Optimal scheduling of household appliances for demand response

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A B S T R A C T
In this paper, residential demand response is studied through the scheduling of typical home appliances in order to minimize electricity cost and earn the relevant incentive. A mixed integer nonlinear optimization model is built under a time-of-use electricity tariff. A case study shows that a household is able to shift consumption in response to the varying prices and incentives, through which the consumer may realize an electricity cost saving of more than 25%. It has also been shown that at different values of the weighting factor α gives varying costs, from which the consumer is able to choose according to their preferences. Therefore a final decision about participation in the program could be made.

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1. Introduction

Demand response (DR) is a highly flexible program that can be customized to the financial and energy usage objectives of participants. It is a program that has been proven over the years to contribute to electrical energy reduction by commercial, industrial and residential consumers. DR is defined as the reduction in the consumption of electric energy by customers from their expected consumption in response to an increase in the price of electric energy or to incentive payments [1,2]. DR options are categorized as price-based and incentive-based programs. These two categories of DR are interconnected and the various programs under each category can be designed to achieve complementary goals, as demonstrated by Aalami et al. [3].

Initial research on DR programs focuses on large consumers, commercial and industrial due to their large consumption [3]. Residential demand response (RDR) has also contributed significantly to energy reduction, as has been proved by some experiments [4–6]. Consumer participation is achieved by varying electricity prices as well as offering incentives [7] and this in turn necessitates system load balancing, which is vital for load reduction during peak times.

The benefits of DR to consumers include financial and system reliability, among others. Financial benefits are gained in the bill savings and incentive payments earned by customers that adjust their electricity demand in response to time-varying electricity rates or incentive-based programs. Reliability benefits are the operational security and adequacy savings that result because DR lowers the likelihood and consequences of forced outages that impose financial costs and inconvenience on customers [8,9].

The general case of a user DR model that provides the basis for RDR has been provided by Chen et al. [10], where home appliances are classified into four major groups and the model is a structure of utility functions and consumption constraints for appliances. The utility functions of these different appliance categories have been provided and the general objective is to maximize social welfare. However, the paper provides a general framework without specific functional formulations and is thus not directly implementable practically. Advancement in RDR has gained momentum recently. A novel energy management system for RDR, the Consumer Automated Energy Management System, which is an online application system, has been proposed in [11]. The system reduces energy cost and the objective is to minimize the electricity cost for operating appliances. In [12] mixed integer linear programming (MILP) is used to schedule home appliances in order to minimize the electricity cost. It is shown that a financial incentive can encourage residential consumers to control their loads, as demonstrated by application of a plug-in hybrid electric vehicle (PHEV) in [13]. The practical model of day-ahead scheduling of appliances is shown in [14]. This model is applied to the scheduling of electric water heaters (EWH) subject to EWH constraints.

The purpose of this paper is to formulate a practical optimization model for a household to determine the optimal scheduling of home appliances under time of use (TOU) electricity prices. In

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addition to the objective of energy cost minimization in [10,13,14], this paper investigates cost minimization with an incentive offered to the consumer during peak times. This paper also looks into the consumer’s inconvenience level; by using the weighting factor of which consumers can take advantage to control how they favor the scheduling inconvenience over the cost, a decision to participate could be made. The inconvenience level is a measure of the disparity between the baseline and optimal schedule. In the literature, comfort level has been used extensively to refer to thermal comfort such as that of EWH and air conditioners [15–17]. The main contribution of this paper is consideration of the inconvenience level and the formulation of the problem as mixed integer nonlinear programming (MINLP) rather than mixed integer programming (MIP).

The remainder of this paper is organized as follows: Section 2 focuses on defining the problem and the optimization model. Section 3 examines a case study used in this paper. The solution methodology and simulation results are discussed in Section 4 and lastly a conclusion is drawn.

2. Problem definition and the model

2.1. Problem definition

An electricity-consuming household’s objective is to minimize its electricity cost. To achieve this, the consumer’s main objective is to be able to schedule appliances as per the electricity price and in our case we also consider the incentive offered during peak times. For a consumer to minimize cost with incentive offered, the objective function is characterized by two components, the electricity cost and the incentive.

2.2. The model

Considering a sampling time (Δt) of 10 min and a study period of 24 h, the mathematical problem is presented below. This model is an improvement of [14] in that in our case we have considered the incentive and the inconvenience.

\[
\begin{align*}
\text{min } J & = \sum_{t=1}^{T} \mathcal{L} \left( \sum_{i=1}^{I} \left( P_i \cdot u_{i,t}^{\text{opt}} - \beta_i \cdot \lambda_i \left( u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}} \right) \right) \right) \cdot \Delta t \\
\Pi_i & \geq 0, \quad \beta_i > 0, \quad i = 1, \ldots, I, \quad t = 1, \ldots, T, \quad I = 10, \quad T = 144.
\end{align*}
\]

\[
u_{i,t} = \begin{cases} 1, & \text{when appliance } i \text{ is on at } t; \\ 0, & \text{when appliance is off at } t. \end{cases}
\]

\[
\lambda_i(u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}}) = \begin{cases} 1, & \text{if } (u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}}) > 0; \\ 0, & \text{if } (u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}}) \leq 0, \end{cases}
\]

where \( t \) is the time index, \( t = 1, \ldots, T, \) \( \Delta t \) is the sampling time and \( t = 24 \) is the horizon, \( i \) is the appliance index and \( I \) is the total number of appliances. \( P_i \) is the rated power of appliance \( i. \) \( u_{i,t}^{\text{opt}} \) is the new on/off status of appliance \( i \) at time \( t \) and \( u_{i,t}^{\text{bl}} \) is the consumer’s baseline on/off status of appliance \( i \) at time \( t. \) \( \rho_i \) is the electricity price at \( t \) and \( \beta_i \) is the incentive for \( t. \) The indicator function \( \lambda_i(u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}}) \) denotes a certain state in a model. It is either 1 or 0 to allow consumers to earn an incentive only when they switch off their appliances during peak times. If it is 1 an incentive is earned, otherwise there is no incentive. The following constraints are formulated to the objective function (1):

\[
\sum_{t=1}^{T} \sum_{i=1}^{I} P_i \cdot \rho_i \cdot \lambda_i(u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}}) \cdot \Delta t \leq 25
\]

This constraint models the maximum cost that the consumer is willing to incur in one day and it is not more than R25 (R denotes the South Africa currency, ZAR or rand).

Note that the inflexibility of many loads is usually not absolute, and they might be flexibly adjusted within a range [18–20]. In this work, we assume that the appliances are flexible within the consumers’ specified time ranges. For each appliance, the user indicates \( d_i \) and \( e_i \) as the beginning and end of the time interval in which the appliance is to be scheduled. \( N_i \) is the allowable time interval or the time duration required to finish the normal operation of appliance \( i. \) Given the predetermined parameters \( d_i, e_i, \) and \( N_i, \) in order to provide the needed consumption for each appliance in times within interval \( [d_i,e_i] \) it is required that,

\[
\sum_{d_i}^{e_i} u_{i,t}^{\text{opt}} \geq N_i
\]

The constraint below ensures continuous operation of appliances [21]. In our case it is applied to all appliances except appliances 3 and 7, the kettle and EWH. Practically the kettle’s normal operation does not exceed the sampling time and the geyser is a continuous on/off appliance.

\[
\sum_{d_i}^{e_i} u_{i,t}^{\text{opt}} - u_{i,t+1}^{\text{opt}} - u_{i,t+2}^{\text{opt}} - \cdots - u_{i,t+(N_i-1)}^{\text{opt}} \geq 1
\]

The current starting time slot of i2 should be after the starting time of i1 plus its run time. For example, the clothes dryer follows the washing machine.

2.3. Scheduling inconvenience

The consumer’s decision to continue participating in the program may not be determined by only the cost saving and the incentive earned, but also by the inconvenience that comes with the new schedule. The scheduling inconvenience (1) seeks to minimize the disparity between the baseline and the optimal schedule. In this paper the postponement and advancement of the schedule are both regarded as an inconvenience. The consumer therefore also minimizes the objective function (5), the inconvenience.

\[
l := \sum_{t=1}^{T} \sum_{i=1}^{I} \left( u_{i,t}^{\text{bl}} - u_{i,t}^{\text{opt}} \right)^2
\]

The inconvenience level is then included in the main objective (1), therefore the modified objective function that the consumer seeks to minimize is expressed as follows:

\[
\text{min } J = J_c + \alpha l
\]

subject to constraints (1)–(5), where \( J_c \) is defined by Eq. (1), \( l \) is the scheduling inconvenience as in (6) and \( \alpha \) is a weighting factor, which represents how the consumer favors the scheduling inconvenience.

This problem has binary control variables \( u_{i,t}^{\text{opt}} \), which are the on/off status of appliances, and the inputs are the appliance’s rated power \( P_i \), the electricity prices \( \rho_i \), the initial appliances’ status \( u_{i,t}^{\text{bl}} \).
Table 1
Eskom’s Homeflex 1 tariff structure.

<table>
<thead>
<tr>
<th>Demand</th>
<th>Charges</th>
<th>Peak energy (R/kWh)</th>
<th>Off-peak (R/kWh)</th>
<th>Environ. levy (R/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Service (R/day)</td>
<td>Network (R/day)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2.45</td>
<td>3.04</td>
<td>1.4452</td>
<td>0.4552</td>
</tr>
<tr>
<td>Low</td>
<td>2.45</td>
<td>3.04</td>
<td>0.5443</td>
<td>0.3627</td>
</tr>
</tbody>
</table>

and the incentive $b_i$. The results are the appliances' optimal schedule, the energy cost, the incentive and the schedule inconvenience.

3. Case study

Eskom’s hourly electricity tariff has been discretized into 10 min, since the model has 10 min' sampling time and the optimization is over a 24-h period. This encourages shorter waiting periods for behaviour change. The scheduling period is achieved by deciding whether to turn on the appliance at the beginning of each 10 min. The tariff is based on Eskom’s TOU Homeflex structure for residential consumers as shown in Table 1. The Homeflex 1 tariff has five charge components. Eskom’s peak times are 07:00–10:00 and 18:00–20:00 [22]. From the information, only energy charges of 144.52c/kWh and 45.54c/kWh for the peak and off-peak high-demand period are used for calculations. A typical household in South Africa has been used as a case study and ten appliances have been selected and studied over a one-month weekday period. These are shown in Table 2. Appliance rated power is specified by the appliance manufacturers and can be obtained from the appliances.

One month’s weekday data on appliance usage in the household under study were collected, as well as the information on the allowable time duration required to finish the normal operation of the appliance, $N_i$, and the highest value is recorded in Table 2. The information on $d_i$ and $e_i$ as the beginning and end of the time interval in which the appliance is to be scheduled is specified by the user, based on the usual or preferred usage and this ties with usage data obtained. This is typical of a working class household where most activities occur in the morning and after work. An incentive of R0.2/kWh is used, which is guided by Refs. [23,24].

Table 2 shows parameters used and typically appliance 1 is scheduled twice in a day at 30 min and 50 min in the morning and evening respectively. It is to be switched on at any time between $t=30$ (05:00) to $t=42$ (07:00) and $t=196$ (16:00) to $t=120$ (20:00) respectively. Appliance 2 is scheduled once a day for at least 50 min any time from $t=108$ (18:00) to $t=114$ (19:00). It is to be noted that the appliance’s baseline schedule is included in the results in Table 3 for ease of comparison.

Table 2
Appliances data.

<table>
<thead>
<tr>
<th>No.</th>
<th>Appliance</th>
<th>Power rating (kW)</th>
<th>Duration $N_i$ (min)</th>
<th>$d_i$</th>
<th>$e_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stove</td>
<td>3.000</td>
<td>30</td>
<td>30</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>Microwave</td>
<td>1.230</td>
<td>10</td>
<td>108</td>
<td>114</td>
</tr>
<tr>
<td>3</td>
<td>Kettle</td>
<td>1.900</td>
<td>10</td>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>Toaster</td>
<td>1.010</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>Iron</td>
<td>1.235</td>
<td>10</td>
<td>48</td>
<td>96</td>
</tr>
<tr>
<td>6</td>
<td>Vacuum cleaner</td>
<td>1.200</td>
<td>10</td>
<td>30</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>EWH</td>
<td>2.600</td>
<td>120</td>
<td>120</td>
<td>48</td>
</tr>
<tr>
<td>8</td>
<td>Dishwasher</td>
<td>2.500</td>
<td>150</td>
<td>120</td>
<td>144</td>
</tr>
<tr>
<td>9</td>
<td>Washing machine</td>
<td>3.000</td>
<td>30</td>
<td>45</td>
<td>96</td>
</tr>
<tr>
<td>10</td>
<td>Tumble dryer</td>
<td>3.300</td>
<td>30</td>
<td>96</td>
<td>122</td>
</tr>
</tbody>
</table>

4. Simulation and results

The formulated model is solved with AIMMS software, which uses AImms outer Approximation Algorithm (AOA) that utilizes CPLEX and CONOPT as mixed integer programming (MIP) and non-linear programming (NLP) as solvers respectively. AOA is written with the AIMMS GMP functions and can be customized by users to fine-tune the algorithm for their specific problems. CPLEX is a very powerful tool for solving large and difficult MIP problems and it uses a branch-and-bound algorithm for solving MIP problems. CONOPT is an efficient large-scale NLP solver. AIMMS computes exact second order derivatives, which are used by CONOPT to solve certain classes of NLP models much more efficiently. It was shown that the algorithm finds a global optimum solution in a finite number of steps [25].

Table 3 shows the consumer’s baseline and the optimal schedule at $\alpha=0.1$. It shows that the consumer can redistribute their load from their baseline schedule to different time slots based of variable prices. According to Table 3 and Fig. 1, the stove’s baseline schedule is 30 min in the morning and evening at $t=37$ (06:10) to $t=39$ (06:30) and $t=108$ (18:00) to $t=112$ (18:40), respectively. The solution suggests a different stove schedule in the evening at times $t=102$ (17:00) to $t=106$ (17:40) and the same time for the morning is maintained. This is logical in that the baseline time is within off-peak times and the inconvenience is minimized. For the second appliance, the microwave, the baseline and optimal switching times are $t=108$ (18:00) and $t=104$ (17:20), as shown in Table 3. The two appliances’ operational times are consecutive, which satisfies the continuous operation constraints (4). The EWH

Table 3
The baseline and optimal appliance on state.

<table>
<thead>
<tr>
<th>No.</th>
<th>Appliance</th>
<th>$u_i^B$ and $u_i^W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stove</td>
<td>$f_{13} = f_{39} = f_{48} = f_{112}$</td>
</tr>
<tr>
<td>2</td>
<td>Microwave</td>
<td>$f_{108}$</td>
</tr>
<tr>
<td>3</td>
<td>Kettle</td>
<td>$f_{20} = f_{104}$</td>
</tr>
<tr>
<td>4</td>
<td>Toaster</td>
<td>$f_{11}$</td>
</tr>
<tr>
<td>5</td>
<td>Iron</td>
<td>$f_{96} = f_{120}$</td>
</tr>
<tr>
<td>6</td>
<td>Vacuum cleaner</td>
<td>$f_{14} - f_{56}$</td>
</tr>
<tr>
<td>7</td>
<td>EWH</td>
<td>$f_{48} - f_{56} = f_{120}$</td>
</tr>
<tr>
<td>8</td>
<td>Dishwasher</td>
<td>$f_{120} - f_{144}$</td>
</tr>
<tr>
<td>9</td>
<td>Washing machine</td>
<td>$f_{66} - f_{130}$</td>
</tr>
<tr>
<td>10</td>
<td>Tumble dryer</td>
<td>$f_{96} - f_{130}$</td>
</tr>
</tbody>
</table>
is excluded from the continuous operation because practically it is a continuous on/off appliance. Fig. 1 shows the switching status of appliances with the highest cost, EWH, dishwasher, clothes dryer and stove. The costs are EWH = R7.2214, dishwasher = R3.401, dryer = R2.3846 and stove = R2.18. The high prices are due to their high-rated power coupled with longer operation time or scheduling within peak times. The solid stems show optimal switching status, while the dotted ones show the baseline appliance commitment. Even though the clothes dryer is only on for three slots, the cost is high because it is scheduled during peak time, possibly because it has to follow the washing machine as per constraint (5). It can be seen that for the EWH, the morning commitment is the same for both baseline and optimal solution, whereas the schedules are different later due to the optimization algorithm trying to schedule it outside the high-priced period. The rest of the results on the remaining appliances are shown in Table 3; others remained in off-peak while a few shifted to overlap both peak and off peak. This is what is expected for the consumer with a wider range of possible starting and ending operational time of appliances, such as in this study.

The simulation results show that baseline cost with $u^{bl}_{i,t}$ is R25.37, while the current minimum electricity cost with $\alpha = 0.1$ is R18.80, a cost reduction of more than 25%. It is emphasized that the amount of savings realized cannot be generalized because the amount of savings may be affected by the price disparity between peak and off-peak prices; in our case off-peak price is more than 30% of the peak price. The value realized can also be affected by the amount by which appliances can be shifted. The results show that the appliances are generally shiftable and this is because of the range that the consumer provided or their level of flexibility. The results also show an earned incentive of R1.87. Fig. 2 shows that the consumer’s morning peak remained relatively the same at 4.5 kWh $t=34$ (05:40). The evening peak of 10.5 kWh has been shifted from peak time $t=113$ (18:50) to off peak time $t=97$ (16:10) at a lower value of 8.4 kWh due to load redistribution. This redistribution of the load can assist the stressed power system at peak times. The schedule inconvenience defined by (5) at $\alpha = 0.1$ is 57.

Table 4 shows the solution of the daily cost ($J_c$) and the inconvenience ($I$) at different weighting factors. A different weighting factor reflects the consumer’s different preferences regarding financial cost and inconvenience. The high value of alpha means a high penalty on inconvenience and the cost will assume a highest value and smallest value of inconvenience. This is in agreement with (6), because $\alpha$ is a tradeoff between the inconvenience and the cost. The weighting factor of appliances can be made to vary between appliances; however for simplicity the authors have assumed a uniform weighting factor. It is also to be noted that the selection of the

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Fig. 1. The baseline and optimal switching status of high cost appliances.

Fig. 2. The load profile under both baseline and optimal schedule.
proper value of $\alpha$ is subjective, as it is dependent on the user’s preference. The different values are only a guide to the consumer. In addition, in Table 3 the value of $J_c$ reaches a ceiling of R24.995, which is commensurate with constraint (2), that the consumer is willing to spend not more than R25 on that day. The weighting factor of 0 means the consumer’s decision is not influenced by the inconvenience and at that time the inconvenience assumes the highest value of 73, while the cost is at its lowest value of R12.87. The higher values of $\alpha$ mean high importance is placed on the inconvenience, for example at $\alpha=100$, $J_c$ is highest at 24.995 and the inconvenience is the smallest value of 3. This tradeoff between cost and inconvenience can help consumers in making a choice on how much they are willing to be inconvenienced, which may affect the level of participation in the program.

4.1. Further discussion

The optimal solution obtained is a suggestion to the consumer on cost minimization against the inconvenience that comes with the new schedule. Once the optimal solution is obtained, the reasonability of the optimal solution is analysed. It is noted that the amount of savings realized is comparatively high and cannot be generalized because the savings may be affected by the price disparity between peak and off-peak prices; in our case off-peak price is 31.5% of the peak price. The value realized is also affected by the amount at which appliances are shifted. In this case study, all appliances were flexible within certain time ranges that were specified by the consumer, hence the relatively higher savings whereas the expectation was that these would be lower when some appliances were considered inflexible. Future work will consider households on a larger scale rather than one household and the study period will be extended to more than one day. In addition, future studies will determine the general household’s acceptable indicator on willingness to participate in a DR program. This will assist in determining the level of customer participation.

5. Conclusion

The paper gives our preliminary results from a study using MINLP in which consumers reschedule their appliances due to variable electricity prices as demonstrated through a TOU tariff and also by offering an incentive. It has been shown that the consumers shift their load to off-peak times and they also limit their usage during peak times by switching off some appliances. The consumer reduced the cost of electricity by more than 25% and also earned some incentives. It is noted that the amount of savings realized cannot be generalized because the savings may be affected by a number of factors, such as shiftable appliances and a price difference between peak and off-peak times. It has also been shown that at different values of the weighting factor $\alpha$, the consumer has varying costs. From this, the consumer is able to know the inconvenience level that comes with the new schedule and is able to adjust it according to his preferences in regard to the cost and the inconvenience. Therefore a final decision about participation in the program could be made.

References


Table 4

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Total cost ($J_c$/R)</th>
<th>Inconvenience ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12.87</td>
<td>73</td>
</tr>
<tr>
<td>0.5</td>
<td>20.800</td>
<td>29</td>
</tr>
<tr>
<td>1</td>
<td>22.561</td>
<td>17</td>
</tr>
<tr>
<td>10</td>
<td>24.786</td>
<td>7</td>
</tr>
<tr>
<td>25</td>
<td>24.995</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>24.995</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>24.995</td>
<td>3</td>
</tr>
</tbody>
</table>