feasibility Hypothesis. After the change of variables in (21) over any dispatch interval $[m, m+N)$ with $m \ge 0$, the set $\Omega_D(P^1, U)$ is not empty.

This hypothesis ensures the solvability of the problem $\text{MPCDED}_{pm+1}(u, [m, m+N))$. Denote the optimal solution of MPCDED for given initial generation $\overline{P}^{m+1}$ by $\overline{\mathbf{u}}^m(\overline{P}^{m+1}) = \left\{\overline{u}_i^{m+j}(\overline{P}^{m+1}), i = 1, 2, \ldots, n, j = 1, 2, \ldots, N-1\right\}$. In the model predictive control method the optimal solution $\overline{\mathbf{u}}^m$ is applied only in the first sampling period $[m, m+1)$ that is, $\overline{u}_i^{m+1}(\overline{P}^{m+1})$ is applied to the state $\overline{P}_i^{m+1}$. Since the $\overline{\mathbf{u}}^m(\overline{P}^{m+1})$ depends on the current state $\overline{P}^{m+1}$, in this way a feedback can be obtained. We define the MPC feedback controller by $v_i^m := \overline{u}_i^{m+1}$. The closed-loop solution $\overline{P}_i^{m+2}$ given by $\overline{P}_i^{m+2} = \overline{P}_i^{m+1} + T v_i^m(\overline{P}^{m+1})$ is actually executed over the time period $[m+1, m+2)$.

The idea of the MPC can be formulated into the following MPC algorithm.

MPC algorithm Initialization: Input the initial status $\overline{P}^1 \underline{\underline{\Delta}} P^1 = (P_1^1, P_2^1, \ldots, P_n^1)$ and let $m = 0$.

(1) Compute the open-loop optimal solution $\overline{\mathbf{u}}^m$ to the problem $\text{MPCDED}_{\overline{P}^{m+1}}(u, [m, m+N))$.
(2) The (closed-loop) *MPC controller* v_i^m is applied to the plant in the sampling interval $[m, m+1)$ to obtain the closed-loop *MPC solution*

$$\overline{P}_i^{m+2} = \overline{P}_i^{m+1} + T v_i^m(\overline{P}^{m+1}) \tag{22}$$

over the period $[m+1, m+2)$.
(3) Let $m := m+1$ and go to step (1).

Theorem 1. [27] *Suppose Feasibility Hypothesis holds, P^* is the globally optimal solution of the DED problem, and the initial power output P^1 at time $t = 1$ satisfies $P_i^1 \in \Omega_{DED}$, then MPC algorithm converges to P^*.*

This theorem tells that the solutions of the MPC algorithm converge to the optimal solution of the DED problem.

Now we consider the inherent robustness properties of the MPC algorithm. The uncertainties in energy demand, price, and reserve demand for the PBDED problem are discussed by fuzzy optimization in [16]. However, no theoretical result is given. For simplicity, we suppose that disturbance happens only in the execution of the controller. That is, the disturbance happens only in Step (2) of MPC algorithm so that when the control v_i^m is applied to the plant in the sampling interval $[m, m+1)$, the system actually execute

$$\overline{P}_i^{m+2} = \overline{P}_i^{m+1} + T v_i^m(\overline{P}^{m+1}) + T w_i^{m+1} \tag{23}$$

over the period $[m+1, m+2)$, where w_i^{m+1} is the disturbance. We assume that, the disturbances satisfy the following bound

$$||w_i^{m+1}|| < e, \quad e > 0, \ i = 1, 2, \ldots, n, \ m \ge 0. \tag{24}$$

Theorem 2. [27] *Suppose Feasibility Hypothesis holds, P^* is the globally optimal solution of the DED problem, the norm of the gradient of the fuel cost function of DED problem has the upper bound L on Ω_{DED}, ϵ is a small enough positive constant, c is a positive constant which is less than ϵ, (23) is executed in Step (2) of MPC algorithm instead of (22), the constant disturbance w_i^k satisfies (24) where e is small enough so that $e < \min\{c/L, (\epsilon - c)/L\}$, then there exists an integer N_0 such that for any $k > N_0$, the optimal MPC solution $\overline{P}^{k+1}$ of the kth loop in MPC algorithm belongs to the domain $\overline{\Omega} := \{P : ||P - P^*|| < c\}$.*

Theorems 1 and 2 are based on the assumption that the objective function C of the DED problem is strictly convex and differentiable over the set Ω_{DED} which is bounded. Since both the fuel cost and emission functions are assumed to be quadratic, then all the objective functions of the DEED, ECDED, PBDEED and PBECDED problems are strictly convex and differentiable over their feasible constraint sets. Also since the demand and energy price and all constraints are assumed to be periodic then all feasible constraint sets, Ω_{DEED}, Ω_{ECDED}, Ω_{PBDEED} and $\Omega_{PBECDED}$ are shift invariant. Therefore, Theorems 1 and 2 are valid for the mentioned problems.

4.1. Advantages of the MPC

The advantages of the MPC algorithm are listed below.

Table 1
Data of the six-unit system.

i	a_i ($/h)	b_i ($/MWh)	c_i ($/MW^{2}h)	α_i (lb/h)	β_i (lb/MWh)	γ_i (lb/MW^{2}h)	P_i^{max} (MW)	P_i^{min} (MW)	UR_i (MW/h)	DR_i (MW/h)
1	240	7.0	0.0070	13.8593	0.32767	0.00419	500	100	80	120
2	200	10.0	0.0095	13.8593	0.32767	0.00419	200	50	50	90
3	220	8.0	0.0090	40.2669	−0.54551	0.00683	300	80	65	100
4	200	11.0	0.0090	40.2669	−0.54551	0.00683	150	50	50	90
5	220	10.5	0.0080	42.8955	−0.51116	0.00461	200	50	50	90
6	190	12.0	0.0075	42.8955	−0.51116	0.00461	120	50	50	90

Table 2
Load demand (MW) of the six-unit system for 24 h.

Time (h)	1	2	3	4	5	6	7	8	9	10	11	12
Demand (MW)	955	942	953	930	935	963	989	1023	1126	1150	1201	1235
Time (h)	13	14	15	16	17	18	19	20	21	22	23	24
Demand (MW)	1190	1251	1263	1250	1221	1202	1159	1092	1023	984	975	960

(1) Reduced dimensions:

The ODD problem for a six units system with a dispatch interval of 1 day, and a sampling period of 1 h. Then $T = 1$ h, $N = 24$, and the corresponding ODD problem must solve an optimization problem with $6 \times 24 = 144$ number of variables. However, in each iteration step of the MPC algorithm, the algorithm starts with any P^{m+1} and solves an optimization problem with $6 \times (24 - 1) = 138$ number of variables which reduces 6 dimensions in the optimization problem and makes the computation easier.

(2) Convergence:

Theorem 1 shows that one can start the MPC algorithm with any P^1 satisfying $P_i^1 \in \Omega_{DED}$ and the optimal solution at each step will converge to the optimal solution of the DED problem. This implies that the reduction of the number of variables in the MPC approach is both reasonable and feasible.

(3) Easy implementation:

Because of the MPC convergence, restarting the MPC algorithm from any time will give rise to the same convergence, which further implies that the MPC algorithm can be executed at any sampling time point. Thus the MPC algorithm is more favorable for practical applications than other open-loop algorithms [27].

(4) Robustness:

Theorem 2 shows that the MPC algorithm is robust against certain disturbances in the execution of the optimal controller. It is shown in [27] that, the MPC algorithm is also robust against the disturbance or uncertainty in the demand which is usually forecast.

4.2. Advantages and disadvantages of the previous works on ODD problem

Since the formulation of the ODD problem, the thrust of research has focused on various optimization techniques and procedures incorporating extended and complex objective functions or constraints. Depending on the type of objective function (nonlinear/linear, smooth/nonsmooth, convex/nonconvex, etc.) as well as the constraints, these optimization techniques can be classified into three main categories. The first category is mathematical programming-based or heuristically-based, such as the lambda iterative method [6], gradient projection method [7], Lagrange relaxation [8], linear programming [9], dynamic programming [1,2] and interior point method [10,5]. The advantages of these methods

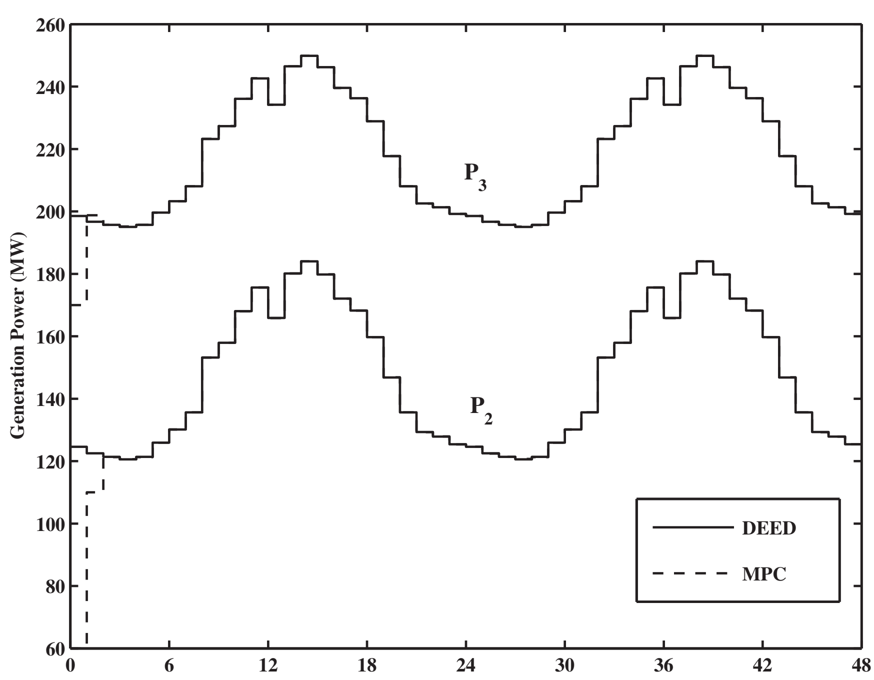

Fig. 1. Convergence of the closed-loop MPC solutions to those of DEED for the six-unit system.

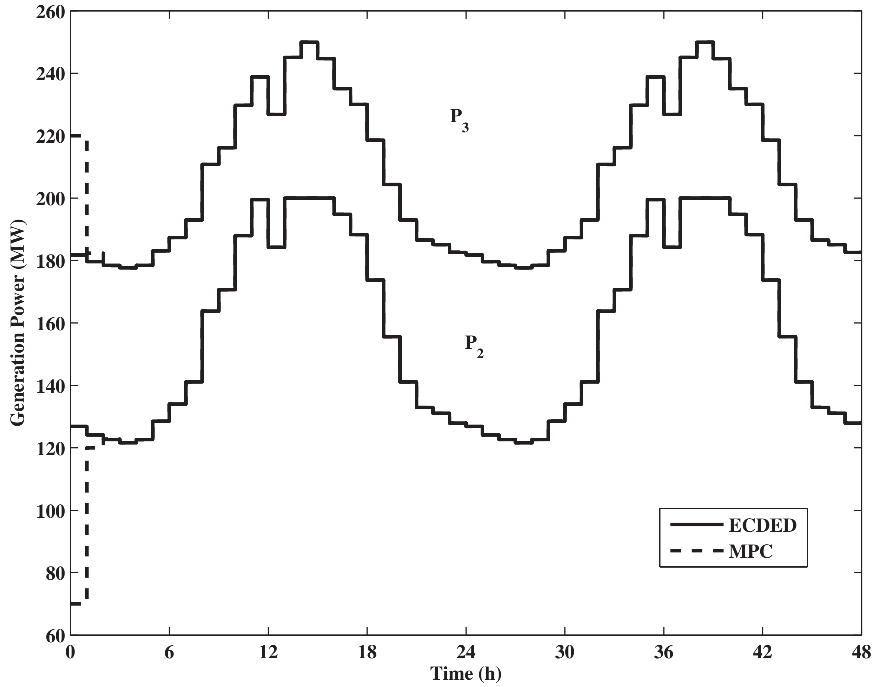

Fig. 2. Convergence of the closed-loop MPC solutions to those of ECDED for the six-unit system.

including: optimality is mathematically proven in some algorithms [32]; they can be applied to large-scale problems [32]; they have no problem-specific parameters to specify; moreover, some of these methods are computationally fast. However, these methods can fail to get global optimal solutions of the ODD with nonsmooth or nonconvex objective functions. Dynamic programming can solve the ODD problems with nonsmooth cost functions; however, it suffers from the "curse of dimensionality" and local optimality. The second category is the stochastic optimization methods such as simulated annealing [11], hybrid genetic algorithms [12], differential evolution [13], particle swarm optimization [4,14] which can get the global optimal solution of the ODD problem without any or fewer restrictions on the shape of the objective function curves. The drawbacks of these methods is the large number of arbitrary or problem-specific parameters and the long computation time [33]. The third category is the hybrid methods such as evolutionary programming with sequential quadratic programming [3], particle swarm optimization with sequential quadratic programming [15], which combine two or more techniques in order to get best features in each algorithm. These methods have been used in

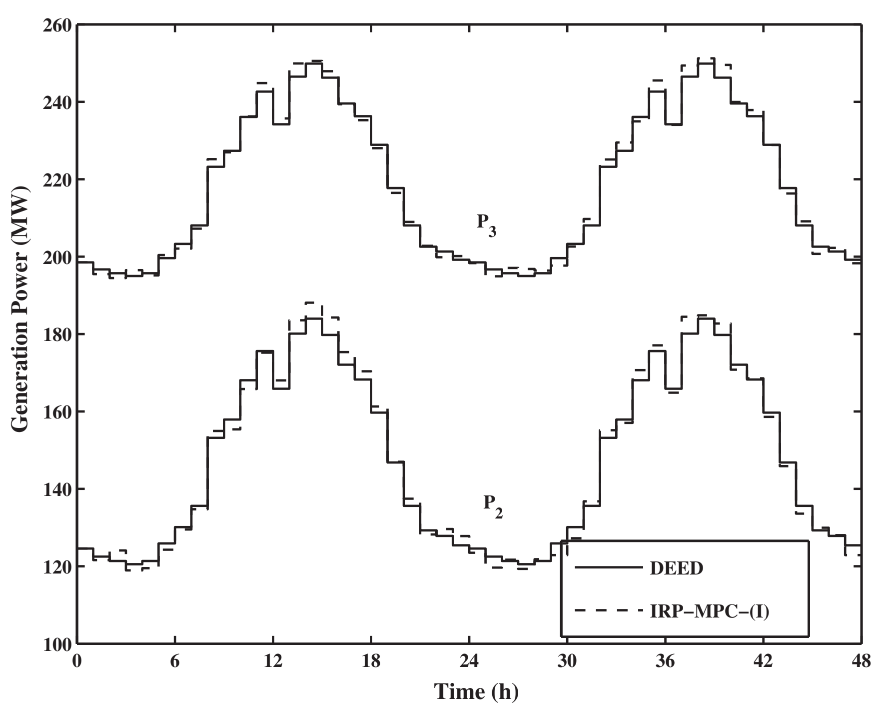

Fig. 3. The generation output of the six-unit system under DEED and IRP-MPC-(I).

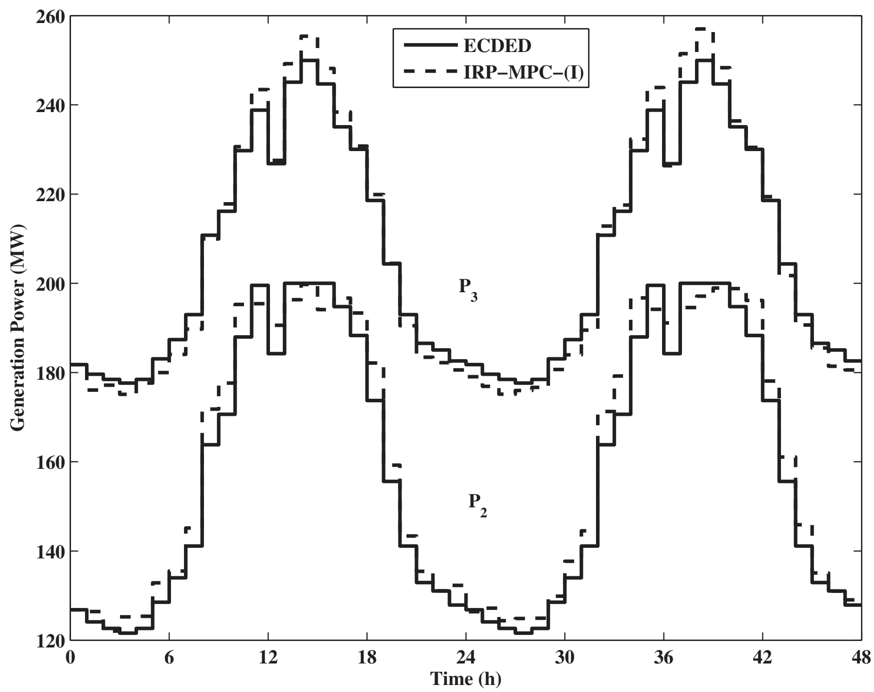

Fig. 4. The generation output of the six-unit system under ECDED and IRP-MPC-(I).

solving the ODD problem and have proven their effectiveness oven other methods.

The works presented in [3–21] focus on introducing efficient optimization methods for the ODD problem under various complex constraints or objectives. However, the obtained optimal solutions have the following drawbacks:

(i) Ramp rate violations in periodic implementation of the solutions:

We note that the conventional ODD problems (i.e. constraints (7) are not included in the optimization problem) presented in [3–21] are formulated over the dispatch interval $[0, N)$ and does not consider the periodic implementations of the optimal solutions over the period $[N, 2N)$, $[2N, 3N)$, ... There is a simple way to periodically implement the optimal solutions: simply repeat the optimal solutions over other periods. However, we have shown in [27] by an example of 10 units that, this simple repetition will possibly cause the ramp rate violations. Also, if we look at the obtained optimal solutions of the DEED problem for a five-unit system given in [17], we can see that the optimal solution of unit 2 is given by $\overline{P}_2^1 = 35.1973$, $\overline{P}_2^{24} = 83.0073$. This solution can not be implemented

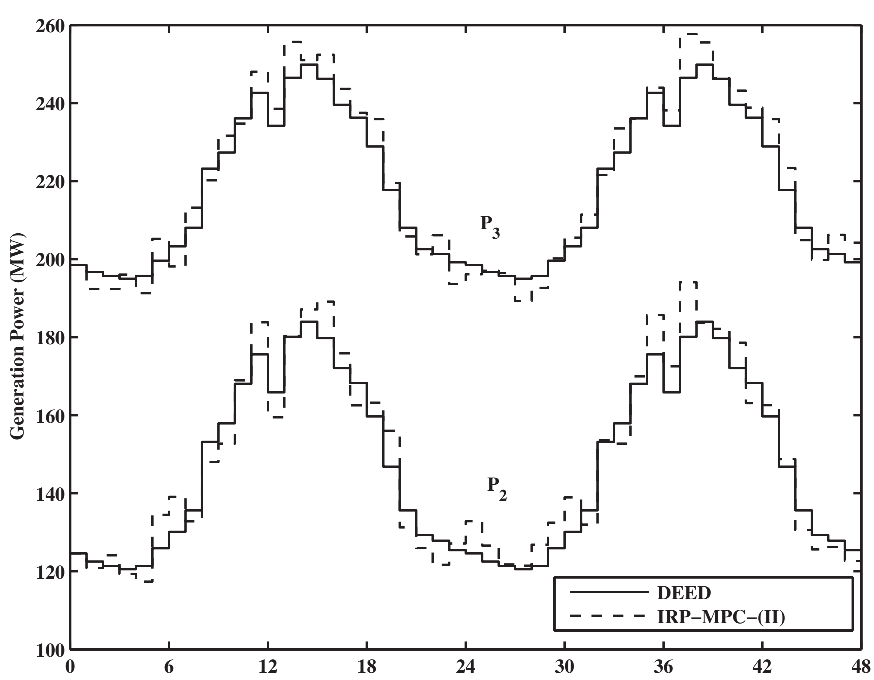

Fig. 5. The generation output of the six-unit system under DEED and IRP-MPC-(II).

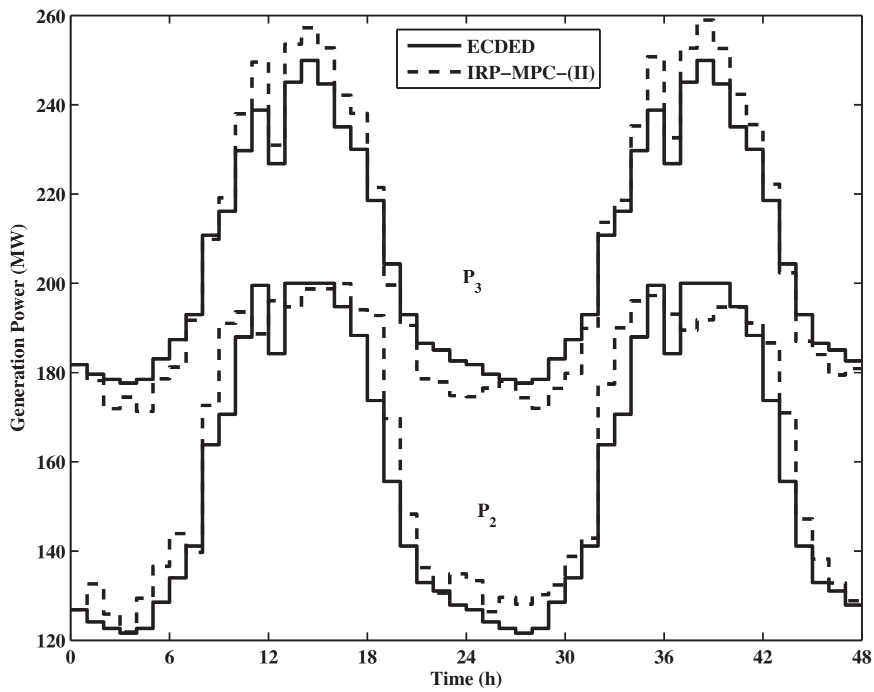

Fig. 6. The generation output of the six-unit system under ECDED and IRP-MPC-(II).

repeatedly every 24 h because $\overline{P}_2^1 - \overline{P}_2^{24} = -47.81 < -DR_2 = -30$. Therefore, the obtained optimal solutions can be implemented over the interval [0, 24], but it can not be implemented over the intervals [24, 48], [48, 72], etc.

(ii) Open-loop optimal solutions:

The ODD problem only provides an open-loop optimal solution to the generation dispatch problem, that is, the optimal solution is predetermined before actual execution, and there is no measurement on the system states which is fed back to the optimization model.

The MPC algorithm applied in this paper does not contradict with the existing ODD methods given in [3–21]. These existing methods provide various optimization solution methods to find the optimal dispatch over a fixed time horizon; while our MPC algorithm provides a periodic implementation framework and does not specify any special optimization method to solve the dispatch problem MPCDED over a fixed time period. Furthermore, the MPC approach in MPC algorithm is in fact a very general philosophy: calculating an optimization problem over a fixed period, implementing the solution only at the beginning part of this fixed period, recalculating the optimization problem over a new time horizon, and repeating these steps. Following this idea, it is possible to incorporate these existing solution methods for dynamic economic and emission dispatch into this MPC framework. That

Table 3
The results of PBDEED for the three price profiles and $\alpha = 1$.

Hour	Price-I ($)	$\sum_{i=1}^{6} P_i^t$ (MW)	Profit ($)	Price-II ($)	$\sum_{i=1}^{6} P_i^t$ (MW)	Profit ($)	Price-III ($)	$\sum_{i=1}^{6} P_i^t$ (MW)	Profit ($)
1	10.314	565.3	800.7918	11.46	747.6	28,868	13.752	**955**	93,941
2	10.888	627		12.00	883.9		14.40	**942**	
3	11.556	753.4		12.84	**935**		15.408	**935**	
4	9.486	485.5		10.54	620.5		12.648	**930**	
5	10.890	630.5		12.10	876.6		14.52	**935**	
6	11.862	845		13.18	**963**		15.816	**963**	
7	12.42	**989**		13.80	**989**		16.56	**989**	
8	11.214	802.8		12.46	1020.9		14.952	**1023**	
9	13.248	**1126**		14.72	**1126**		17.664	**1126**	
10	13.446	**1150**		14.94	**1150**		17.928	**1150**	
11	13.374	**1201**		14.86	**1201**		17.832	**1201**	
12	13.914	**1235**		15.46	**1235**		18.552	**1235**	
13	11.736	814.5		13.04	**1190**		15.648	**1190**	
14	12.420	1009		13.80	**1251**		16.56	**1251**	
15	10.980	651.4		12.20	943.5		14.64	**1263**	
16	10.188	549.3		11.32	734.1		13.584	**1250**	
17	11.25	704.8		12.50	1012.4		15.00	**1221**	
18	12.258	906.1		13.62	**1202**		16.344	**1202**	
19	9.792	545.6		10.88	696.2		13.056	**1159**	
20	11.034	661.1		12.26	952.2		14.712	**1092**	
21	11.196	692.8		12.44	1014.9		14.928	**1023**	
22	12.222	941		13.58	**984**		16.296	**984**	
23	11.43	740.4		12.70	**975**		15.24	**975**	
24	10.278	560.7		11.42	738		13.704	**960**	

Table 4
The results of PBDEED and PBECDED for price-II.

Hour	PBDEED			PBECDED		
	$\alpha = 0.5$ $\sum_{i=1}^{6} P_i^t$	$\alpha = 0.7$ $\sum_{i=1}^{6} P_i^t$	$\alpha = 1$ $\sum_{i=1}^{6} P_i^t$	$E_1^{\max}$ $\sum_{i=1}^{6} P_i^t$	$E_2^{\max}$ $\sum_{i=1}^{6} P_i^t$	$E_3^{\max}$ $\sum_{i=1}^{6} P_i^t$
1	549.5	617.9	747.6	500.2	684.9	747.6
2	593.2	723.9	883.9	565.1	806.5	883.9
3	752.4	923.0	**935**	714.5	**935**	**935**
4	441.0	516.2	620.5	426.3	568.8	620.5
5	610.4	745.0	876.6	580.7	817.5	876.6
6	829.0	**963**	**963**	786.5	**963**	**963**
7	968.7	**989**	**989**	917.8	**989**	**989**
8	748.8	834.1	1020.9	700.8	929.5	1020.9
9	109.38	**1126**	**1126**	1045.8	**1126**	**1126**
10	**1150**	**1150**	**1150**	1142	**1150**	**1150**
11	1188.3	**1201**	**1201**	1128.2	**1201**	**1201**
12	**1235**	**1235**	**1235**	1224.3	**1235**	**1235**
13	797.4	984.8	**1190**	756.8	1102.5	**1190**
14	968.7	1203.6	**1251**	917.8	**1251**	**1251**
15	628.8	770.9	943.5	598	860	943.5
16	503.3	596.6	734.1	485	666.4	734.1
17	684.0	835.8	1012.4	649.9	930.4	1012.4
18	922.9	1088.8	**1202**	879.7	1187.9	**1202**
19	468.1	572.1	696.2	450.6	657.5	696.2
20	639.8	785.0	952.2	608.3	876	952.2
21	673.0	827.4	1014.9	639.5	924.2	1014.9
22	919.1	**984**	**984**	871.2	**984**	**984**
23	720.8	888.5	**975**	684.8	**975**	**975**
24	514.9	610.2	738	495.8	676.2	738
Profit ($)	23, 427	27, 426	28, 868	21, 866	28, 571	28, 868

is, by adding constraints like (7) to avoid ramp rate violations in existing ODD models given in [3–21], then it is possible to apply the above-mentioned optimization methods at each loop of the MPC algorithm and thus the obtained results will not violate any ramp rate constraint and may also be robust against disturbances.

5. Simulation results

In this section we present an example consisting of six units for the application of MPC to DEED, ECDED, PBDEED and PBECDED problems. The coefficients of the cost function a_i, b_i and c_i as well as the demand are taken from [4], and the emission coefficients α_i,

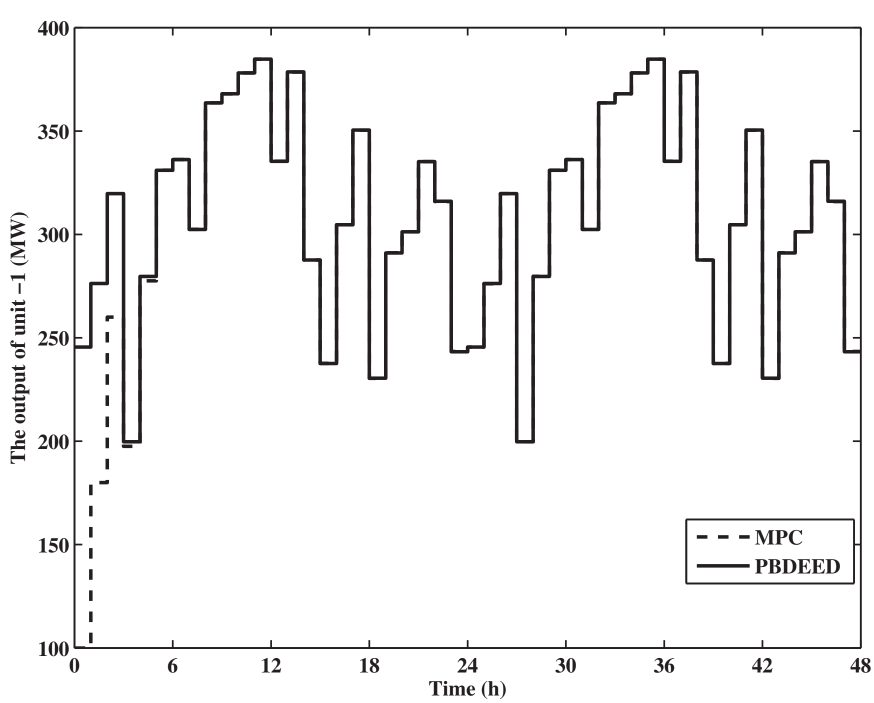

Fig. 7. Convergence of the MPC solution of the six-unit system to those of PBDEED for $\alpha = 0.7$ and price-II.

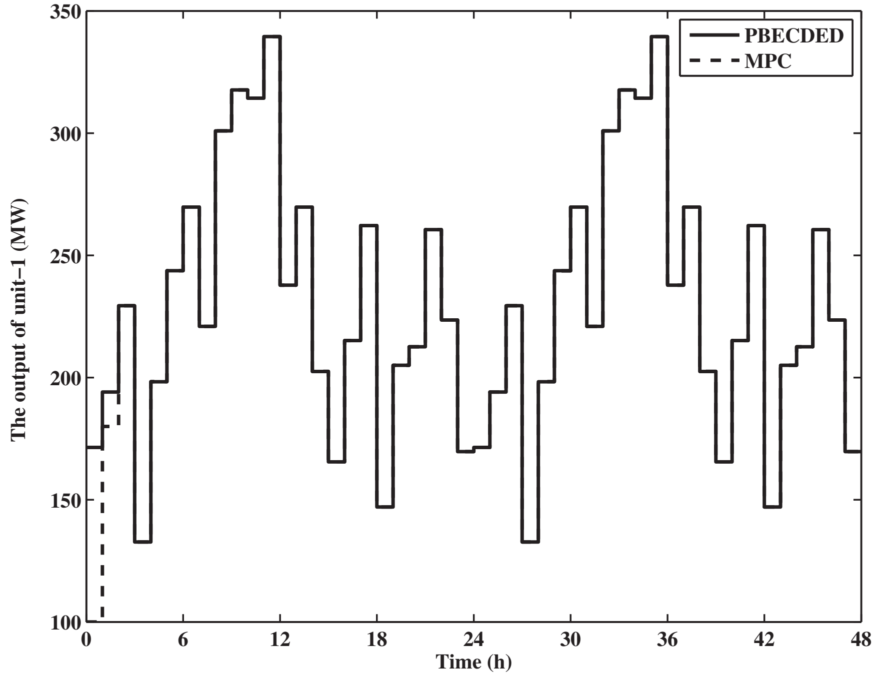

Fig. 8. Convergence of the MPC solution of the six-unit system to those of PBECDED for $E^{max} = 16,200$ lb and price-II.

β_i and γ_i are taken from [31] and are given in Tables 1 and 2. The load demand as well as the energy price are assumed to be periodic over a dispatch period of 1 day and the sampling period is chosen to be 1 h. The solution of the ODD problem as well as the optimal control sequence of the MPC are computed by the `fmincon` code of the MATLAB Optimization Toolbox.

5.1. DEED and ECDED problems

Here we show that the solutions of the MPC converge to the optimal solutions of the DEED and ECDED problems. The inherent robustness properties of the model predictive control (IRP-MPC) algorithm is also shown. The initial P^1 of the MPC to be applied to the DEED and ECDED problems are chosen, respectively, as $P^1 = (340, 60, 170, 140, 195, 50)$ and $P^1 = (290, 70, 220, 140, 185, 50)$ such that $\sum_{i=1}^{6} P_i^1 = D^1 = 955$. The weighting factor and the allowable emission limit are chosen as $\alpha = 0.5$ and $E^{max} = 25,900$ lb. The proposed MPC strategy is implemented over 48 h. In Figs. 1 and 2 , we present the optimal outputs of units 2 and 3 for the DEED and ECDED problems, respectively. It is observed that, the optimal outputs of these units is decreasing with time in the interval [0, 5), and it increase to reach its maximum at $t = 15$ (corresponds to the pick

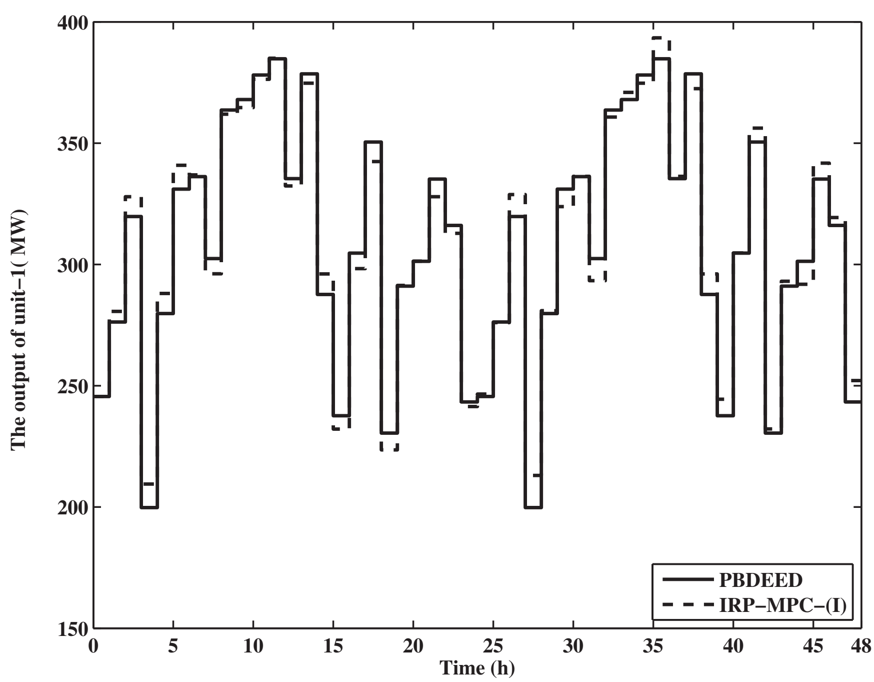

Fig. 9. The generation output of the six-unit system under PBDEED for $\alpha = 0.7$, price-II and IRP-MPC-(I).

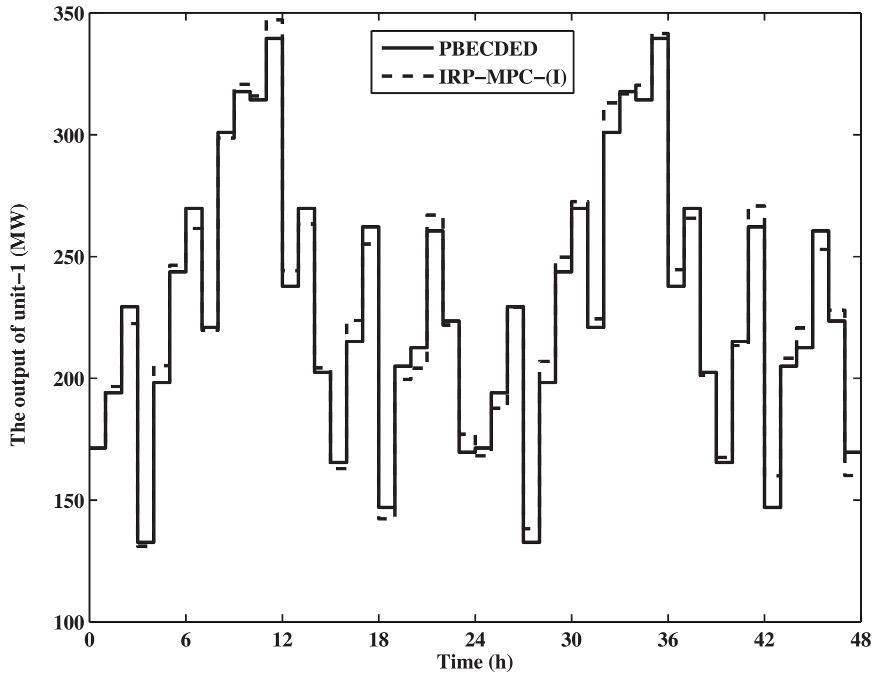

Fig. 10. The generation output of the six-unit system under PBECDED for $E^{\max} = 16,200$ lb, price-II and IRP-MPC-(I).

of the demand), then it sequentially decreasing during the interval [16, 24). We can see that, the MPC closed-loop solutions starting with different initial P^1 approach the optimal solution of the DEED and ECDED problems after 2 h.

To show the IRP-MPC, let (23) be executed, and the disturbance w_i^m is generated by

$$w_i^m = -\varepsilon_i + 2\varepsilon_i r(m), \tag{25}$$

where the parameters $r(m)$'s are uniformly distributed random numbers on [0, 1]. Denote $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_6)$. We choose different ranges of disturbances. For the DEED problem we have two cases:

IRP-MPC-(I): $\varepsilon = (5, 3, 2, 2, 3, 2)$,
IRP-MPC-(II): $\varepsilon = (15, 9, 6, 6, 9, 6)$.
For the ECDED problem we have also two cases:
IRP-MPC-(I): $\varepsilon = (5, 3, 2, 2, 3, 2)$,
IRP-MPC-(II): $\varepsilon = (10, 6, 4, 4, 6, 4)$.

In these cases the initial P_i^1 for the MPC are chosen as the optimal solution of the DEED and ECDED problems at $t = 1$, i.e., $P_i^1 = \overline{P}_i^1$. From Figs. 3 and 4 , we can see that, the IRP-MPC-(I) can keep the disturbed system around the optimal solutions of these problems.

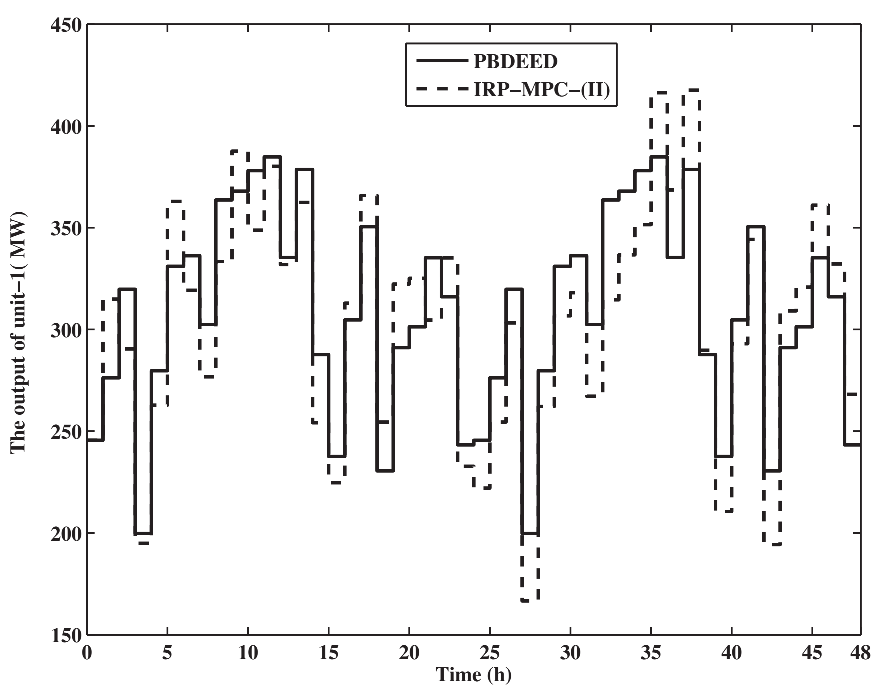

Fig. 11. The generation output of the six-unit system under PBDEED for $\alpha = 0.7$, price-II and IRP-MPC-(II).

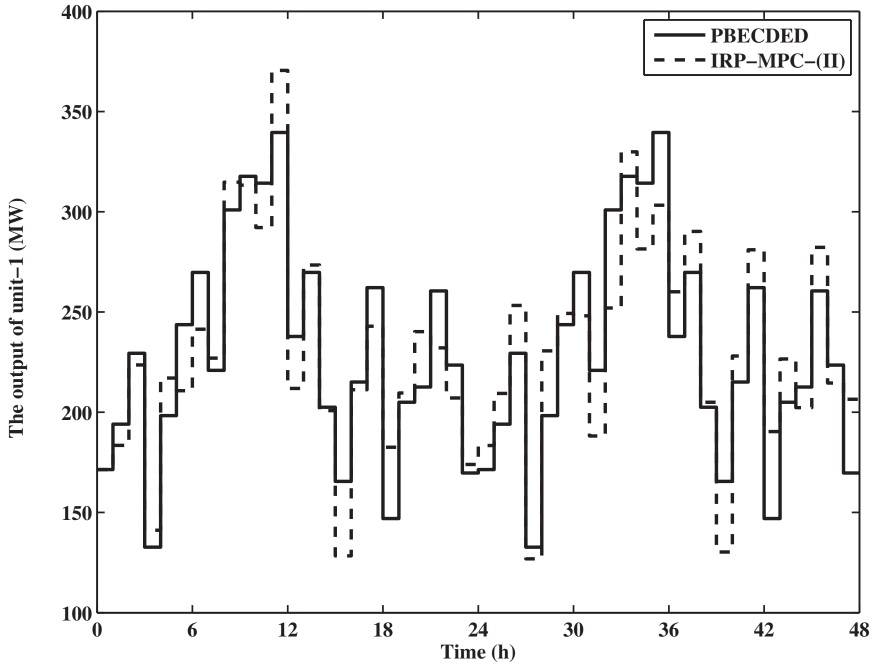

Fig. 12. The generation output of the six-unit system under PBECDED for $E^{max} = 16,200$ lb, price-II and IRP-MPC-(II).

In Figs. 5 and 6, we use the disturbance bounds which are given in IRP-MPC-(II). We can see that, although the disturbance increased, the IRP-MPC-(II) still keep the disturbed system in the neighborhood of the optimal solution of the DEED and ECDED problems. From these cases we observe that, the size of this neighborhood depends on the bound of the disturbance.

5.2. PBDEED and PBECDED problems

In this section we first present the optimal solutions of the PBDEED and PBECDED with different energy price profiles, different weighting factor α, and different allowable emission limit E^{max}. Then we show the convergence of the MPC and IRP-MPC algorithm through the problems PBDEED and PBECDED. The initial P^1 is chosen as $P^1 = (100, 170, 240, 110, 100, 100)$ such that $\sum_{i=1}^{n} P_i^1 \le D^1$. The result of the optimization is dependent on the energy price data. Indeed, minor changes in the energy price may give a significant change in the power generation of the units as well as the profit. We consider different energy price profiles which are given in Table 3.

The effect of the energy price over the time horizon on the total power which produced by the committed units for the PBDEED problem for $\alpha = 1$ is shown in Table 3. It is observed that, for price-I and price-II, the total power is less than the demand except in some intervals which are shown by bold font. It means that, GENCO choose to supply power less than the demand in some hours of the day and equal to the demand in other hours of the day. For price-III, the optimal solution of the PBDEED satisfy the demand over all intervals. In this case GENCO will supply power to satisfy the demand over the whole day since this will maximize its profit. We also note that, the profit increases according to the energy prices.

The effect of the emission on the total amount of power and the total profit in case of price-II for the PBDEED problem is given in Table 4. It can be seen that, as the weighting factor α is increased (i.e. the importance of the emission is decreased), both the profit and the total power are increased. The effect of the allowable emission limit E^{max} on the total amount of power and the total profit in case of price-II for the PBECDED problem is also shown in Table 4. We can see that, as E^{max} is increased both the profit and the total power are increased. We can also observe that, when E^{max} takes the values $E_1^{max} = 16,200$ and $E_2^{max} = 27,232$, the emission constraint (12) is active in the optimization problem and when $E_3^{max} = 30,621$, the emission constraint (12) is not active and the solution of the PBECDED is the same of that of the PBDEED for $\alpha = 1$ which is given in Table 3.

Figs. 7 and 8 present the optimal outputs of unit 1 for the PBDEED and PBECDED problems as well as the MPC closed-loop solutions. It is seen that, the closed-loop MPC solutions asymptotically converge the optimal solutions of the PBDEED and PBECDED problems, in case of price-II. We note that, the MPC closed-loop solutions starting from the same initial P_i^1 approach the optimal solutions of the DEED and ECDED problems after 4 h and 2 h, respectively. To show the IRP-MPC for the PBDEED and PBECDED problems, let the disturbance w_i^m is generated by (25) and the disturbance bound ε is chosen in two cases:

IRP-MPC-(I): $\varepsilon = (10, 5, 4, 2, 3, 2)$,
IRP-MPC-(II): $\varepsilon = (40, 20, 16, 8, 12, 8)$.

In these cases the initial P_i^1 for the MPC is chosen as the optimal solution of the PBDEED and PBECDED problems. From Figs. 9–12, we can see that, both the IRP-MPC-(I) and IRP-MPC-(II) can keep the disturbed system around the optimal solution of the PBDEED and PBECDED problems.

6. Conclusions

This paper presents some ODD formulations before and after the deregulation of the electric power market taking into consideration the emission of gaseous pollutants from fossil-fueled plants. Both the demand and energy price are assumed to be periodic. We applied MPC approach to the periodic implementation of the optimal solutions of these problems. The convergence and robustness of the MPC algorithms are demonstrated through the application of MPC to these with a six-unit system.

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