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# Hybrid DE-SQP and hybrid PSO-SQP methods for solving dynamic economic emission dispatch problem with valve-point effects



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#### ABSTRACT

The dynamic economic emission dispatch (DEED) problem taking into consideration valve-point effects is a complicated non-linear constrained multi-objective optimization problem with non-smooth and nonconvex characteristics. DEED determines the optimal generation schedule of committed generating units by minimizing both fuel cost and emission simultaneously under a set of constraints. This paper presents two hybrid optimization methods to solve the DEED problem. The first method combines differential evolution (DE) and sequential quadratic programming (SQP). The second one is hybrid particle swarm optimization (PSO) and SQP. DE or PSO is used as a global optimizer and SQP is used as fine tuning to determine the final optimal solution. Two test systems consisting of five and ten generating units with non-smooth fuel cost functions have been used to illustrate the effectiveness of the proposed methods compared with other methods. The second purpose of this paper is to extend the DEED problem in such a way that its optimal solution can be periodically implemented.

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#### 1. Introduction

Dynamic economic dispatch (DED) is a real-time power system problem that is used to schedule the committed generating units' outputs so as to meet the load demand over a dispatch period at minimum operating cost while satisfying ramp rate constraints and other constraints (see the review paper [1]). DED is a more complicated problem than the static economic dispatch (SED) problem where the ramp rate constraint is neglected. The ramp rate constraint is a dynamic constraint, which is important to maintain the life of the generators [2]. In the literature, several optimization techniques have been used to solve the DED problem with complex objective functions or constraints [1]. Mathematical programming based or heuristically based methods have been used to solve the DED problem, such as, gradient projection method [3], Lagrange relaxation [4], linear programming [5], nonlinear programming [2], dynamic programming [6,7] and interior point method [8]. These methods can be used for solving the DED problem with smooth and convex cost functions. However, in reality, the input-output characteristics of generating units are non-smooth and non-convex

owing to steam valves in large steam turbines. Neglecting such valve-point effects often introduces inaccuracy into the resulting dispatch. A DED problem with valve point effects is a non-smooth and non-convex optimization problem with multiple local optimal points, which makes finding the global optimal challenging. Most of the above-mentioned optimization methods may fail to solve such problems, as they are sensitive to initial estimates and converge into a local optimal solution and computational complexity [1]. In [9], Maclaurin series based Lagrangian method (MSL) has been used to solve the DED problem with valve-point effects. Over the past few years, in order to solve this problem, many computational intelligence methods have been developed, such as simulated annealing (SA) [10], genetic algorithms (GA) [11], differential evolution (DE) [12], particle swarm optimization (PSO) [13,14], artificial immune system (AIS) [15,16], artificial bee colony (ABC) algorithm [17] and harmony search (HS) algorithm [18]. Many of these methods have proven their effectiveness in solving DED problems without any restriction or fewer restrictions on the shape of the cost function curves. Hybrid methods that combine two or more optimization methods such as EP-SQP [19,20], PSO-SQP [21], hybrid bee colony optimization and sequential quadratic programming (BCO-SQP) [22] and hybrid Hopfield neural network and quadratic programming (HNN-QP) [23] have been successfully applied to DED problems with valve point effects.

The DED problem plays an important role in power system operation. However, the traditional DED strategies are designed in such a way that the fuel cost is minimized, neglecting emission

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constraints. The emission of gaseous pollutants including  $SO_2$ ,  $NO_x$ , CO and  $CO_2$  from fossil fuel fired thermal plants affects human health directly or indirectly. Therefore, electric utilities are requested to reduce emission from their plants. Emission dispatching is one of the preferred options to reduce emissions [24].

The emission can be taken into the static/dynamic economic dispatch formulation following three main approaches [25]. The first approach is to minimize the fuel cost and treat the emission as a constraint with a permissible limit (see e.g. [26-29]). This formulation, however, presents severe difficulty in establishing the trade-off relations between cost and emission [25]. The second approach handles both fuel cost and emission simultaneously as competing objectives [30-32]. The third approach treats the emission as another objective in addition to the fuel cost objective. However, the multi-objective optimization problem is converted to a single-objective optimization problem by a linear combination of both objectives [26,33-35]. This approach yields meaningful results to the decision maker when solved many times for different values of the weighting factor. This paper focuses on the third approach. In the second and third approaches, the dynamic dispatch problem is referred to as dynamic economic emission dispatch (DEED), which is a multi-objective optimization problem that minimizes both fuel cost and emission simultaneously under the ramp rate constraint and other constraints.

In some of the pioneering works on the topic of economic dispatch with emission considerations (see e.g. [26,32,36,37]), both generation costs and pollution levels are modeled as analytical, usually quadratic functions of the power unit output. Some other works have taken into account non-smooth and non-convex cost functions due to valve-point effects and a highly nonlinear emission function (see e.g. [30,31,33-35]). The traditional gradient based optimization methods may not converge to feasible solutions for such complex problems where the objective functions are not continuously differentiable and/or are discontinuous in nature. Alternatively, many computational intelligence methods have been proposed for solving such problems. In [34], by assuming that the decision maker has goals for each of the two objective functions, the multi-objective optimization problem is transformed into a single-objective optimization by the goal-attainment method and is solved by the PSO method. In [30], it was assumed that the decision maker had a fuzzy goal for each of the objective functions. The optimal non-inferior generation schedule is determined by the EP-based fuzzy satisfying method. In [31], the multi-objective optimization problem is solved by the non-dominated sorting genetic algorithm-II (NSGA-II). In [33,35], an improved pattern search (PS) based algorithm and an improved bacterial foraging algorithm (IBFA) are, respectively, used to solve the DEED problem, where the multi-objective optimization problem is converted into a singleobjective optimization.

The DE algorithm which was proposed by Storn and Price [38], is a population-based stochastic parallel search technique. DE uses a rather greedy and less stochastic approach to problem solving compared to other evolutionary algorithms. DE has the ability to handle optimization problems with non-smooth/non-convex objective functions [38]. Moreover, it has a simple structure and good convergence property, and it requires only a few robust control parameters [38]. DE has been successfully applied to the DED problem with non-smooth and non-convex cost functions (see [12,39]).

PSO, which was proposed by Kennedy and Eberhart [40], is one of the forms of computational intelligence that can be used for solving optimization problems with non-smooth and nonconvex characteristics without requiring derivative information. The PSO shares many similarities with evolutionary computation techniques such as DE and GA techniques. The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike DE and GA, PSO has no evolution operators such as crossover and mutation. So the PSO algorithm is simple and easy to implement, since its working mechanism only involves two fundamental updating rules. PSO has fewer operators to adjust in the implementation, and it can be flexibly combined with other optimization techniques to build a hybrid algorithm. PSO has been applied to the DED problem in [13]. In [41], quantum-inspired particle swarm optimization (QPSO) has been successfully used for solving the SED problem with valve-point effects.

Although both DE and PSO seem to be good methods to solve the DED problem with non-smooth and non-convex cost functions, solutions obtained are just near global optimum with long computation time. Therefore, hybrid methods such as DE-SQP and PSO-SQP can be effective in solving the DED and DEED problems with valve-point effects.

The main contributions of the paper are: (1) A multi-objective optimization problem is formulated using the DEED approach. The multi-objective optimization problem is converted into a single-objective optimization one using the weighting method. (2) Two hybrid methods, DE-SQP and PSO-SQP, are proposed and validated for solving the DEED problem with valve-point effects. DE or PSO is used as a base level search for global exploration and SQP is used as a local search to fine-tune the solution obtained from DE or PSO. (3) The effectiveness of the proposed methods is shown for two test systems consisting of five and ten units with non-smooth cost functions. (4) The results of the hybrid DE-SQP and PSO-SQP methods are compared with other methods given in the literature. (5) The DEED problem is extended in such a way that its optimal solution can be periodically implemented.

#### 2. Formulation of the DEED problem

The objective of the DEED problem is to determine the generation levels for the committed units, which simultaneously minimize the total fuel cost and pollutants emission over a dispatch period, while satisfying a set of constraints. The following objectives and constraints are taken into account in the formulation of DEED problem.

#### 2.1. Objective functions

(i) Cost: Traditionally, the cost function curve of a thermal unit is approximated by a quadratic function [36,37]. Power plants commonly have multiple valves, which are used to control the power output of the unit. When steam admission valves in thermal units are first opened, a sudden increase in losses is registered, which results in ripples in the cost function [10]. This phenomenon is called valve-point effects. The generator with valve-point effects has a very different input–output curve compared with the smooth cost function. Taking the valvepoint effects into consideration, the fuel cost is expressed as the sum of quadratic and sinusoidal functions [19,31,41]. Therefore, the total fuel cost over the dispatch period [0, *N*] is given by:

$$C = \sum_{t=1}^{N} \sum_{i=1}^{n} C_i(P_i^t) = \sum_{t=1}^{N} \sum_{i=1}^{n} a_i + b_i P_i^t + c_i(P_i^t)^2 + |e_i \sin(f_i(P_i^{\min} - P_i^t))|, \quad (1)$$

where  $a_i$ ,  $b_i$  and  $c_i$  are positive constants, and  $e_i$  and  $f_i$  are the coefficients of unit *i* reflecting valve-point effects; *n* is the number of committed units; *N* is the number of intervals in the time horizon;  $P_i^t$  is the generation of unit *i* during the *t*th time interval [t - 1, t);  $P_i^{\min}$  is the minimum capacity of unit *i*;  $C_i(P_i^t)$  is the generation cost for unit *i* to produce  $P_i^t$ .

(ii) *Emission:* The most important emissions considered in the power generation industry, because of their effects on the environment, are SO<sub>2</sub>, CO<sub>2</sub> and NO<sub>x</sub>. These emissions can be modeled through functions that associate emissions with power production for each unit. The emissions of both SO<sub>2</sub> and CO<sub>2</sub> can be modeled as quadratic polynomial functions [32]. NO<sub>x</sub> emissions are more difficult to model since they come from different sources and their production is associated with several factors, such as boiler temperature and air content. One approach to represent NO<sub>x</sub> emissions is using a combination of polynomial and exponential terms [42]. The emission of SO<sub>2</sub>, CO<sub>2</sub> and NO<sub>x</sub> can be modeled separately. However, for comparison purposes, the total pollutants emission over the dispatch period [0, *N*] can be expressed as [31,43]:

$$E = \sum_{t=1}^{N} \sum_{i=1}^{n} E_i(P_i^t) = \sum_{t=1}^{N} \sum_{i=1}^{n} \alpha_i + \beta_i P_i^t + \gamma_i (P_i^t)^2 + \eta_i \exp(\delta_i P_i^t),$$
(2)

where constants  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\eta_i$  and  $\delta_i$  are the coefficient of the *i*th unit emission characteristics and  $E_i(P_i^t)$  is the amount of emission from unit *i* from producing power  $P_i^t$ .

#### 2.2. Constraints

Three kinds of constraints are considered in the DEED problem, i.e., the generation capacity of each generator, power balance, and ramp rate limits.

(i) Power balance constraint

$$\sum_{i=1}^{n} P_{i}^{t} = D^{t} + Loss^{t}, \qquad t = 1, \dots, N$$
(3)

where  $D^t$  and  $Loss^t$  are the demand and transmission line loss at time t (i.e., the tth time interval), respectively.  $Loss^t$  is calculated by using B-coefficients, which can be expressed as a quadratic function of the unit's power outputs as:

$$Loss^{t} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{i}^{t} B_{ij} P_{j}^{t}, \qquad t = 1, \dots, N$$
(4)

where *B<sub>ij</sub>* is the *ij*th element of the loss coefficient square matrix of size *n*.

(ii) Generation limits

$$P_i^{\min} \le P_i^t \le P_i^{\max}, \qquad i = 1, \dots, n, \quad t = 1, \dots, N$$
 (5)

where  $P_i^{\min}$  and  $P_i^{\max}$  are the minimum and maximum capacity of unit *i*, respectively.

(iii) Generating unit ramp rate limits

$$-DR_{i} \leq P_{i}^{t} - P_{i}^{t-1} \leq UR_{i}, \qquad i = 1, \dots, n, \quad t = 2, \dots, N$$
(6)

where  $UR_i$  and  $DR_i$  are the maximum ramp up/down rates for unit *i*.

#### 2.3. The optimization problem

Aggregating the objectives and constraints, the DEED problem can be mathematically formulated as a nonlinear constrained multi-objective optimization problem, which can be converted into a single-objective optimization using the weighting method as:

$$\min F = wC + (1 - w)E \tag{7}$$

subject to constraints (3)-(6)

where  $w \in [0, 1]$  is a weighting factor. It will be noted that, when w = 1, the problem (7) determines the optimal amount of the generated power by minimizing the cost regardless of emission and the

DEED problem leads to the DED problem. If w = 0, then the DEED problem determines the optimal amount of the generated power by minimizing the emission regardless of cost and the DEED problem leads to pure dynamic emission dispatch (PDED) [31].

#### 3. Differential evolution method

DE is a simple yet powerful heuristic method for solving nonlinear, non-convex and non-smooth optimization problems. The DE algorithm is a population based algorithm using three operators, mutation, crossover and selection, to evolve from the randomly generated initial population to the final individual solution [38]. In the initialization a population of  $N_P$  target vectors (parents)  $X_i = \{x_{1i}, x_{2i}, \ldots, x_{Di}\}, i = 1, 2, \ldots, N_P$  is randomly generated within user-defined bounds, where D is the dimension of the optimization problem. Let  $X_i^G = \{x_{1i}^G, x_{2i}^G, \ldots, x_{Di}^G\}$  be the individual *i* at the current generation G. A mutant vector  $V_i^{G+1} = (v_{1i}^{G+1}, v_{2i}^{G+1}, \ldots, v_{Di}^{G+1})$  is generated according to

$$V_{i}^{G+1} = X_{r_{1}}^{G} + \mathcal{F} \times (X_{r_{2}}^{G} - X_{r_{3}}^{G}), \qquad r_{1} \neq r_{2} \neq r_{3} \neq i, \quad i = 1, 2, \dots, N_{P}$$
(8)

with randomly chosen integer indexes  $r_1, r_2, r_3 \in \{1, 2, ..., N_P\}$ . Here  $\mathcal{F}$  is the mutation factor.

According to the target vector  $X_i^G$  and the mutant vector  $V_i^{G+1}$ , a new trial vector (offspring)  $U_i^{G+1} = \left\{ u_{1i}^{G+1}, u_{2i}^{G+1}, \dots, u_{Di}^{G+1} \right\}$  is created with

$$u_{ji}^{G+1} = \begin{cases} v_{ji}^{G+1} & \text{if } (\operatorname{rand}(j) \le CR) & \text{or } j = \operatorname{rnb}(i) \\ x_{ji}^G & \text{otherwise} \end{cases},$$
(9)

where j = 1, 2, ..., D,  $i = 1, 2, ..., N_P$  and rand(j) is the jth evaluation of a uniform random number between [0, 1].  $CR \in [0, 1]$  is the crossover constant that has to be determined by the user. rnb(i) is a randomly chosen index from 1, 2, ..., D which ensures that  $U_i^{G+1}$  gets at least one parameter from  $V_i^{G+1}$  [38].

The selection process determines which of the vectors will be chosen for the next generation by implementing one-to-one competition between the offsprings and their corresponding parents. If f denotes the function to be minimized, then

$$X_{i}^{G+1} = \begin{cases} U_{i}^{G+1} & \text{if } f(U_{i}^{G+1}) \le f(X_{i}^{G}) \\ X_{i}^{G} & \text{otherwise} \end{cases},$$
(10)

where  $i = 1, 2, ..., N_P$ . The value of f of each trial vector  $U_i^{G+1}$  is compared with that of its parent target vector  $X_i^G$ . The above iteration process of reproduction and selection will continue until a user-specified stopping criterion is met.

#### 4. Particle swarm optimization method

Particle swarm optimization (PSO) is one of the evolutionary computations, which can be used for solving optimization problems with non-smooth and non-convex objective functions [40]. The PSO algorithm searches in parallel using a swarm of particles. Considering the *i*th particle in the swarm, its position and velocity at iteration *G* are denoted by  $X_i^G = \{x_{1i}^G, x_{2i}^G, \ldots, x_{Di}^G\}$  and  $V_i^G = \{v_{1i}^G, v_{2i}^G, \ldots, v_{Di}^G\}$ , respectively. The particle updates its position and velocity repeatedly by the following equations:

$$\nu_{ji}^{G+1} = \omega \nu_{ji}^{G} + c_1 r_1 (Pbest_{ji}^G - x_{ji}^G) + c_2 r_2 (Gbest_j^G - x_{ji}^G),$$
  

$$j = 1, 2, \dots, D, \quad i = 1, 2, \dots, N_P,$$
(11)

$$x_{ji}^{G+1} = x_{ji}^G + K v_{ji}^{G+1}, \qquad j = 1, 2, \dots, D, \quad i = 1, 2, \dots, N_P,$$
 (12)

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until the stopping criterion is reached. Here,  $Pbest_i^G = (Pbest_{1i}^G, Pbest_{2i}^G, \dots, Pbest_{Di}^G)$  is the best known position of the particle *i* until the current iteration *G* and  $Gbest^G = (Gbest_1^G, Gbest_2^G, \dots, Gbest_D^G)$  is the best particle in the swarm at iteration *G*;  $r_1$  and  $r_2$  are uniform random numbers in [0, 1];  $c_1$  and  $c_2$  are acceleration factors;  $\omega$  and *K* are the inertial and constriction factors that have been defined in [44,45], respectively, as:

$$\omega = \omega_{\max} - \frac{(\omega_{\max} - \omega_{\min})G}{G_{\max}}, \qquad K = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|}, \quad (13)$$

where  $\omega_{\text{max}}$  and  $\omega_{\text{min}}$  are initial and final weights,  $G_{\text{max}}$  is the maximum iteration number, and  $\varphi = c_1 + c_2 \ge 4$ . The fitness of each particle can be evaluated according to the objective function to be minimized.

In this paper, we define the evaluation function for evaluating the fitness of each individual in the population in DE and PSO algorithms as follows:

$$f = F + \lambda \sum_{t=1}^{N} \left( \sum_{i=1}^{n} P_i^t - (D^t + Loss^t) \right)^2,$$
 (14)

where  $\lambda$  is a penalty value. Then the objective is to find  $f_{\min}$ , the minimum evaluation value of all the individuals in all iterations. The penalty term reflects the violation of the equality constraint. Once the minimum of *f* is reached, the equality constraint is satisfied. Also, the generation power output of each unit at time *t* should be adjusted to satisfy the following constraints, which combine constraints (5) and (6) as:

$$P_{i}^{t} = \begin{cases} P_{i}^{t,\min} & \text{if } P_{i}^{t} < P_{i}^{t,\min}, \\ P_{i}^{t} & \text{if } P_{i}^{t,\min} \le P_{i}^{t} \le P_{i}^{t,\max}, \\ P_{i}^{t,\max} & \text{if } P_{i}^{t} > P_{i}^{t,\max}, \end{cases}$$
(15)

$$P_{i}^{t,\min} = \begin{cases} P_{i}^{\min} & \text{if } t = 1, \\ \max(P_{i}^{\min}, P_{i}^{t-1} - DR_{i}) & \text{others,} \end{cases},$$

$$P_{i}^{t,\max} = \begin{cases} P_{i}^{\max} & \text{if } t = 1, \\ \min(P_{i}^{\max}, P_{i}^{t-1} + UR_{i}) & \text{others.} \end{cases}$$
(16)

#### 5. Sequential quadratic programming method

The SQP method can be considered as one of the best nonlinear programming methods for constrained optimization problems [46]. It outperforms every other nonlinear programming method in terms of efficiency, accuracy and percentage of successful solutions over a large number of test problems. The method closely resembles Newton's method for constrained optimization, just as is done for unconstrained optimization. At each iteration, an approximation is made of the Hessian of the Lagrangian function using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton updating method. The result of the approximation is then used to generate a quadratic programming (QP) sub-problem whose solution is used to form a search direction for a line search procedure.

In a general form the optimization problem (7) with constraints (3)–(6) can be rewritten as

$$\min_{\mathbf{x}} F(\mathbf{x}),\tag{17}$$

$$g_i(x) = 0, \qquad i = 1, 2, \dots, m_e$$
 (18)

$$g_i(x) \le 0, \qquad i = m_e + 1, 2, \dots, m$$
 (19)

where  $x = (P_1^1, P_2^1, \dots, P_n^1, P_1^2, P_2^2, \dots, P_n^2, \dots, P_1^N, P_2^N, \dots, P_n^N)^I$ , g(x) represents the constraints from (3)–(6),  $m_e$  is the number of equality constraints and m is the number of constraints.

The formulation of the SQP subroutine is taken from [19,22]. For each iteration, a QP is solved to obtain the search direction for updating the control variables. The QP problem can be described as follows:

Minimize the following

$$\nabla F(\mathbf{x}_k)^T d_k + \frac{1}{2} d_k^T H_k d_k \tag{20}$$

subject to

$$g_i(x_k) + [\nabla g_i(x_k)]^T d_k = 0, \qquad i = 1, 2, \dots, m_e$$
 (21)

$$g_i(x_k) + [\nabla g_i(x_k)]^T d_k \le 0, \qquad i = m_e + 1, \dots, m$$
 (22)

where  $d_k$  is the basis for a search direction at iteration k, and  $H_k$  is the Hessian matrix of the Lagrangian function defined by:

$$L(x, \lambda) = F(x) + \lambda^T g(x)$$
 at  $x = x_k$ ,

where  $\lambda$  is the vector of the Lagrangian multiplier.

At each iteration,  $H_k$  is approximated by  $B_k$  and calculated using the BFGS quasi-Newton method as:

$$B_{k+1} = B_k + \frac{q_k q_k^T}{q_k^T S_k} - \frac{(B_k S_k)(B_k S_k)^T}{S_k^T B_k S_k},$$
(23)

where  $S_k = x_{k+1} - x_k$  and  $q_k = \nabla L(x_{k+1}, \lambda_{k+1}) - \nabla L(x_k, \lambda_{k+1})$ .

At each iteration k, a search direction  $d_k$  is calculated by solving the QP sub-problem (20)–(22). The calculated  $d_k$  is used to form a new iteration

$$x_{k+1} = x_k + \alpha_k d_k. \tag{24}$$

The step length value  $\alpha_k$  is determined to produce a considerable reduction in an augmented Lagrangian merit function, which is given by:

$$L(x, \lambda, \rho) = F(x) - \sum_{i=1}^{m} \lambda_i (g_i(x) - s_i) + \frac{1}{2} \sum_{i=1}^{m} \rho_i (g_i(x) - s_i)^2,$$
(25)

where  $\rho_i$  is a non-negative penalty parameter and  $s_i$  is slack variable with  $s_i = 0$  for  $i = 1, 2, ..., m_e$  and  $s_i \ge 0$  for  $i = m_e + 1, ..., m$ . The procedure is repeated until the value of  $x_k$  has reached some tolerance value.

Since the objective function of the DEED problem is non-convex and non-smooth, SQP ensures a local minimum for an initial solution. SQP has been combined with computational intelligence methods to constitute hybrid methods for solving the DED problem with non-smooth cost functions (see [19,20]). In this paper, DE or PSO is used as a global search and finally the best solution obtained from DE or PSO is given as initial condition for the SQP method as a local search to fine tune the solution.

#### 6. Simulation results

In this section we show the effectiveness of the two proposed hybrid methods DE-SQP and PSO-SQP for solving the DEED problem. The optimal solutions of this problem are performed over 24 h (N=24). For comparison purposes, we shall solve the DEED problem with w = 1 and w = 0.5. Two test systems consisting of five units and ten units with valve point effects are used to investigate the effectiveness of the proposed techniques in solving these problems. The technical data of the units, as well as the demand for the five and ten units systems, are taken from [34,31], respectively. In the DE-SQP algorithm, the control parameters of the DE

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Tabla	1

Hourly generation (MW) schedule obtained from DEED ( $w = 1$ ) for 5-unit system	em.

Н	DE-SQP						PSO-SQP					
	$P_1$	P <sub>2</sub>	P <sub>3</sub>	$P_4$	$P_5$	Loss	$\overline{P_1}$	$P_2$	$P_3$	$P_4$	$P_5$	Loss
1	19.6671	99.3285	30.0002	125.0290	139.7957	3.8205	11.4332	24.5937	112.6649	124.8255	139.9578	3.4751
2	10.0000	98.1460	66.5553	124.8249	139.6003	4.1266	10.0000	54.5848	110.6628	124.8145	138.8630	3.9251
3	10.0000	98.5929	106.5553	124.7446	139.8895	4.7823	16.2577	84.5848	113.3256	125.2555	140.2897	4.7135
4	10.0403	98.5493	112.5986	124.9357	189.8537	5.9775	10.0000	98.1664	112.5099	125.2409	190.0600	5.9772
5	10.0000	92.6878	108.5367	124.1238	229.3590	6.7073	10.0000	89.1455	111.6724	124.9280	228.9414	6.6874
6	39.9759	98.9154	113.2218	134.1017	229.6400	7.8548	39.9997	99.0995	113.0730	134.0337	229.6498	7.8558
7	10.0000	98.3087	112.7789	183.6985	229.6611	8.4472	10.0000	98.2029	112.8499	183.9454	229.4484	8.4467
8	12.9827	98.3039	112.7953	209.5425	229.6303	9.2548	13.1107	98.4023	112.6127	209.5222	229.6075	9.2553
9	42.9827	104.9387	112.5785	210.0177	229.6819	10.1996	43.1107	102.2953	114.2371	210.7439	229.7988	10.1858
10	64.5563	98.3868	112.4795	209.7516	229.3849	10.5592	64.9041	98.3302	112.4586	209.6534	229.2119	10.5583
11	75.0000	102.8000	113.2107	210.4401	229.5899	11.0407	75.0000	101.8810	114.4250	209.8585	229.8665	11.0311
12	75.0000	98.6214	138.0875	210.0776	229.7356	11.5223	75.0000	124.3129	112.4607	210.2713	229.6763	11.7213
13	64.0106	98.5220	112.7331	209.9040	229.3893	10.5591	64.4200	98.2824	112.3974	209.8474	229.6131	10.5603
14	49.3375	98.4142	112.8479	209.9967	229.5720	10.1682	49.8501	98.8815	112.1553	209.8009	229.4835	10.1713
15	52.7524	98.4470	112.5436	209.6813	189.6658	9.0900	36.0767	98.4420	112.8759	185.9252	229.8031	9.1230
16	25.8887	99.0026	112.7026	209.7523	139.8940	7.2402	10.0000	98.5938	112.7123	135.9252	230.0032	7.2346
17	10.0000	94.9531	110.2569	209.8458	139.7151	6.7708	10.0000	88.7826	112.1244	124.8638	228.9142	6.6851
18	10.0000	98.6774	112.6931	209.9107	184.7110	7.9922	40.0000	105.2586	114.9867	125.4454	230.1804	7.8711
19	13.0402	98.2030	112.5940	209.9028	229.5160	9.2561	67.5146	98.4770	112.6392	154.8824	229.5365	9.0497
20	43.0402	109.8703	121.8582	210.0922	229.7166	10.5777	68.9177	98.7513	112.6800	204.8824	229.3120	10.5435
21	38.7787	98.9611	112.6070	210.0195	229.5387	9.9050	38.9725	99.0798	112.5816	209.8709	229.3994	9.9043
22	10.0000	98.5891	112.3623	162.3947	229.5263	7.8724	10.0493	99.0665	112.7445	209.8076	181.2464	7.9143
23	10.0000	98.6173	112.3728	125.0027	186.9141	5.9070	10.0000	98.6181	112.7340	171.8330	139.7544	5.9395
24	10.0000	97.7303	95.5132	124.6522	139.6716	4.5673	10.0000	81.1715	112.0820	124.3974	139.8392	4.4901

are chosen as:  $N_p = 60$ ,  $\mathcal{F} = 0.423$  and CR = 0.885. The maximum number of iterations of the DE are selected as 20,000. In the PSO-SQP algorithm, we have chosen the following parameters:  $N_p = 60$ ,  $V^{\text{max}} = 0.5 * P^{\text{max}}$ ,  $V^{\text{min}} = -0.5 * P^{\text{min}}$ ;  $\omega_{\text{max}} = 0.9$ ,  $\omega_{\text{min}} = 0.4$ ,  $c_1 = 2.25$ ,  $c_2 = 2.25$  and  $G_{\text{max}} = 20,000$ . The results represent the average of 30 runs of the two proposed methods. All computations were carried out using the MATLAB program.

**1 – Five-unit test system:** This example presents an application of the DE-SQP and PSO-SQP methods to the DEED problem consisting of five units. The best solutions of the DEED problem with w = 1 and w = 0.5 using the DE-SQP and PSO-SQP methods are given in Tables 1 and 2, respectively. The best cost and emission obtained

by the DE-SQP and PSO-SQP methods are given in Table 7. Comparisons between our proposed methods (DE-SQP and PSO-SQP) and other methods are given in Table 7.

**2** – **Ten-unit test system:** This example presents an application of DE-SQP and PSO-SQP to the DEED problem consisting of ten units. The best solutions of the DEED problem with w = 1 and w = 0.5 are given, respectively, in Tables 3 and 4 for the DE-SQP method. Tables 5 and 6 present the best solutions of the DEED problem with w = 1 and w = 0.5 using the PSO-SQP method. Comparisons between our proposed methods (DE-SQP and PSO-SQP) and other methods are given in Table 7.

#### Table 2

Hourly generation (N	V) schedule obtained	from DEED ( $w = 0.5$	) for 5-unit system.
	,		,

Н	DE-SQP						PSO-SQP					
	P <sub>1</sub>	P <sub>2</sub>	$P_3$	$P_4$	$P_5$	Loss	$P_1$	P <sub>2</sub>	$P_3$	$P_4$	$P_5$	Loss
1	14.8268	20.0315	113.2445	125.1900	140.1782	3.4710	15.6625	20.0919	112.9755	125.0083	139.7310	3.4692
2	11.3400	50.0072	112.6916	125.3360	139.5412	3.9160	11.3322	50.0154	112.7441	125.1023	139.7219	3.9159
3	29.6429	73.0206	112.7897	124.7228	139.4841	4.6601	29.8746	72.7001	112.6780	124.8987	139.5081	4.6594
4	59.6429	98.0413	112.8276	125.4323	139.8899	5.8341	59.8746	98.6802	112.8091	124.9241	139.5476	5.8355
5	74.4937	98.1315	112.5976	139.5185	139.7374	6.4788	74.4171	98.5776	112.6890	139.3872	139.4086	6.4795
6	74.7550	98.4822	112.8156	189.5185	140.2139	7.7854	75.0000	98.7700	112.9609	189.3872	139.6673	7.7854
7	73.7378	98.5901	112.6739	209.8052	139.5098	8.3169	74.2444	98.2495	112.5627	209.6976	139.5617	8.3159
8	75.0000	102.2437	122.3423	209.7080	153.7361	9.0302	75.0000	100.0961	122.8983	209.9946	155.0342	9.0231
9	75.0000	98.2743	113.0444	210.0436	203.7361	10.0985	74.9938	98.5553	112.5349	209.5036	204.5135	10.1011
10	72.5317	97.7322	112.4313	209.5599	222.2837	10.5388	65.9680	98.1380	112.4105	209.2977	228.7411	10.5554
11	75.0000	99.1655	117.5165	209.8143	229.5115	11.0078	75.0000	99.1912	117.3544	209.9258	229.5377	11.0091
12	75.0000	99.1177	137.7685	210.0998	229.5383	11.5243	75.0000	98.8629	138.0187	210.1676	229.4735	11.5228
13	70.7405	97.7635	112.4161	209.7667	223.8564	10.5432	73.5830	98.5391	112.4882	209.8168	220.1095	10.5366
14	75.0000	98.6271	126.2220	210.0466	190.1296	10.0253	75.0000	98.7871	126.4764	209.9777	189.7829	10.0242
15	75.0000	99.1669	138.8852	209.7932	140.1322	8.9776	74.9911	98.4001	139.6294	209.9255	140.0282	8.9742
16	75.0000	98.8458	112.9378	160.4643	139.7791	7.0270	44.9911	95.9191	112.1534	194.5233	139.5422	7.1292
17	74.5398	97.9513	112.8684	139.2430	139.8750	6.4776	15.0730	88.7350	112.2965	208.9761	139.6424	6.7230
18	75.0000	98.6407	112.7645	189.2430	140.1369	7.7852	45.0730	99.4790	121.3833	209.8743	140.0477	7.8574
19	75.0000	98.5272	139.9118	209.8308	139.7040	8.9738	75.0000	104.6133	133.2673	210.1777	139.9514	9.0097
20	75.0000	114.1831	175.0000	210.4904	139.6863	10.3597	75.0000	98.4747	141.1332	210.2208	189.5471	10.3759
21	75.0000	111.4441	153.0303	210.2858	139.9392	9.6994	75.0000	98.7589	166.2569	209.8422	139.7809	9.6388
22	70.0320	98.2309	113.0303	191.6325	139.7893	7.7150	74.9999	98.5569	126.2569	173.0516	139.7644	7.6297
23	40.0338	98.4037	113.1673	141.6325	139.5617	5.7990	56.7019	98.6673	112.6278	124.9395	139.8345	5.7710
24	10.0338	80.6922	112.3798	124.8631	139.5199	4.4887	26.7021	98.4645	112.7146	124.6583	104.9598	4.4992

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**Table 3** Hourly generation (MW) schedule obtained from DEED (w = 1) using DE-SQP for 10-unit system.

Н	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$	$P_{10}$	Loss
1	150.0000	135.0000	73.0000	70.3333	222.9974	155.1682	99.2918	120.0000	20.0000	10.0000	19.7912
2	150.0000	135.0000	101.9485	120.3333	222.6154	123.7029	129.2918	90.0000	48.7980	10.7150	22.4058
3	150.0000	135.0000	181.9485	170.3333	174.2621	130.9190	129.6896	120.0000	53.5785	40.7150	28.4468
4	150.0000	135.0000	183.1516	218.2899	223.5485	160.0000	129.3947	120.0000	80.0000	42.0564	35.4415
5	150.0000	135.0000	258.8414	249.7412	224.0147	160.0000	128.5373	120.0000	80.0000	13.2136	39.3484
6	150.0000	135.0000	315.1962	299.7412	243.0000	160.0000	129.8624	120.0000	80.0000	43.2136	48.0136
7	150.0000	176.9470	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	52.9470
8	178.2448	228.3049	340.0000	300.0000	243.0000	160.0000	129.9436	120.0000	80.0000	54.9118	58.4054
9	258.2448	308.3049	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5500
10	289.0490	384.5331	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	79.5821
11	368.7363	397.1230	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	87.8595
12	374.8564	439.5807	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	92.4378
13	342.1737	386.2429	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	84.4166
14	262.1737	306.2429	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	53.1527	70.5693
15	182.1737	226.2429	340.0000	299.9639	243.0000	160.0000	130.0000	120.0000	80.0000	53.0342	58.4148
16	150.0000	146.2429	294.7660	249.9639	223.6700	160.0000	129.6353	120.0000	80.0000	43.3613	43.6398
17	150.0000	135.0000	258.1720	249.5279	223.9121	160.0000	128.8682	120.0000	80.0000	13.8650	39.3459
18	150.0000	151.6366	298.4749	299.5279	243.0000	160.0000	129.7933	120.0000	80.0000	43.6183	48.0511
19	227.2425	231.6366	299.3393	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	43.5728	58.7914
20	307.2425	311.6366	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	74.8793
21	265.4293	301.1183	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5476
22	185.4293	221.1183	263.3759	250.0000	225.8767	160.0000	129.8685	120.0000	80.0000	41.1109	48.7801
23	150.0000	141.1183	183.3759	200.0000	223.4887	155.9437	128.7427	120.0000	50.0000	11.1109	31.7806
24	150.0000	135.0000	173.1056	180.5739	173.7249	118.1382	128.6826	120.0000	20.0000	10.0000	25.2260

The results of the five-unit and ten-unit test systems for the DEED problem show that our proposed methods are efficient, yielding a cheaper cost and a lower amount of emission than the other methods.

## 6.1. Periodic implementation of the optimal solution of the DEED problem

In this section we show a technical deficiency that may arise when the optimal solutions for the DEED problem are implemented repeatedly and periodically in response to periodic demand. Then we show how to overcome this deficiency. The periodicity assumption comes from the fact that the demand is periodic due to cyclic consumption behavior and seasonal changes [47]. Note that the DEED problem is formulated over the dispatch interval [0, *N*) and does not consider the periodic implementations of the optimal solution over the period [*N*, 2*N*), [2*N*, 3*N*), . . .. Sometimes such an optimal solution cannot be implemented practically, in other words, the solution is not practically feasible. The solution has not taken into consideration the consistency of the unit ramp rate constraints for all the units. Our results for the DEED problem show that the optimal solutions can be only implemented over the interval [0, 24]. These solutions cannot be implemented for the following 24 h by a simple repetition, since the ramp rate constraint is violated when the generating units are moved from the 24th hour of a day to the first hour of the next day. If we look at the optimal solution of the DEED problem (*w* = 1) obtained using DE-SQP for the five-unit system given in Table 1, we can see that the optimal solution of unit 3 is given by  $P_3^1 = 30.0002$ and  $P_3^{24} = 95.5132$ . This solution cannot be implemented repeatedly every 24 h because  $P_3^1 - P_3^{24} = -65.513 < -DR_3 = -40$ . Also, for the DEED problem (*w* = 0.5) we can see from Table 2 that,

Table 4

Hourly generation (MW) schedule obtained from DEED ( $w = 0.5$ ) using DE-SQP for 10-unit syst	tem.
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Н	$P_1$	<i>P</i> <sub>2</sub>	P <sub>3</sub>	P4	$P_5$	$P_6$	P <sub>7</sub>	$P_8$	$P_9$	P <sub>10</sub>	Loss
1	150.0000	135.0000	73.0000	120.3833	171.9264	121.8871	99.0894	120.0000	51.9715	12.3269	19.5848
2	150.0000	135.0000	83.7433	120.6711	171.2469	128.1038	129.0894	120.0000	52.2245	42.3269	22.4065
3	150.0000	135.0000	162.5193	169.0630	174.6565	120.9375	129.5651	120.0000	80.0000	44.7980	28.5399
4	150.0000	135.0000	207.1277	191.7171	224.6565	160.0000	128.7881	120.0000	80.0000	44.1712	35.4606
5	150.0000	135.0000	215.5927	240.7025	243.0000	160.0000	129.9008	120.0000	80.0000	45.1414	39.3376
6	150.0000	157.4427	295.4659	285.1625	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	48.0732
7	150.0000	221.0206	296.1557	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	53.1764
8	178.9906	227.5589	339.8541	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	58.4036
9	258.9906	307.5589	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5495
10	301.9478	371.5987	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	79.5467
11	369.6977	396.1633	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	87.8611
12	395.4459	419.0055	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	92.4515
13	345.4506	382.9677	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	84.4183
14	265.4506	302.9677	338.1488	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5671
15	185.4506	222.9677	337.9894	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	58.4078
16	150.0000	142.9677	278.5398	250.0000	243.0000	160.0000	129.1928	120.0000	80.0000	43.8688	43.5692
17	150.0000	135.0000	216.0680	241.8310	243.0000	160.0000	129.7597	120.0000	80.0000	43.6770	39.3358
18	150.0000	151.7799	294.9569	291.3094	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	48.0463
19	227.0990	231.7799	288.8850	299.0316	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	58.7956
20	307.0990	311.7799	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	74.8790
21	266.1429	300.4048	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5478
22	186.1429	220.4048	262.2225	250.0000	223.7053	160.0000	130.0000	120.0000	80.0000	44.3094	48.7850
23	150.0000	140.4048	184.0217	200.0000	222.3715	124.9570	129.4908	90.0000	80.0000	42.7995	32.0455
24	150.0000	135.0000	161.9013	150.0000	174.5532	123.0137	129.1198	120.0000	52.8354	12.7995	25.2231

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Table 5						
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Hour	ly generation	(MW)	) schedule obta	ained from	DEED (N	v = 1) i	using PSO-	SQP for 1	10-unit systen	n.
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Н	$P_1$	$P_2$	P <sub>3</sub>	P <sub>4</sub>	P <sub>5</sub>	$P_6$	P <sub>7</sub>	P <sub>8</sub>	P <sub>9</sub>	P <sub>10</sub>	Loss
1	150.0000	135.0000	161.1311	116.0850	172.2860	72.4783	128.6751	90.0000	20.0000	10.0000	19.6561
2	150.0000	135.0000	155.6167	166.0850	124.1501	122.4783	128.8833	120.0000	20.0000	10.0000	22.2136
3	150.0000	135.0000	183.5401	174.5330	174.1501	160.0000	129.0625	120.0000	50.0000	10.0000	28.2862
4	150.0000	135.0000	242.6098	189.7078	224.1501	160.0000	130.0000	120.0000	80.0000	10.0000	35.4677
5	150.0000	135.0000	264.8208	239.7078	225.6501	160.0000	130.0000	120.0000	80.0000	14.1749	39.3542
6	150.0000	135.0000	324.1902	289.7078	243.0000	160.0000	129.9610	120.0000	80.0000	44.1749	48.0338
7	150.0000	176.9470	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	52.9470
8	178.2983	228.2515	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	54.8549	58.4049
9	258.2983	308.2515	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5500
10	285.3452	388.2515	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	79.5967
11	305.6975	460.3508	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	88.0489
12	374.6758	439.7622	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	92.4380
13	340.6099	387.8061	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	84.4163
14	260.6099	307.8061	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	53.1538	70.5701
15	180.6099	227.8061	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	53.0003	58.4166
16	150.0000	147.8061	296.7007	250.0000	221.6825	160.0000	129.7722	120.0000	80.0000	41.6921	43.6541
17	150.0000	135.0000	257.3405	250.0000	225.3079	160.0000	130.0000	120.0000	80.0000	11.6921	39.3408
18	150.0000	149.6340	301.7197	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	41.6921	48.0461
19	229.2505	229.6340	298.7420	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	44.1686	58.7956
20	309.2505	309.6340	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	74.8849
21	264.4495	302.0980	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5475
22	184.4495	222.0980	266.9432	250.0000	222.8129	160.0000	128.7893	120.0000	80.0000	41.7004	48.7935
23	150.0000	142.0980	186.9432	200.0000	222.9809	121.9721	130.0000	120.0000	50.0000	40.0000	31.9945
24	150.0000	135.0000	165.8788	177.3048	177.2730	124.8055	128.9304	120.0000	20.0000	10.0000	25.1962

 $P_2^1 - P_2^{24} = 20.0315 - 80.6922 = -60.6607 < -DR_2 = -30$ . A similar observation can be seen for the optimal solutions of the DEED problem given in Tables 1 and 2 using PSO-SQP.

From the results of the ten-unit system using the DE-SQP technique, we also observe from Tables 3 and 4 that  $P_3^1 - P_3^{24} = 73 - 173.1056 = -100.1056 < -DR_3 = -80$  and  $P_3^1 - P_3^{24} = 73 - 161.9013 = -88.9013 < -DR_3 = -80$ , respectively. This ramp rate violation also happened for the DEED problem with w = 1 and w = 0.5 using the PSO-SQP technique (see Tables 5 and 6).

To eliminate the violation of the unit ramp rate constraints between the last hour in a day and the first hour of the next day, we include the ramp limit on the difference between  $P_i^{24}$  and  $P_i^{25} = P_i^1$ . This can be achieved by adding the next constraint to the conventional DEED problem (see [47,26]).

$$-DR_{i} \leq P_{i}^{1} - P_{i}^{N} \leq UR_{i}, \qquad i = 1, 2, \dots, n.$$
(26)

The extended version of the DEED problem after adding constraints (26) will be referred to as EDEED, respectively.

In the DE or PSO algorithm, the up and down limits of the generating unit i at time t should be modified to take into account constraints (26) as:

$$P_{i}^{t,\min} = \begin{cases} P_{i}^{\min} & \text{if } t = 1\\ \max(P_{i}^{\min}, P_{i}^{1} - UR_{i}, P_{i}^{N-1} - DR_{i}) & \text{if } t = N\\ \max(P_{i}^{\min}, P_{i}^{t-1} - DR_{i}) & \text{others} \end{cases}$$

$$P_{i}^{t,\max} = \begin{cases} P_{i}^{\max} & \text{if } t = 1\\ \min(P_{i}^{\max}, P_{i}^{1} + DR_{i}, P_{i}^{N-1} + UR_{i}) & \text{if } t = N\\ \min(P_{i}^{\max}, P_{i}^{t-1} + UR_{i}) & \text{others} \end{cases}$$

#### Table 6

Hourly generation (MW) schedule obtained from DEED (w = 0.5) using PSO-SQP for 10-unit system.

Н	$P_1$	P <sub>2</sub>	P <sub>3</sub>	$P_4$	$P_5$	$P_6$	P <sub>7</sub>	$P_8$	$P_9$	P <sub>10</sub>	Loss
1	150.0000	135.0000	73.0000	64.9607	171.6816	123.9156	128.8882	85.1571	80.0000	43.2457	19.8490
2	150.0000	135.0000	153.0000	114.9607	173.0230	123.1067	100.0000	90.0000	80.0000	13.2457	22.3362
3	150.0000	135.0000	183.1071	148.1744	173.3021	124.4116	130.0000	120.0000	80.0000	42.5649	28.5602
4	150.0000	135.0000	202.2424	198.1744	223.3021	160.0000	129.7374	120.0000	80.0000	42.9886	35.4449
5	150.0000	135.0000	220.9820	248.1744	231.2658	160.0000	129.7282	120.0000	80.0000	44.1826	39.3331
6	150.0000	138.8466	300.9820	298.1744	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	48.0031
7	150.0000	218.8466	298.3133	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	53.1600
8	177.8564	228.6940	339.8549	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	58.4053
9	257.8564	308.6940	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5504
10	303.0647	370.4800	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	79.5448
11	369.2799	396.5805	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	87.8604
12	395.3086	419.1426	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	92.4512
13	345.3921	383.0261	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	84.4183
14	265.3921	303.0261	338.1488	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5671
15	185.3921	223.0261	337.9895	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	58.4078
16	150.0000	143.0261	275.3372	250.0000	243.0000	160.0000	129.7236	120.0000	80.0000	46.4772	43.5641
17	150.0000	135.0000	217.1604	241.2473	243.0000	160.0000	130.0000	120.0000	80.0000	42.9256	39.3333
18	150.0000	199.6263	297.1604	241.6045	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	48.3912
19	226.5688	232.3088	296.3173	291.6045	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	58.7994
20	306.5688	312.3088	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	74.8776
21	265.5009	301.0467	340.0000	300.0000	243.0000	160.0000	130.0000	120.0000	80.0000	55.0000	70.5477
22	185.5009	221.0467	263.0794	250.0000	223.1614	160.0000	130.0000	120.0000	80.0000	43.9977	48.7862
23	150.0000	141.0467	185.0360	200.0000	182.6752	160.0000	128.5268	120.0000	53.5833	42.9194	31.7877
24	150.0000	135.0000	105.0360	177.9183	173.3264	123.0331	128.9683	120.0000	52.4596	43.5990	25.3409

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Table 7
Comparison results for 5-unit and 10-unit systems.

	State	w = 1		<i>w</i> = 0.5		
		Cost (\$)	Emission (lb)	Cost (\$)	Emission (lb)	
	SA [10]	47,356	-	-	-	
	APSO [14]	44,678	-	-	-	
	AIS [16]	44,385.43	-	-	-	
	GA [17]	44,862.42	-	-	-	
	PSO [17]	44,253.24	-	-	-	
	ABC [17]	44,045.83	-	-	-	
	MSL [9]	49,216.81	-	-	-	
5-Unit system	HS [18]	44,376.23	-	-	-	
	DE [12]	45,800	-	-	-	
	PS [33]	46,530	-	47,911	18,927	
	EP [33]	46,777	-	-	-	
	PSO [34]	47,852	22,405	50,893	20,163	
	DE-SQP	43,161	23,080	44,450	19,616	
	PSO-SQP	43,263	23,180	44,542	19,772	
	EP [15]	2.5854	-	-	-	
	PSO [15]	2.5722	-	-	-	
10 11 10 10 10 106	AIS [15]	2.5197	_	-	-	
10-Unit system $\{(\cos t \times 10^{\circ})$	NSGA-II [31]	2.5168	3.1740	-	-	
\$) and (emission $\times 10^3$ lb)}	IBFA [35]	2.4817	3.2750	-	-	
	DE-SQP	2.4659	3.2405	2.4688	3.1564	
	PSO-SQP	2.4668	3.3023	2.4701	3.1507	

#### Table 8

Hourly generation (MW) schedule obtained from EDEED (w = 0.5) for 5-unit system.

Н	DE-SQP					PSO-SQP						
	P <sub>1</sub>	$P_2$	<i>P</i> <sub>3</sub>	$P_4$	$P_5$	Loss	$P_1$	$P_2$	P <sub>3</sub>	$P_4$	$P_5$	Loss
1	10.0000	98.8213	40.6007	125.0302	139.3378	3.7900	10.9359	98.4749	112.7579	124.9377	66.5260	3.6324
2	10.0000	98.1632	66.3751	124.9899	139.5992	4.1275	39.5616	98.5940	112.6301	124.9116	63.3138	4.0113
3	13.6958	95.4032	106.3750	124.7227	139.5644	4.7611	30.3783	98.4893	112.6479	124.8913	113.3138	4.7207
4	43.6958	98.7875	116.3219	137.2926	139.7517	5.8496	60.0625	98.4961	112.7098	124.9337	139.6331	5.8352
5	26.6251	98.5905	112.6699	187.2926	139.4540	6.6321	74.3294	98.4702	112.5516	139.3432	139.7851	6.4796
6	55.4754	98.5196	112.4171	209.8120	139.6443	7.8684	75.0000	98.7959	112.9160	189.3432	139.7304	7.7855
7	73.6738	98.5358	112.5937	209.6423	139.8705	8.3163	73.9414	98.5456	112.4844	209.7231	139.6226	8.3171
8	75.0000	99.0914	114.3621	210.2124	164.3883	9.0543	75.0000	98.8651	125.3535	209.7026	154.0893	9.0105
9	65.9592	98.2241	112.4256	209.8442	213.6665	10.1195	74.9946	98.5046	112.5892	209.9673	204.0457	10.1015
10	65.3037	98.3852	112.7004	209.6943	228.4718	10.5554	68.8070	98.5110	112.7020	209.4201	225.1058	10.5458
11	75.0000	98.8688	118.0133	209.6808	229.4413	11.0041	75.0000	98.5798	118.0216	209.7182	229.6845	11.0041
12	75.0000	98.5815	138.5759	209.9115	229.4503	11.5192	75.0000	98.5992	138.4204	209.7781	229.7224	11.5201
13	70.3789	98.4723	112.7987	209.7579	223.1338	10.5415	72.5169	98.5570	112.5674	209.6985	221.1984	10.5383
14	57.5706	98.5306	112.5466	209.6997	221.7940	10.1416	75.0000	98.6605	112.6534	209.7291	204.0577	10.1008
15	56.9918	98.8290	112.7356	209.9866	184.5381	9.0812	74.9962	98.5886	125.4948	209.8723	154.0580	9.0099
16	26.9918	98.2502	112.8616	209.5833	139.5466	7.2335	44.9962	86.7787	112.6130	203.1816	139.5659	7.1354
17	14.1106	88.9647	112.8414	209.2885	139.5216	6.7268	15.0154	89.8303	112.6326	207.6745	139.5669	6.7197
18	44.1106	98.5698	123.6061	209.8037	139.7595	7.8496	45.0154	98.8270	122.5759	209.7455	139.6878	7.8516
19	74.0673	98.3710	112.4661	209.7506	168.4051	9.0600	74.6821	98.5548	112.7098	209.8057	167.3068	9.0592
20	75.0000	98.6640	112.6066	209.8573	218.4051	10.5330	75.0000	98.9707	113.0863	210.1663	217.3068	10.5301
21	75.0000	98.6274	116.1478	210.1283	189.8792	9.7827	75.0000	98.5227	116.2610	209.9957	190.0021	9.7817
22	75.0000	98.6590	112.6880	186.4758	139.8792	7.7020	74.9889	98.4575	112.6807	186.5710	140.0037	7.7017
23	45.3623	98.1720	112.6845	136.4758	140.0919	5.7864	45.0671	98.5472	112.7545	136.5710	139.8480	5.7878
24	23.7076	98.8696	80.6007	124.6145	139.7819	4.5743	15.0741	98.4564	112.6903	124.7759	116.5260	4.5228

Because of lack of space we show only the solutions of the EDEED problem (w = 0.5) using the DE-SQP and PSO-SQP for the five-unit system, which are given in Table 8. It can be seen that, the unit ramp rate constraint has not been violated. In other words, the units can be operated after the 24th hour to the first hour in the next day without the need to be concerned about the unit ramp rate limits of the units. The best cost and emission for the EDEED problem (w = 0.5) using DE-SQP are given by C = 43991, E = 14345 and using PSO-SQP they are given by C = 44348, E = 13659.

#### 7. Conclusion

In this paper we have proposed two hybrid approaches to solve the DEED problem with valve-point effects. The first approach integrates the DE with the SQP, while the second one integrates the PSO with the SQP. In these approaches DE or PSO is used as a base level search and SQP as a local level search. Hence DE or PSO is first applied to the DEED problem to find the best solution. This best solution is given to SQP as an initial condition to fine-tune the optimal solution in the end. The feasibility and efficiency of the DE-SQP and PSO-SQP are illustrated by conducting two study cases consisting of five and ten units with valve-point effects. Our results have been compared with other methods. The results demonstrate that for the DEED problem with valve-point effects, the solution obtained by DE-SQP or PSO-SQP is better than that obtained by other methods in terms of fuel cost and emission. The periodic implementation of the optimal solutions of the DEED problem has been discussed. A.M. Elaiw et al. / Electric Power Systems Research 103 (2013) 192-200

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