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# Implementing a model predictive control strategy on the dynamic economic emission dispatch problem with game theory based demand response programs

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ABSTRACT

In this paper, a game theory demand response program is incorporated into two problems; the dynamic economic emission dispatch problem and the price based dynamic economic emission dispatch problem. The game theory demand response program is an incentive based program which provides monetary incentives for willing customers who agree to curtail their demand, with the incentive greater than or equals to the their cost of curtailment. Both mathematical problems are multi-objective optimization problems and for the first model, the objectives are to minimize fuel costs and emissions and determine the optimal incentive and load curtailment for customers. The second model seeks to minimize emissions, maximize profits and also determine the optimal incentive and load curtailment for customers. Model predictive control, which is known as a closed loop approach from a control perspective is deployed to solve both proposed mathematical models and a comparison is provided with solutions obtained via an open loop approach. Obtained results validate the superiority of the closed loop approach over the open loop controller. For instance the closed loop approach yields 4.36 MWh and 11.35 MWh higher customer energy curtailments than the open loop approach for the first and second models respectively. Furthermore, obtained results also prove that the closed loop control approach shows better robustness against uncertainties and disturbance.



## 1. Introduction

The DEED problem is a multi-objective mathematical optimization problem with two conflicting objectives of minimizing fuel cost and emissions of thermal generators. The aim is to determine the optimal output of thermal generators under several practical constraints [1]. Some of the often considered constraints include: power balance constraints [2], ramp rate constraints, generator output limit constraints [3], line flow limit constraints, spinning reserve constraints [4], etc. The problem has received considerable interest by engineers and scientists alike due to increasing environmental consciousness and the need to curtail harmful emissions from thermal generators. In recent years, as many nations of the world have shifted from a regulated power system and embraced deregulation, this has given rise to the development of a new variant of the DEED problem. In this new variant, maximizing profit has replaced the former objective of minimizing cost. This has given birth to the PBDEED problem with the dual objectives of maximizing profit and minimizing emissions of thermal generators under the same or similar constraints as the DEED problem [5]. The DEED or PBDEED problem is solved depending on if it is in a regulated or deregulated climate. Another feature of modern power system operations is the drive or push to encourage a more responsible use of electrical power and minimization of electric power consumption. This has given rise to demand response programs. These programs are classified into two kinds: price based programs or incentive based programs.

In this work, we integrate a GTDR program which is an incentive based demand response program into the DEED and PBDEED problem. The resulting models determines the optimal output of thermal generator, optimal load to be curtailed by participating customers and the incentive to be paid to them. The resulting models are known as GTDR-DEED and GTDR-PBDEED. It has been shown in Ref. [6] that integrating DEED/PBDEED and demand response programs and solving the resultant integrated model

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**Nomenclature**

*Sets and indices:*
$I$ time
$T$ generators
$J$ customers

*Variables:*
$P_{i,t}$ power generated from generator $i$ at time $t$
$x_{j,t}$ amount of power curtailed by a customer $j$ at time $t$
$y_{j,t}$ incentive of a participating customer $j$ at time $t$

*Parameters:*
$C_i$ fuel cost of generator $i$
$E_i$ emissions for generator $i$
$D_t$ total system demand at time $t$
$loss_t$ total system losses at time $t$
$P_{i,min}$ and $P_{i,max}$ minimum and maximum capacity of generator $i$
$DR_i$ and $UR_i$ maximum ramp down and up rates of generator $i$
$a_i$, $b_i$ and $c_i$ fuel cost coefficients of generator $i$
$e_i$, $f_i$ and $g_i$ emission coefficients of generator $i$
$B_{i,k}$ $ik$th element of the loss coefficient square matrix of size $I$
$EP_t$ forecast energy price at time $t$
$K_{1,j}$ and $K_{2,j}$ cost function coefficients of customer $j$
$UB$ utilitys total budget
$CM_j$ daily limit of interruptible energy for customer $j$
$\lambda_{j,t}$ value of power interruptibility of customer $j$ at time $t$.
$\theta_j$ customer type
$m$ switching interval of the MPC controller
$w_1$, $w_2$ and $w_3$ objective function weights

*List of abbreviations:*
*GTDR* game theory based demand response
*SED* static economic dispatch
*DED* dynamic economic dispatch
*DEED* dynamic economic emission dispatch
*PBDEED* price based dynamic economic emission dispatch
*MPC* model predictive control
*PBDR* price based demand response
*IBDR* incentive based demand response
*TOU* time of use rates
*RTP* real time pricing
*CPP* critical peak pricing
*EDP* extreme day pricing
*EDCPP* extreme day critical peak pricing
*DLC* direct load control
*IS* interruptible services
*EDRP* emergency demand response programs
*CMP* capacity market programs
*DB* demand bidding/buyback programs
*AMS* ancillary market services
*ISO* independent system operator
*PJM* Pennsylvania-New Jersey–Maryland
*AIMMS* advanced interactive multidimensional modelling system
*MADM* multi attribute decision making
*MINLP* mixed integer non linear programming
*CHP* combined heat and power
*CSA* Cuckoo search algorithm
*TLBO* teaching learning based optimization
*BSA* backtracking search algorithm
*CSADHS* chaotic self adaptive differential harmony search
*NSGAII* nondominated sorting genetic algorithm II
*FFA* hybrid fire fly algorithm
*HS* harmony search
*RE* renewable energy

yields better results than independent consideration of either DEED/PBDEED or DR [7] as it introduces optimality at both the supply and demand side of the power system [8]. However, solving the GTDR-DEED and GTDR-PBDEED problem only determines open loop control solutions when viewed from a control systems perspective. The disadvantage of this is that the model cannot compensate for inaccuracies and disturbances arising from modelling uncertainties. This is due to the fact that there is no way for the inaccurate system solutions to be fed back to the system and updated to obtain accurate solutions.

Closed-loop systems on the other hand are inherently able to give feedback to the optimization model [5] and update solutions [9]. Due to the superiority of closed-loop systems over open loop systems, MPC which is a prominent closed-loop approach is used in this work. MPC has found wide applications in a number of engineering applications and has recently been used in power system applications like in Ref. [10] where MPC was applied to generator maintenance scheduling [5], where MPC was applied to economic dispatch problems [11], where MPC was applied to a solar, wind, diesel battery hybrid power system. A complete introduction to MPC is provided in Ref. [12].

In view of the successful application of the MPC strategy in power system applications and its ability to handle disturbances and uncertainties, MPC is used in this work to also solve the GTDR-DEED and GTDR-PBDEED mathematical problems. MPC is utilized because in practical applications of GTDR-DEED and GTDR-PBDEED, there might be variations in system parameters like load demand or the price of energy. This can introduce a whole lot of uncertainty or disturbance in the system. MPC overcomes the aforementioned problems. The proposed MPC approach is shown to handle uncertainties and disturbance well and exhibit convergence and robustness which further makes it extremely suitable for real time and practical applications.

This paper is an extension of [8] where the GTDR-DEED model was presented. One of the additions in this work is the development of a GTDR-PBDEED model. The GTDR-PBDEED problem shows a practical framework for the integration of an incentive based demand response program with economic dispatch in a deregulated environment where one of the objectives is to maximize profit. Another addition is the application of the MPC strategy in solving both GTDR-DEED and GTDR-PBDEED. It is shown that the MPC strategy handles uncertainty and disturbance better than open loop approaches.

The rest of this paper is organized as follows: Section 2 presents a literature review of DEED, PBDEED, DR and MPC. Section 3 gives the DEED and PBDEED formulations. Section 4 introduces the Game Theory based Demand Response Program formulation. Section 5 details both the GTDR-DEED and GTDR-PBDEED mathematical models and the proposed MPC formulations applied to both models. Section 6 focuses on numerical simulations using the developed mathematical models and presents obtained results. The paper is concluded in Section 7.

## 2. Literature review

The DEED problem is a problem that has been prevalent in the literature and has had many evolving variants over the years. It's initial variant was known as the SED problem [4]. The SED problem is concerned with minimizing fuel costs and determining the optimal output of thermal generators to satisfy a particular load demand at a specific time instant. The SED later metamorphosed into the DED. The DED problem is concerned with determining the optimal output of the committed thermal generators to satisfy a particular load demand over a pre-determined time horizon with minimal operating costs amongst other constraints. Typically, the DED problem is solved by dividing the total time horizon into smaller time intervals (usually 1 h), solving the SED problem at the smaller time intervals and enforcing ramp rate constraints between consecutive intervals. As general environmental awareness increased, researchers became interested in ways of reducing emission and environmental effects in energy modelling. This led to the development of various environmentally conscious energy modelling approaches like in Ref. [13] which employs a fuzzy based approach, [14] which employs a MADM approach, [17] which employs mathematical modelling approaches to reduce harmful emissions [15] and greenhouse gases [16]. Zeroing in on the DED problem, increasing environmental awareness led researchers to consider the problem of emission dispatching [4] and this led to the further evolvement of the DED problem into the DEED problem. The DEED problem seeks to minimize fuel costs and emissions of thermal generators subject to load demand constraints, ramp rate constraints, maximum and minimum capacity constraints amongst others. This is essentially a multi-objective optimization problem with the dual objectives of minimizing both fuel costs and emissions. The optimization problem is either solved by transforming it into a single objective optimization problem by the goal attainment method [8] or by multi-objective optimization techniques.

Recently the DEED problem has evolved due to the advent of deregulation and liberalization of the power industry. This has expanded the objective of the generator operators from fuel cost and emissions minimization to include profit maximization. This new set up is known as PBDEED. Typically, the DEED/PBDEED problem is solved either by conventional mathematical optimization techniques or by artificial intelligence/meta-heuristic optimization techniques. Both approaches have their respective advantages and disadvantages. Conventional mathematical techniques have the advantage of being able to guarantee optimal solutions, don't have the need to define domain specific parameters and have short computational times [4]. Their main disadvantage is that they often are unable to handle non-convex cost function [8]. Meta-heuristic techniques on the other hand, can handle non convex cost functions effectively but have the disadvantage of the need for a definition of a large number of domain specific parameters. Examples of the use of conventional mathematical techniques is in Ref. [28] where MINLP was used to obtain the optimal dispatch schedule for a CHP plant and in Ref. [27] where linear, mixed integer and non-linear programming methods were investigated for energy dispatch modelling. In recent years, various meta-heuristic techniques have been applied to the DED/DEED/PBDEED problem. Examples of such techniques include CSA [26], TLBO [25], BSA [24], CSADHS [18], NSGA-II [19], FFA [20], HS [22] amongst others.

There have been three major research thrusts in the literature concerning DEED/PBDEED. The first is the development of novel meta-heuristic techniques as shown with the examples given above and the second is the integration of RE sources into the DEED formulations like in Ref. [21]. Both of these research directions are essentially concerned with introducing optimality at the supply end of the power system. The third research direction concerns integrating DR programs into the DEED problem.

In Ref. [32] demand response is defined as a change in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized. Generally, demand response programs are broadly classified into two: PB-DR [34] and IB-DR [35]. In PB-DR, the electricity tariffs vary with time, i.e., different electricity tariffs for various peak times. The aim is to encourage consumers to curtail their energy use to take advantage of favourable prices. Examples of PB-DR include TOU, RTP, CPP, EDP, and ED-CPP. In IB-DR, incentives are simply offered to consumers to reduce or curtail their electricity use when the power system is stressed. The incentives can be in form of rebates or lower electricity tariffs [36]. It should be noted that unlike PB-DR, consumers can be penalized if their load is not curtailed when the system is stressed. Examples of IB-DR include DLC (direct load control), IS, EDRP, CMP, DB and AMS.

Demand response can be implemented in either regulated and deregulated set-ups. In both set-ups, demand response programs can lead to reduction in harmful emissions and operational costs which brings about environmental and power system benefits [33,37]. Demand response programs also reduce wholesale market prices [41]. In IB-DR, the incentive can either be monetary or in the form of reduced electricity tariffs [38] and in order to ensure that customers participate, the incentive offered to consumers should be greater than or equal to the customer outage cost [39]. In Ref. [40], voluntary incentive game theory based demand response contracts are defined as: an agreement between utility and customer wherein the customer agrees to willingly shed load and in return receive monetary compensation.

Unlike the other two research thrusts of DEED that are solely concerned with the supply end of the power system, integrating DR into the DEED problem introduces optimality at the supply and demand side of the power system. Examples of such works include [7] where the optimal dispatch strategy for renewable energy sources in a micro grid was presented. Another similar work is [6] where the dispatch strategy for renewable energy sources was presented, this time using GA. In both works, the customers cost of interruptions were not factored in, neither was there a consideration of incentives to be offered to the customers to entice them to participate in DR programs. To this end, in Ref. [8], the mathematical formulations of game theory based demand response contracts are modified and extended over multiple time intervals and integrated into the DEED mathematical problem. The extended mathematical formulation includes the modification of the individual rationality constraint and the incentive compatibility constraint. Both constraints are structured over the total optimization horizon (a day) instead of a single time interval (every hour). Other realistic and practical constraints like budgetary and maximum power constraints were also incorporated into the model. For a full description of the modified GTDR program and DEED model, the reader is referred to [8]. The DR program is structured in such a manner that the customers do not operate at a loss (in other words, the program is attractive) and integrating it into the DEED problem enables us to obtain optimal solutions at both the supply side and demand side.

This paper extends the work in Ref. [8]. The GTDR-DEED model presented in Ref. [8] was under a regulated environment. In this work, we present a GTDR-PBDEED model which is under a deregulated environment. From a control system perspective solving the DED, DEED, PBDEED, GTDR-DEED or GTDR-PBDEED problems through either conventional mathematical techniques or meta-heuristic techniques only provides open loop solutions.

Open loop systems despite their merits are unable to compensate for inaccuracies and disturbances arising from modelling uncertainties. This is due to the fact that open loop systems have no feed back mechanisms for inaccurate system solutions (in the presence or disturbances and inaccuracies) to be fed back to the system and updated in order to obtain accurate solutions [12].

Closed-loop systems are able to give feedback and update inaccurate solutions [5]. In GTDR-DEED and GTDR-PBDEED, there might be variations in system parameters like load demand or the price of energy, thus MPC [9] a closed-loop control mechanism is used to solve both GTDR-DEED and GTDR-PBDEED models. The proposed MPC approach is shown to handle uncertainties and disturbance well and exhibit convergence and robustness which further makes it extremely suitable for solving the developed models.

## 3. DEED and PBDEED model formulations

### 3.1. The dynamic economic emission dispatch model

The DEED problem is concerned with minimizing the fuel costs and emission of thermal generators and determining their optimal power output. The mathematical formulation is presented below [5]:

$$min\sum_{t=1}^{T}\sum_{i=1}^{I}C_i(P_{i,t}), \tag{1}$$

$$min\sum_{t=1}^{T}\sum_{i=1}^{I}E_i(P_{i,t}), \tag{2}$$

with

$$C_i(P_{i,t}) = a_i + b_iP_{i,t} + c_iP_{i,t}^2, \tag{3}$$

$$E_i(P_{i,t}) = d_i + e_iP_{i,t} + f_iP_{i,t}^2, \tag{4}$$

subject to the following network constraints:

$$\sum_{i=1}^{I}(P_{i,t}) = D_t + loss_t, \tag{5}$$

$$P_{i,min} \le P_{i,t} \le P_{i,max}, \tag{6}$$

$$-DR_i \le P_{i,t+1} - P_{i,t} \le UR_i, \tag{7}$$

where

$$loss_t = \sum_{i=1}^{I}\sum_{k=1}^{K}P_{i,t}B_{i,k}P_{k,t}, \tag{8}$$

$P_{i,t}$ is the power generated from generator $i$ at time $t$;
$C_i$ is the fuel cost of generator $i$;
$E_i$ is the emissions for generator $i$;
$D_t$ is the total system demand at time $t$;
$loss_t$ is the total system losses at time $t$;
$P_{i,min}$ and $P_{i,max}$ are the minimum and maximum capacity of generator $i$ respectively;
$DR_i$ and $UR_i$ are the maximum ramp down and up rates of generator $i$ respectively;
$a_i$, $b_i$ and $c_i$ are the fuel cost coefficients of generator $i$ respectively;
$e_i$, $f_i$ and $g_i$ are the emission coefficients of generator $i$ respectively;
$B_{i,k}$ is the $ik$ th element of the loss coefficient square matrix of size $I$;
$I$ and $T$ are the number of generators and the dispatch interval respectively.

Equation (3) gives the fuel cost function of the thermal generators. This cost function is typically obtained from heat run tests [19]. In these tests, the thermal generator unit is varied through it's normal operating limits and measurements of output power and fuel consumption costs are obtained. The fuel cost function thus give the fuel costs in $/h of the thermal generator unit as a function of its output power. This tests also enables the fuel cost coefficients of individual generator units to be calculated from the measured data. There are number of different fuel cost functions like the linear cost function [30], piecewise linear cost function [31], quadratic cost function [8], valve point effect cost function [19]. However, the quadratic cost is the most prevalent cost function in the literature [4] and is used in this paper.

Similarly, equation (4) gives the emission function for the thermal generator units. These emissions can be modelled through functions that associate emissions with real power production for each unit. These functions are also obtained through measured tests like the heat run tests [19]. These tests enable the emission coefficients to be calculated. The emission functions gives the total emissions in lb/h of a thermal generator unit as a function of its output power [4]. In this paper, the quadratic emissions function is used to represent this relationship [8].

The following is a brief description of the constraints:

- The first constraint (5) is the power balance constraint and ensures that at any time $t$, the total power generated equals the demand and the transmission losses. The transmission losses occur because the power stations are typically sited away from where the power is needed and there are losses in the course of the power being transmitted. The most common and widely accepted method for calculating these losses is by the B-coefficient method which is a method where the network losses are represented as a quadratic function of the generators output [1] and is given in equation (8). As stated before, $B_{i,k}$ is the $ik$ th element of the loss coefficient square matrix $B$ of size $I$. $P_{i,t}$ and $P_{j,t}$ are the output power of generator $i$ and $j$ respectively. This method has been used in Ref. [8]. The B - coefficient method has also been used in Ref. [5]. Another work it has been used in is [9] amongst others.
- The second constraint is the generation limits constraint (6) and ensures that the generator limits are not exceeded; and
- The final constraint (7) is the generator ramp rate limits constraints and ensures that the generator ramp rate limits are not violated.

The multi-objective optimization can be transformed into a single objective function using a weighting factor $w$ subject to the same constraints (5)–(7):

$$min\left[w\sum_{t=1}^{T}\sum_{i=1}^{I}C_i(P_{i,t}) + (1-w)\sum_{t=1}^{T}\sum_{i=1}^{I}E_i(P_{i,t})\right]. \tag{9}$$

where $w$ and $1-w$ are two non-negative weighting factors. When converting multi-objective optimization problems into single objective functions, it is required that weighting factors satisfy the following condition [9]:

$$w_1 + w_2 = 1. \tag{10}$$

Typically, the choice of weighting factors determines which objective is given preference. If the aim is to solely minimize costs then $w = 1$, whilst if the aim is to solely minimize emissions, then $w = 0$. In this work, since the aim is to simultaneously minimize fuel costs and emissions [22], equal values are given to the weighting factors [18].

### 3.2. Profit based dynamic economic emission dispatch model

In a deregulated market environment, the objective is to maximize profit and minimize emissions. Let us assume that the forecast energy price at time $t$ is given by $EP_t$, the revenue is given by $\sum_{t=1}^{T}\sum_{i=1}^{I} EP_t * P_{i,t}$ [5] and the cost by $\sum_{t=1}^{T}\sum_{i=1}^{I} C_i(P_{i,t})$. Thus, the profit is given by:

$$\sum_{t=1}^{T}\sum_{i=1}^{I} EP_t * P_{i,t} - \sum_{t=1}^{T}\sum_{i=1}^{I} C_i(P_{i,t}), \tag{11}$$

The final optimization problem is given by:

$$max \sum_{t=1}^{T}\sum_{i=1}^{I} EP_t * P_{i,t} - \sum_{t=1}^{T}\sum_{i=1}^{I} C_i(P_{i,t}), \tag{12}$$

$$min \sum_{t=1}^{T}\sum_{i=1}^{I} E_i(P_{i,t}), \tag{13}$$

with

$$C_i(P_{i,t}) = a_i + b_i P_{i,t} + c_i P_{i,t}^2, \tag{14}$$

$$E_i(P_{i,t}) = d_i + e_i P_{i,t} + f_i P_{i,t}^2, \tag{15}$$

subject to the following network constraints:

$$\sum_{i=1}^{I} (P_{i,t}) \le D_t + loss_t, \tag{16}$$

$$P_{i,min} \le P_{i,t} \le P_{i,max}, \tag{17}$$

$$-DR_i \le P_{i,t+1} - P_{i,t} \le UR_i. \tag{18}$$

It is observed that the constraints for both DEED and PBDEED are quite similar, the only difference being the power balance constraint. In DEED, the generated power must equal total demand, while in PBDEED, the generated power can be less than the total demand as the aim is to maximize total profit. Again, it is instructive to mention that just like in the DEED formulations, the fuel cost and emissions are both assumed to be quadratic functions of the generators active power output and other transmission and distribution line constraints are ignored. The multi-objective optimization can be transformed into a single objective function using a weighting factor $w$ subject to the same constraints (16)–(18):

$$min\left[w\left[\sum_{t=1}^{T}\sum_{i=1}^{I} C_i(P_{i,t}) - EP_t * P_{i,t}\right] + (1-w)\sum_{t=1}^{T}\sum_{i=1}^{I} E_i(P_{i,t})\right]. \tag{19}$$

## 4. Game theory based demand response formulations

The objective of the GTDR formulations is to maximize the utility benefit [8]:

$$max_{x,y} \sum_{t=1}^{T}\sum_{j=1}^{J}\left[\lambda_{j,t} x_{j,t} - y_{j,t}\right] \tag{20}$$

s.t.

$$\sum_{t=1}^{T}\left[y_{j,t} - \left(K_{1,j} x_{j,t}^2 + K_{2,j} x_{j,t} - K_{2,t} x_{j,t}\theta_j\right)\right] \ge 0, for\, j = 1, \ldots, J, \tag{21}$$

$$\begin{aligned}&\sum_{t=1}^{T}\left[y_{j,t} - \left(K_{1,j} x_{j,t}^2 + K_{2,j} x_{j,t} - K_{2,t} x_{j,t}\theta_j\right)\right] \ge \\ &\sum_{t=1}^{T}\left[y_{j-1,t} - \left(K_{1,j-1} x_{j-1,t}^2 + K_{2,j-1} x_{j-1,t} - K_{2,j-1} x_{j-1,t}\theta_{j-1}\right)\right], \\ &\quad for\, j = 2, \ldots, J,\end{aligned} \tag{22}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J} y_{j,t} \le UB, \tag{23}$$

$$\sum_{t=1}^{T} x_{j,t} \le CM_j, \tag{24}$$

The customer outage cost function is assumed to be quadratic and is given by:

$$\left(K_{1,j} x_{j,t}^2 + K_{2,j} x_{j,t} - K_{2,t} x_{j,t}\theta_j\right). \tag{25}$$

Where

$K_{1,j}$ and $K_{2,j}$ are the cost function coefficients of customer $j$;
$x_{j,t}$ is the amount of power curtailed by a customer $j$ at time $t$;
$y_{j,t}$ is the incentive of a participating customer $j$ at time $t$;
$UB$ is the utility's total budget;
$CM_j$ is the daily limit of interruptible energy for customer $j$;
$J$ and $T$ are the total number of customers and the total time interval respectively;
$\lambda_{j,t}$ is the value of power interruptibility of participating customer $j$ at time $t$. This parameter gives the cost of the electric utility not delivering electric power to a particular location on the grid. $\lambda_{j,t}$ can be calculated from OPF routines;
$\theta_j$ is the customer type [38]. $\theta$ is normalized in the interval $0 \le \theta \le 1$ and categorizes the different kinds of customers based on their willingness or readiness to shed power, with $\theta = 0$ being the least willing and $\theta = 1$ the most willing customer.

The following is a concise description of the constraints:

- Constraint (21) is the modified Individual rationality constraint and seeks to make the contract attractive to customers, by making sure that the total daily incentive received by a customer equals or exceeds the daily cost of interruption.
- Constraint (22) is the modified Incentive compatibility constraint and seeks to ensure that consumers are compensated commensurate to their level of power curtailed. Thus, the greater the daily power curbed, the greater the customer benefit.
- Constraint (23) ensures that the total incentive paid by the utility is less than the utilitys budget.
- Constraint (24) ensures that the total daily power curtailed by each customer is less than its daily limit of interruptible power. In the next section, the combined DEED/PBDEED and game theory based demand response models are detailed.

## 5. Mathematical model of DEED/PBDEED combined with game theory based demand response formulations

### 5.1. GTDR-DEED

The weighted single objective GTDR-DEED mathematical formulation is:

$$\begin{aligned} \min w_1\left[\sum_{t=1}^{T}\sum_{i=1}^{I}C_i(P_{i,t})\right]+w_2\left[\sum_{t=1}^{T}\sum_{i=1}^{I}E_i(P_{i,t})\right] \\ +w_3\left[\left[\sum_{t=1}^{T}\sum_{j=1}^{J}\left[y_{j,t}-\lambda_{j,t}x_{j,t}\right]\right]\right] \end{aligned} \tag{26}$$

subject to the following network constraints:

$$\sum_{i=1}^{I}P_{i,t}=D_t+loss_t-\sum_{j=1}^{J}x_{j,t}, \tag{27}$$

$$P_{i,min}\le P_{i,t}\le P_{i,max}, \tag{28}$$

$$-DR_i\le P_{i,t+1}-P_{i,t}\le UR_i, \tag{29}$$

$$\sum_{t=1}^{T}\left[y_{j,t}-\left(K_{1,j}x_{j,t}^2+K_{2,j}x_{j,t}-K_{2,t}x_{j,t}\theta_j\right)\right]\ge 0,\ for\ j=1,\ldots,J, \tag{30}$$

$$\begin{aligned} &\sum_{t=1}^{T}\left[y_{j,t}-\left(K_{1,j}x_{j,t}^2+K_{2,j}x_{j,t}-K_{2,t}x_{j,t}\theta_j\right)\right]\ge \\ &\sum_{t=1}^{T}\left[y_{j-1,t}-\left(K_{1,j-1}x_{j-1,t}^2+K_{2,j-1}x_{j-1,t}-K_{2,j-1}x_{j-1,t}\theta_{j-1}\right)\right], \\ &\qquad for\ j=2,\ldots,J, \end{aligned} \tag{31}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}y_{j,t}\le UB, \tag{32}$$

$$\sum_{t=1}^{T}x_{j,t}\le CM_j, \tag{33}$$

$$loss_t=\sum_{i=1}^{I}\sum_{k=1}^{K}P_{i,t}B_{i,k}P_{k,t}, \tag{34}$$

where $w_1, w_2$ and $w_3$ are the weights and the following condition is required to be satisfied when choosing weights:

$$w_1+w_2+w_3=1. \tag{35}$$

The variables to be determined by the optimization model are $x_{j,t}$, $y_{j,t}$ and $P_{i,t}$.

### 5.2. GTDR-PBDEED

For the GTDR-PBDEED, we assume that the utility or the Independent System Operator (ISO) wants to maximize its profit and benefit and minimize emissions as it is operating in a deregulated environment. This can be given as:

$$\begin{aligned} \min w_1\left[\sum_{t=1}^{T}\sum_{i=1}^{I}\left[C_i(P_{i,t})-EP_t{*}P_{i,t}\right]+\sum_{t=1}^{T}\sum_{j=1}^{J}\left[y_{j,t}-\lambda_{j,t}x_{j,t}\right]\right] \\ +w_2\sum_{t=1}^{T}\sum_{i=1}^{I}E_i(P_{i,t}) \end{aligned} \tag{36}$$

subject to the following network constraints:

$$\sum_{i=1}^{I}P_{i,t}\le D_t+loss_t-\sum_{j=1}^{J}x_{j,t}, \tag{37}$$

$$P_{i,min}\le P_{i,t}\le P_{i,max}, \tag{38}$$

$$-DR_i\le P_{i,t+1}-P_{i,t}\le UR_i, \tag{39}$$

$$\sum_{t=1}^{T}\left[y_{j,t}-\left(K_{1,j}x_{j,t}^2+K_{2,j}x_{j,t}-K_{2,t}x_{j,t}\theta_j\right)\right]\ge 0, for\ j=1,\ldots,J, \tag{40}$$

$$\begin{aligned} &\sum_{t=1}^{T}y_{j,t}-\left(K_{1,j}x_{j,t}^2+K_{2,j}x_{j,t}-K_{2,t}x_{j,t}\theta_j\right)\ge \\ &\sum_{t=1}^{T}\left[y_{j-1,t}-\left(K_{1,j-1}x_{j-1,t}^2+K_{2,j-1}x_{j-1,t}-K_{2,j-1}x_{j-1,t}\theta_{j-1}\right)\right], \\ &\qquad for\ j=2,\ldots,J, \end{aligned} \tag{41}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}y_{j,t}\le UB, \tag{42}$$

$$\sum_{t=1}^{T}x_{j,t}\le CM_j, \tag{43}$$

$$loss_t=\sum_{i=1}^{I}\sum_{k=1}^{K}P_{i,t}B_{i,k}P_{k,t}, \tag{44}$$

where $w_1$ and $w_2$ are the weights and the following condition is required to be satisfied when choosing weights:

$$w_1+w_2=1. \tag{45}$$

The variables to be determined by the optimization model are $x_{j,t}$, $y_{j,t}$ and $P_{i,t}$.

### 5.3. Model predictive control

The open loop GTDR-DEED model and GTDR-PBDEED model are defined over the time interval $T$ with optimization variables $x_{1,t}$, $y_{1,t}$, $P_{1,t}$ …, $x_{1,T}$, $y_{1,T}$, $P_{1,T}$ ($i=1,2,\ldots,I$ and $j=1,2,\ldots,J$). When the same problem is considered over a time interval ($m+1$ $m+T$), the variables are $x_{1,m+1}$, $y_{1,m+1}$, $P_{1,m+1}$, …, $x_{1,m+T}$, $y_{1,m+T}$, $P_{1,m+T}$.

Therefore the closed-loop (MPC) GTDR-DEED problem is given below:

$$\min w_1\left[\sum_{t=m+1}^{T}\sum_{i=1}^{I}C_i(P_{i,t})\right]+w_2\left[\sum_{t=m+1}^{T}\sum_{i=1}^{I}E_i(P_{i,t})\right]+w_3\left[\sum_{t=m+1}^{T}\sum_{j=1}^{J}\left[y_{j,t}-\lambda_{j,t}x_{j,t}\right]\right], \tag{46}$$

where $m$ is the switching interval of the MPC controller. The constraints of the closed-loop system are the same as that of the open loop solution and at each iteration, the constraints of the closed-loop model are updated. The optimal solution is utilized only in the first sample interval $(m+1, m+2)$ and this solution is applied as the input over the second sample interval $(m+2, m+3)$. This provides a closed feed back scheme. A full description of the MPC algorithm is provided in Ref. [12]. Similarly, the closed-loop (MPC) GTDR-PBDEED problem is given as:

$$\min w_1\left[\sum_{t=m+1}^{T}\sum_{i=1}^{I}\left[C_i(P_{i,t})-EP_t*P_{i,t}\right]+\sum_{t=m+1}^{T}\sum_{j=1}^{J}\left[y_{j,t}-\lambda_{j,t}x_{j,t}\right]\right]+w_2\left[\sum_{t=m+1}^{T}\sum_{i=1}^{I}E_i(P_{i,t})\right]. \tag{47}$$

## 6. Numerical simulations, obtained results and discussions

To verify the proposed GTDR-DEED and GTDR-PBDEED mathematical formulations, a case study of six generator units and five industrial customers is used. This case study has hitherto been used in Ref. [8]. The data for the generator units has also been used in Ref. [5] and was originally obtained from Ref. [29]. Table 1 shows the fuel cost coefficients and the emission coefficients [5]. The system consists of six thermal units, twenty six buses, and forty six transmission lines [29]. The maximum load demand is 1263 MW. Table 2 gives the initial hourly demand [29], which has one mid-day peak synonymous with industrial customers. Table 3 gives the hourly values of power interruptibility ($\lambda_{j,t}$) obtained from the PJM Market [42] LMP prices on the 30th of April 2014. For PBDEED, the energy price ($EP_t$) is assumed to be the highest LMP price. Table 4 details the cost function coefficients, customer type and daily customer energy limit [8]. The assumption is that the utility knows the customers daily limit of interruptible energy ($CM_j$) which it then uses to rank the customers in order of increasing willingness to curb electric power. Furthermore, the utility knows the outage cost function coefficients of participating customers ($K_{1,j}$ and $K_{2,j}$). The customer cost function coefficients, customer type and daily energy limit were originally obtained and modified from Ref. [38] which contains practical data from a US case study. The transmission loss formula coefficients for the six unit test system [29] are given by equation (48) and the utility daily budget ($UB$) is $ 50000.

The decision variables for both GTDR-DEED and GTDR-PBDEED are the optimal customer power to be curtailed ($x_{j,t}$), optimal incentive to be paid to customers ($y_{j,t}$) and power generated from all generators ($P_{i,t}$). The entire dispatch period is 24 h ($T = 24$) [8] and the sampling period is 1 h [29] as has always been used in the literature [4]. The Advanced Interactive Multidimensional Modelling System (AIMMS) [43] is utilized to build and solve both

**Table 2**
Total initial hourly demand.

| Time(h) | Total demand (MW) |
|---|---|
| 1 | 955 |
| 2 | 942 |
| 3 | 935 |
| 4 | 930 |
| 5 | 935 |
| 6 | 963 |
| 7 | 989 |
| 8 | 1023 |
| 9 | 1126 |
| 10 | 1150 |
| 11 | 1201 |
| 12 | 1235 |
| 13 | 1190 |
| 14 | 1251 |
| 15 | 1263 |
| 16 | 1250 |
| 17 | 1221 |
| 18 | 1202 |
| 19 | 1159 |
| 20 | 1092 |
| 21 | 1023 |
| 22 | 984 |
| 23 | 975 |
| 24 | 960 |

**Table 3**
Hourly values of power interruptibility.

| $\lambda_{j,t}$ ($) | | | | | |
|---|---|---|---|---|---|
| | $j$=1 | $j$=2 | $j$=3 | $j$=4 | $j$=5 |
| $t = 1$ | 27.61 | 28.30 | 28.79 | 26.93 | 27.60 |
| $t = 2$ | 29.41 | 30.07 | 30.53 | 28.79 | 29.44 |
| $t = 3$ | 28.24 | 28.87 | 29.28 | 27.66 | 28.32 |
| $t = 4$ | 26.69 | 28.76 | 29.14 | 27.74 | 28.24 |
| $t = 5$ | 29.01 | 32.24 | 32.64 | 31.20 | 31.66 |
| $t = 6$ | 33.96 | 36.67 | 37.15 | 35.38 | 35.99 |
| $t = 7$ | 83.97 | 89.46 | 90.65 | 85.71 | 87.70 |
| $t = 8$ | 81.10 | 82.88 | 83.79 | 79.06 | 81.06 |
| $t = 9$ | 110.60 | 112.93 | 114.11 | 107.72 | 110.44 |
| $t = 10$ | 74.12 | 75.43 | 76.09 | 72.40 | 73.95 |
| $t = 11$ | 78.95 | 80.19 | 80.65 | 77.29 | 78.93 |
| $t = 12$ | 66.85 | 67.55 | 67.76 | 65.75 | 66.67 |
| $t = 13$ | 47.98 | 48.58 | 48.63 | 47.10 | 47.93 |
| $t = 14$ | 66.82 | 67.74 | 68.07 | 65.55 | 66.74 |
| $t = 15$ | 48.50 | 49.35 | 49.69 | 47.41 | 48.47 |
| $t = 16$ | 49.21 | 50.28 | 50.87 | 47.94 | 49.19 |
| $t = 17$ | 66.65 | 69.36 | 70.29 | 66.05 | 67.71 |
| $t = 18$ | 61.49 | 66.57 | 67.19 | 59.69 | 66.24 |
| $t = 19$ | 56.19 | 57.67 | 58.25 | 54.48 | 56.53 |
| $t = 20$ | 57.92 | 59.38 | 59.98 | 55.58 | 57.98 |
| $t = 21$ | 49.16 | 49.86 | 50.36 | 48.31 | 48.96 |
| $t = 22$ | 54.00 | 54.38 | 54.84 | 53.46 | 53.63 |
| $t = 23$ | 34.37 | 34.67 | 34.96 | 33.98 | 34.21 |
| $t = 24$ | 30.30 | 30.71 | 31.00 | 29.89 | 30.20 |

**Table 1**
Data of the six-unit system.

| $i$ | $a_i$ | $b_i$ | $c_i$ | $e_i$ | $f_i$ | $g_i$ | $P_{i,min}$ | $P_{i,max}$ | $DR_i$ | $UR_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 240 | 7 | 0.007 | 13.8593 | 0.32767 | 0.00419 | 100 | 500 | 120 | 80 |
| 2 | 200 | 10 | 0.0095 | 13.8593 | 0.32767 | 0.00419 | 50 | 200 | 90 | 50 |
| 3 | 220 | 8.5 | 0.009 | 40.2669 | −0.54551 | 0.00683 | 80 | 300 | 100 | 65 |
| 4 | 200 | 11 | 0.009 | 40.2669 | −0.54551 | 0.00683 | 50 | 150 | 90 | 50 |
| 5 | 220 | 10.5 | 0.008 | 42.8955 | −0.51116 | 0.00461 | 50 | 200 | 90 | 50 |
| 6 | 190 | 12 | 0.0075 | 42.8955 | −0.51116 | 0.00461 | 50 | 150 | 90 | 50 |

**Table 4**
Customer cost function coefficients, customer type and daily customer energy limit.

| $j$ | $K_{1,j}$ | $K_{2,j}$ | $\theta_j$ | $CM_j$(MWh) |
|---|---|---|---|---|
| 1 | 1.847 | 11.64 | 0 | 200 |
| 2 | 1.378 | 11.63 | 0.1734 | 280 |
| 3 | 1.079 | 11.32 | 0.4828 | 410 |
| 4 | 0.9124 | 11.5 | 0.7208 | 500 |
| 5 | 0.8794 | 11.21 | 1 | 700 |

GTDR-DEED and GTDR-PBDEED models using the CONOPT solver on a computer with Intel (R) core processor and 4 GB of RAM.

$$B = 10^{-4} \times \begin{bmatrix} 0.420 & 0.051 & 0.045 & 0.057 & 0.078 & 0.066 \\ 0.051 & 0.180 & 0.039 & 0.048 & 0.045 & 0.060 \\ 0.045 & 0.039 & 0.195 & 0.051 & 0.072 & 0.057 \\ 0.057 & 0.048 & 0.051 & 0.213 & 0.090 & 0.075 \\ 0.078 & 0.045 & 0.072 & 0.090 & 0.207 & 0.096 \\ 0.066 & 0.060 & 0.057 & 0.075 & 0.096 & 0.255 \end{bmatrix} perMW \tag{48}$$

### 6.1. Simulation results without disturbance

The MPC strategy is implemented on both the GTDR-DEED and the GTDR-PBDEED problem. As stated before, for multi-objective problems in order to solve the problem with minimal computational complexity, it is often necessary to use the goal attainment method or weighted sum approach and convert the objectives into a single objective [5]. Thus, for GTDR-DEED, $w_1 = w_2 = w_3 = \frac{1}{3}$ while for GTDR-PBDEED, $w_1 = w_2 = 0.5$. The values for the weights are chosen so that equal preference is given to all the objectives and in both cases, the sum of the weights equals 1.

Figs. 1 and 2 show the results obtained from the MPC implementations on GTDR-DEED and the GTDR-PBDEED respectively. Each figure shows the optimal power generated from all generators, the total demand profile, optimal power curtailed by the customers and the optimal customer incentive. For comparison purposes, we also show obtained results of the GTDR-DEED and GTDR-PBDEED with open loop control in Figs. 3 and 4 respectively. A careful comparison of the figures shows that both open loop and closed-loop control yield similar results.

Table 5 provides a numerical results comparison between both approaches. From the results it shows that the closed-loop returns better results than the open loop approach. This is because comparing GTDR-DEED under open loop and closed-loop, it is seen that the closed-loop approach returns lower fuel costs ($ 290554.50 to the open loop's $ 291898.16). The closed-loop approach again returns lower emissions (24,332.84 lb to open loop's 24,474.04 lb). Even though both approaches yield the same amount of customer incentive ($ 50000), the closed-loop approach to GTDR-DEED yields better energy curtailment (1957.38 MWh to open loop's 1953.02 MWh) and also lower energy loss (264 MWh to open loop's 266 MWh). Going further to compare GTDR-PBDEED under the closed-loop approach and the open loop approach, from Table 5 we see that the closed-loop approach again yields lower total fuel costs, emissions, energy generated and energy losses. The closed-loop approach also yields a higher total customer incentive ($ 32228.39 to the open loop's $ 31954.89) and higher total profits ($ 1119751.99 to the open loop's $ 1095533.01). Furthermore results from the closed-loop approach converge to that of the open loop solution both under GTDR-DEED and GTDR-PBDEED as evidenced by Figs. 5 and 6 respectively, thereby demonstrating the convergence ability of the MPC algorithm.

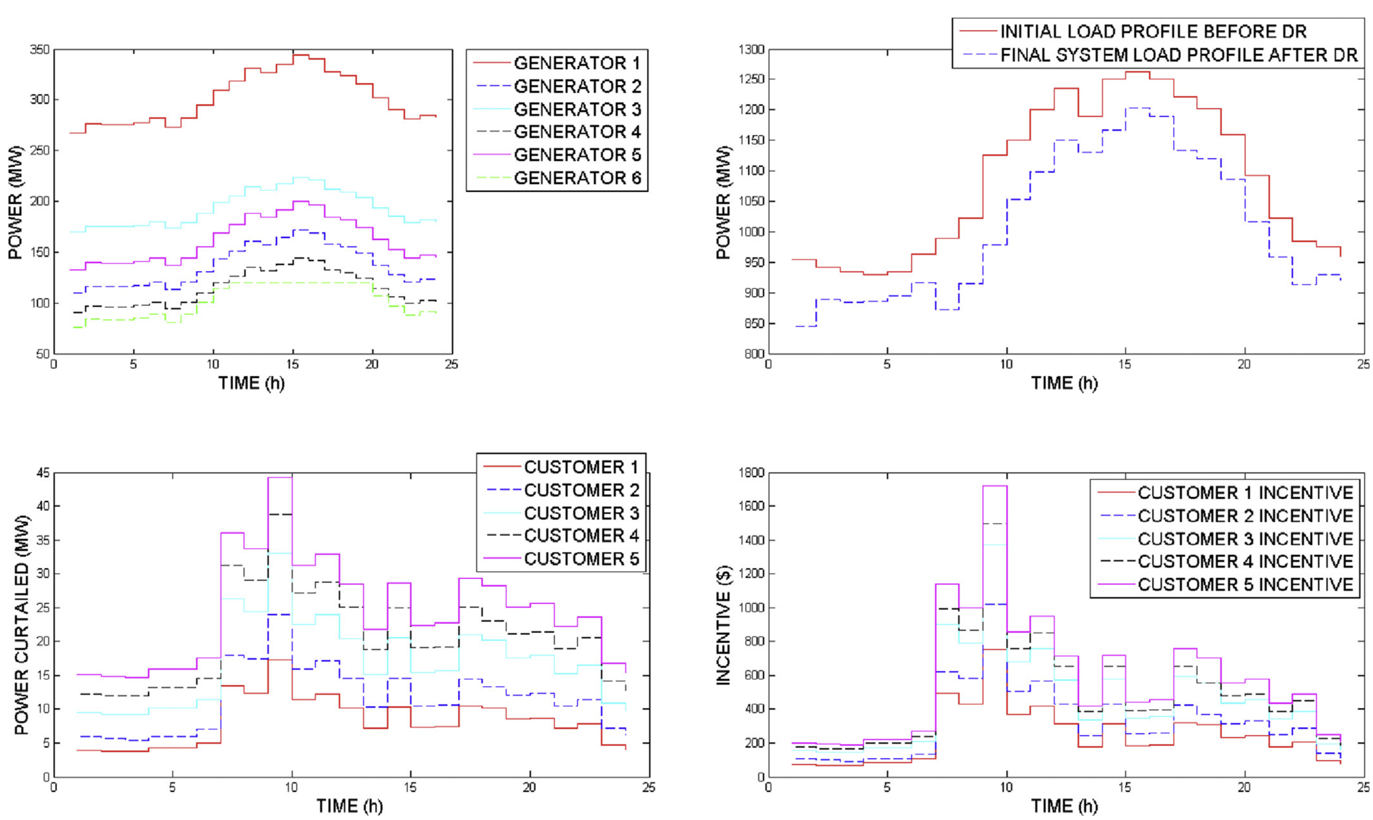


**Fig. 1.** GTDR-DEED closed-loop results with no disturbance.

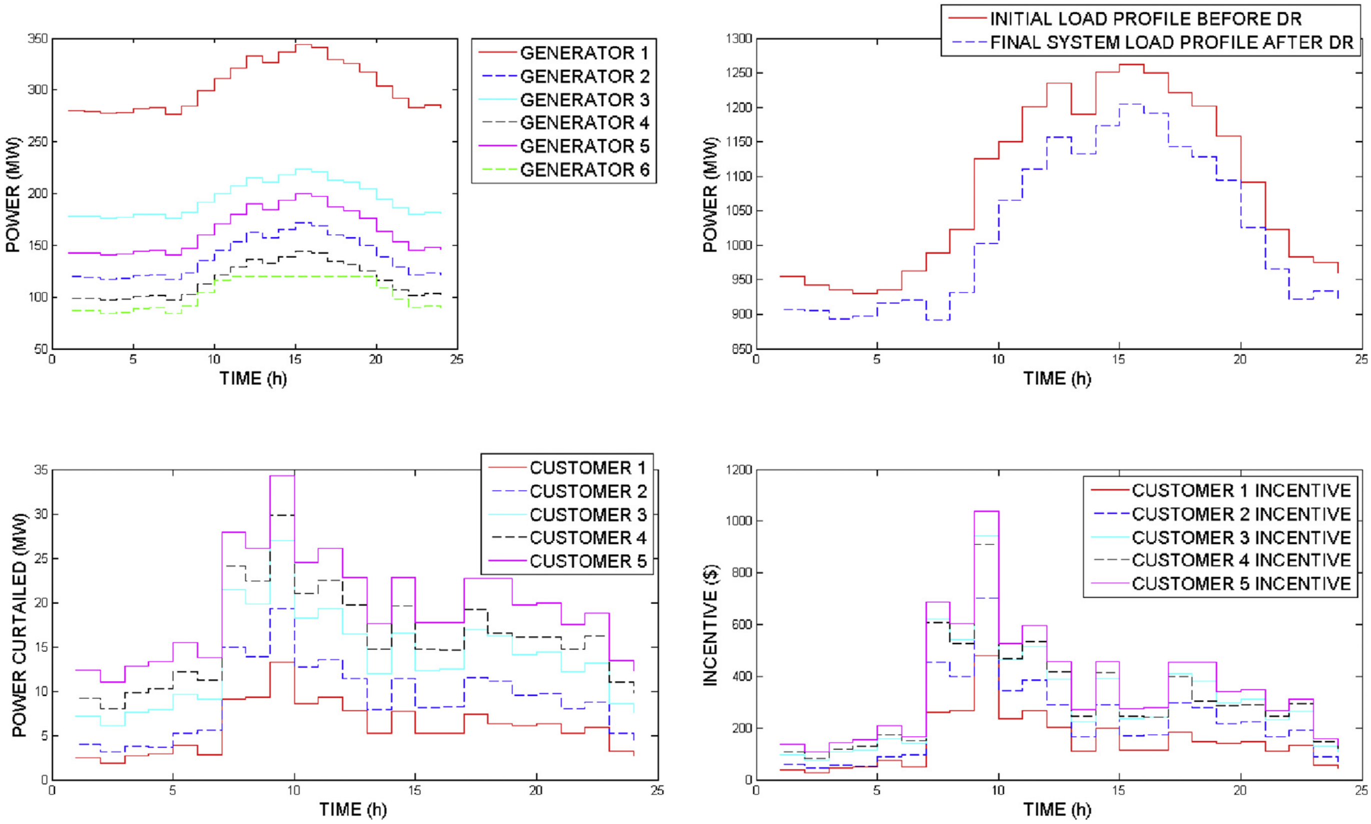


**Fig. 2.** GTDR-PBDEED closed-loop results with no disturbance.

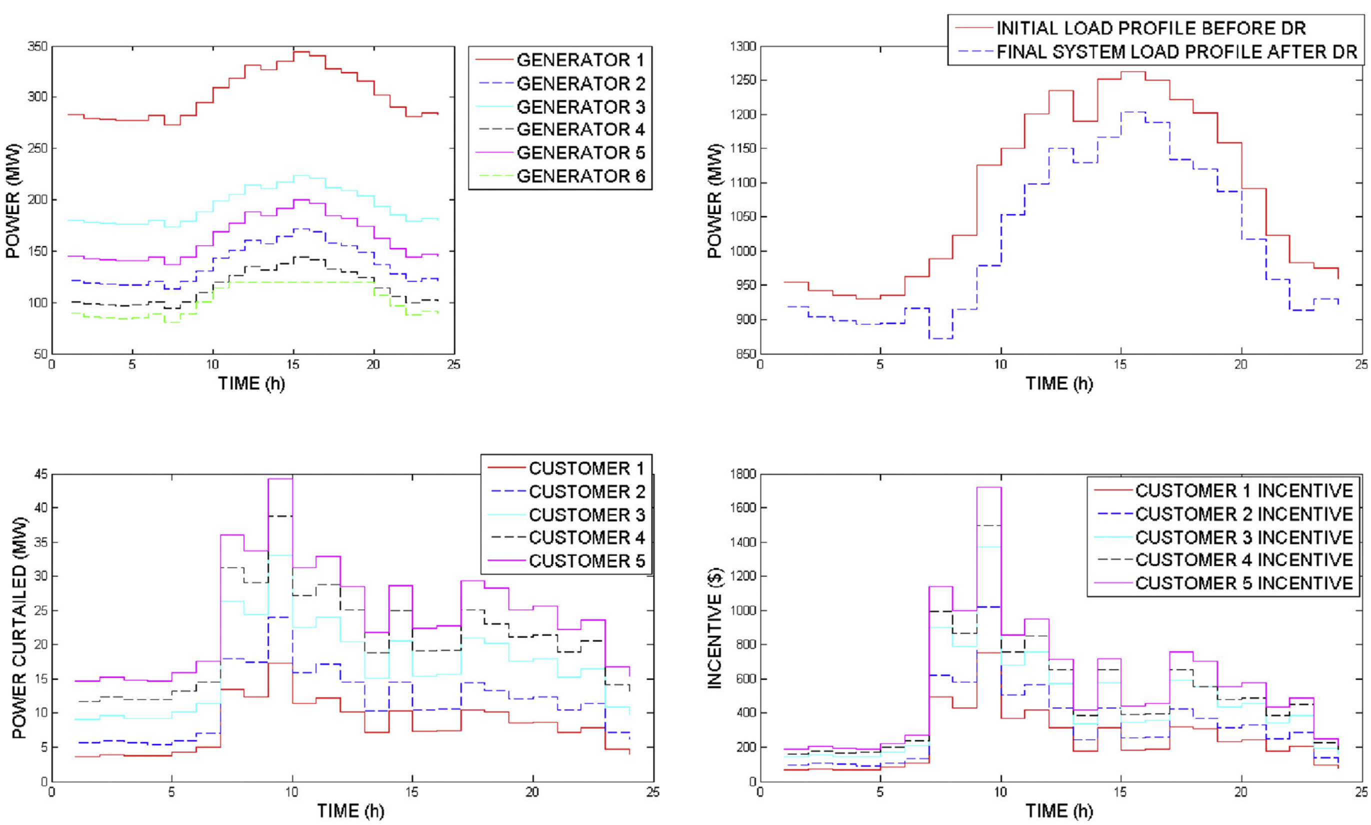


**Fig. 3.** GTDR-DEED open loop results with no disturbance.

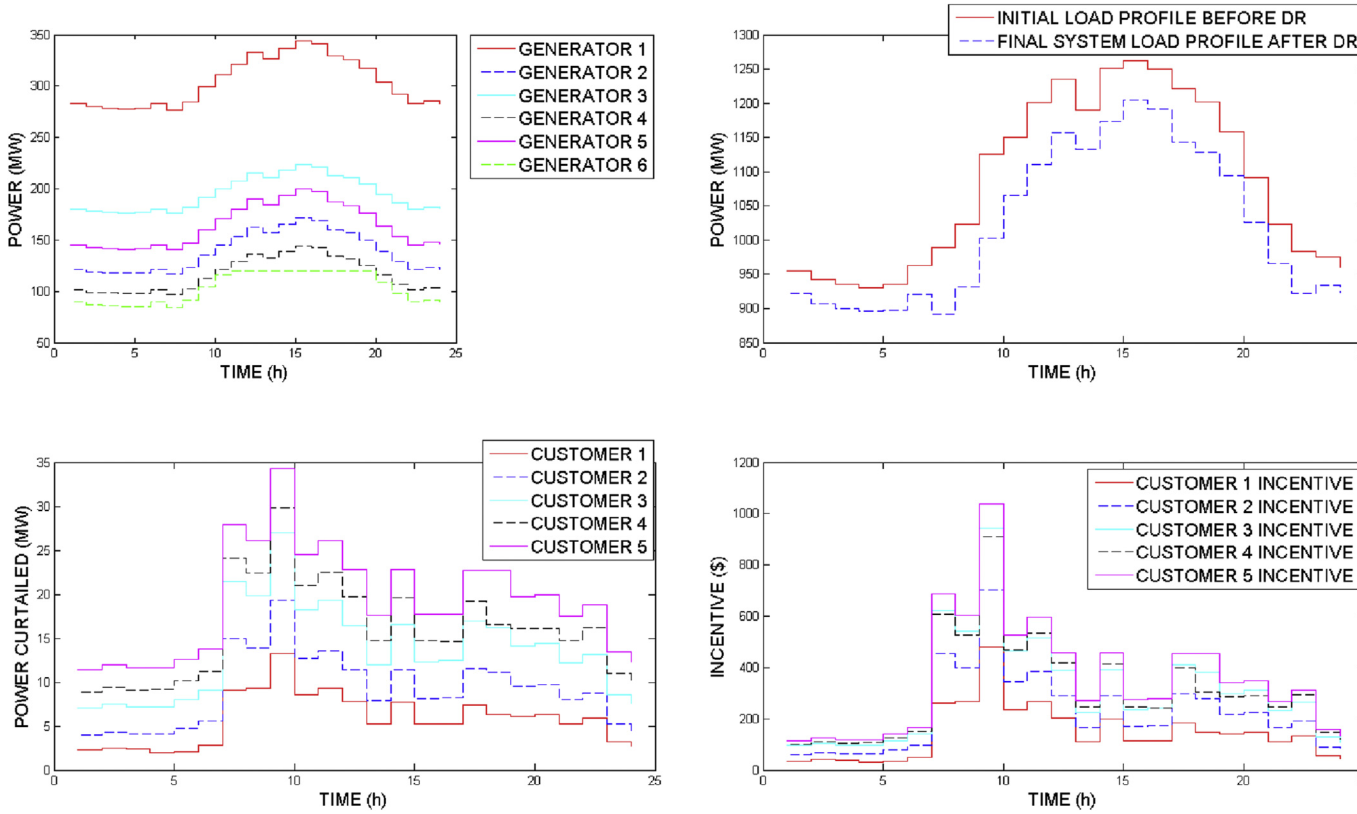


**Fig. 4.** GTDR-PBDEED open loop results with no disturbance.

### *6.2. Simulation results with disturbance*

To test the robustness of the MPC algorithm against uncertainties and disturbance, we assume that for GTDR-DEED and GTDR-PBDEED the demand randomly increases between 3.5% and 10% of the initial demand. Also the energy price is similarly randomly varied between −5% and 5% of the initial energy price. Similarly for GTDR-DEED, $w_1 = w_2 = w_3 = \frac{1}{3}$ while for GTDR-PBDEED, $w_1 = w_2 = 0.5$. Figs. 7 and 8 shows the results obtained for GTDR-DEED and GTDR-PBDEED respectively. For comparison purposes we also show results for GTDR-DEED and GTDR-PBDEED under open loop control (See Figs. 9 and 10) respectively. Table 6 shows a numerical comparison between both open loop and closed-loop control.

From the results it shows that the closed-loop returns better results than the open loop approach and handles disturbances and uncertainties better. This is because comparing GTDR-PBDEED under open loop and closed-loop, it is seen from Table 6 that the closed-loop approach returns lower fuel costs ($ 316258.72 to the open loop's $ 318937.26). The closed-loop approach again returns lower emissions (27,865.93 lb to open loop's 28,204.86 lb). Again, the closed-loop approach to GTDR-DEED yields better energy curtailment (1508.77 MWh to open loop's 1504.47 MWh) and also lower energy loss (307.11 MWh to open loop's 311.41 MWh). Finally, the closed-loop approach also yields a higher total customer incentive ($ 35889.02 to the open loop's $ 31533.21) and higher total profits ($ 1214384.57 to the open loop's $ 1208383.41) Comparing GTDR-DEED under the closed-loop approach and the open loop approach, from Table 6 we see that the closed-loop approach again yields lower total fuel costs, emissions, energy generated and energy losses. Both approaches yield the same amount of customer incentive ($50000). Figs. 11 and 12 shows the performance of the open loop controller and the closed-loop controller with disturbance.

**Table 5**
Results of the open loop and closed-loop approach without disturbance.

| | Open loop | | Closed-loop | |
|---|---|---|---|---|
| | GTDR DEED | GTDR PBDEED | GTDR DEED | GTDR PBDEED |
| Total fuel cost ($) | 291898.16 | 294006.12 | 290554.50 | 293964.84 |
| Total emissions (lb) | 24474.04 | 24,739.27 | 24,332.84 | 24,734.78 |
| Total customer incentive ($) | 50000 | 31954.89 | 50000 | 32228.39 |
| Total customer energy curtailed (MWh) | 1953.02 | 1518.68 | 1957.38 | 1530.03 |
| Total energy generated (MWh) | 24,266.59 | 24,435.32 | 24,156.62 | 24,431.98 |
| Total energy loss (MWh) | 266 | 269 | 264 | 268.94 |
| Total profit ($) | | 1095533.01 | | 1119751.99 |

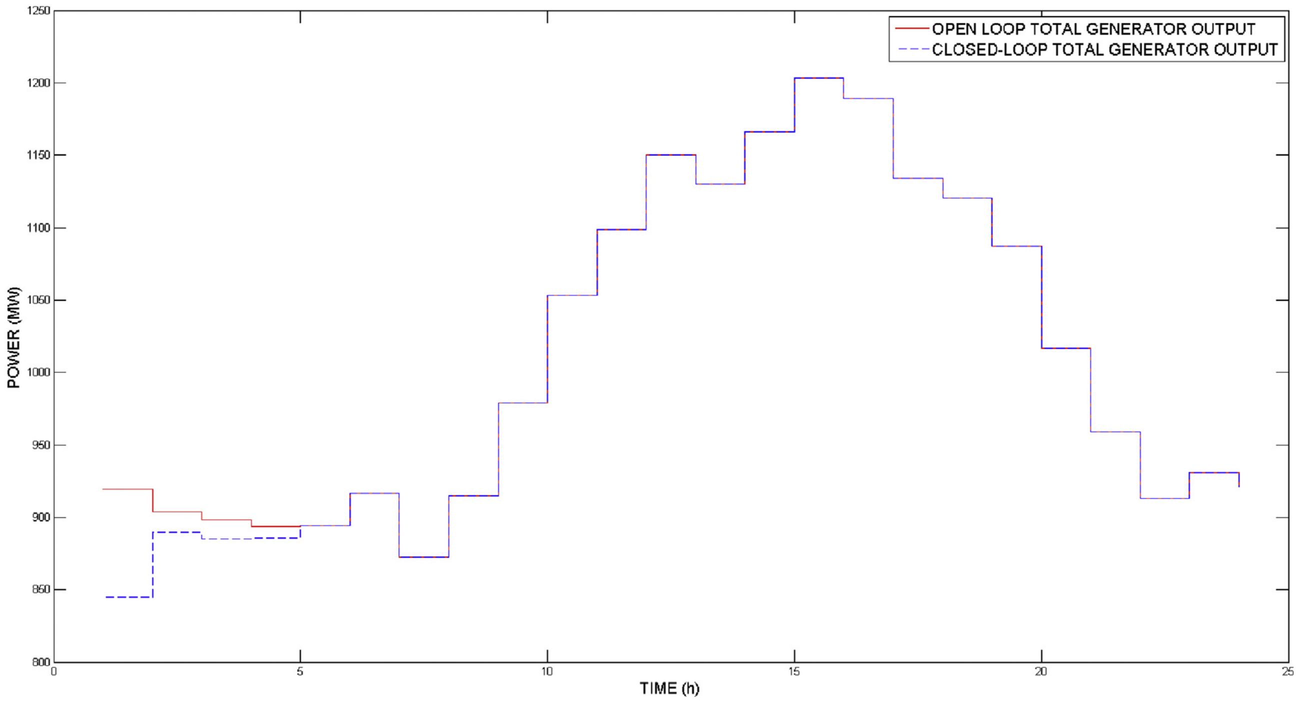


**Fig. 5.** Convergence of the closed-loop solutions to that of the open loop solutions for GTDR-DEED.

### 6.3. *Discussion of results*

The results obtained can be discussed along two lines. Results from GTDR-DEED and GTDR-PBDEED will be discussed and analysed. Also discussions can be done comparing results obtained under open loop and closed-loop control strategies. The discussion would focus on the following economic and power system parameters: Total Fuel Cost ($), Total Emissions (lb), Total Customer Incentive ($), Total Customer Energy Curtailed (MWh), Total Energy Generated (MWh), Total Energy Loss (MWh) and Total Profit ($). In simulations done, we gives equal preference to all the objectives and thus give them equal weights (see equations (35) and (45)). We ignore investigating the effect of varying the weights (and hence the objectives) as this and the effect of using a larger power system has been done in Ref. [8]. As stated earlier, GTDR-PBDEED is for a deregulated environment, whilst GTDR-DEED is for a regulated environment.

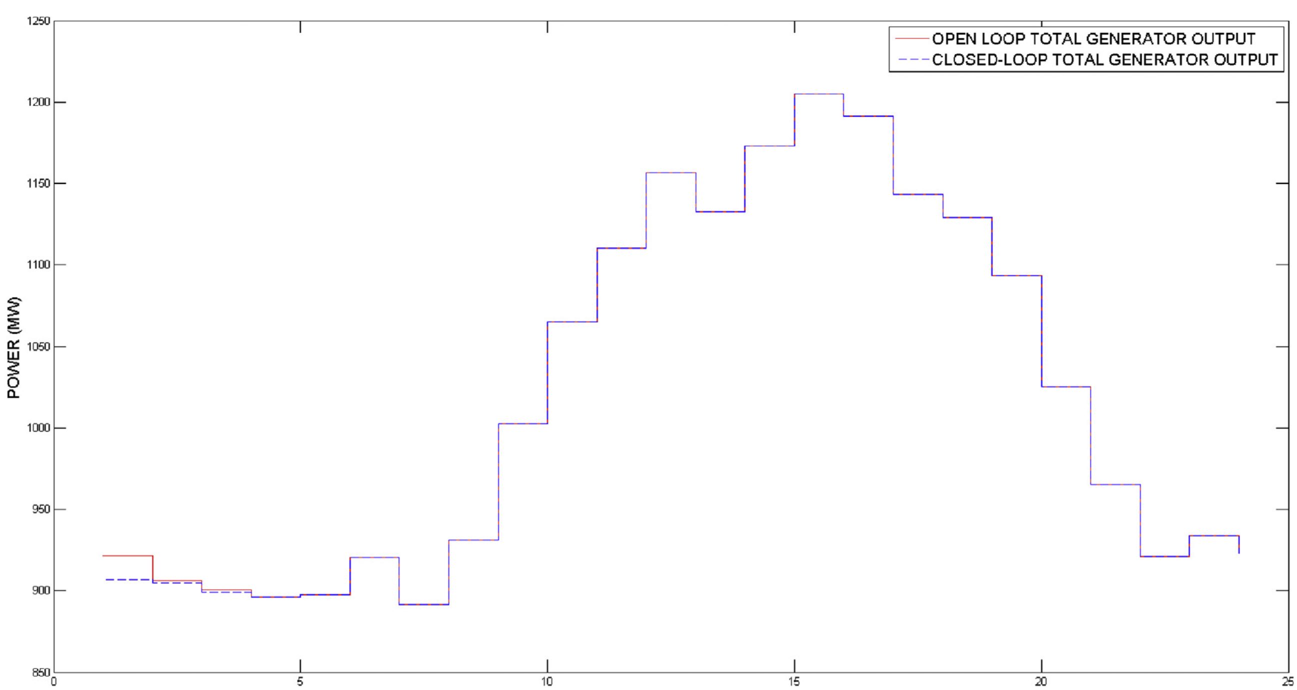


**Fig. 6.** Convergence of the closed-loop solutions to that of the open loop solutions for GTDR-PBDEED.

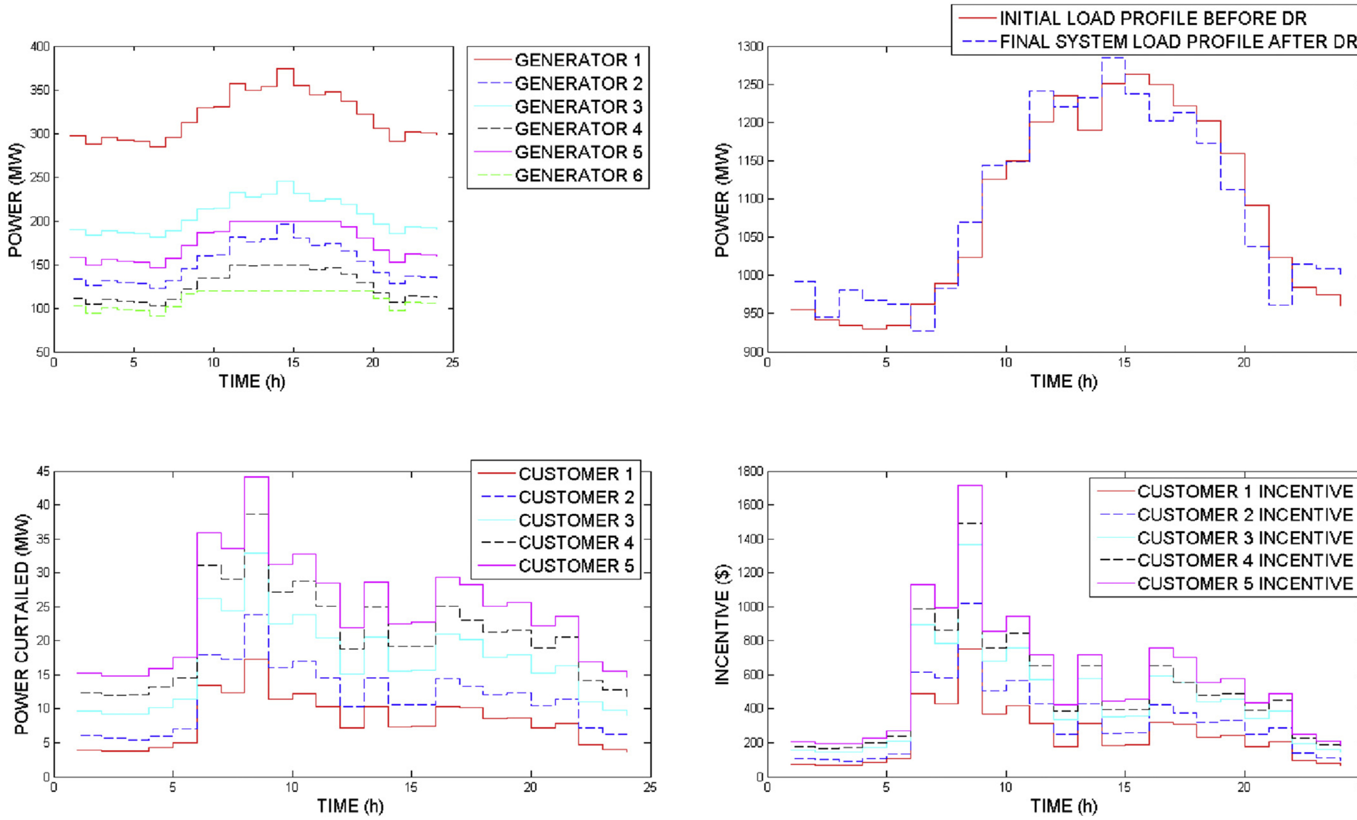


**Fig. 7.** GTDR-DEED closed-loop results with disturbance.

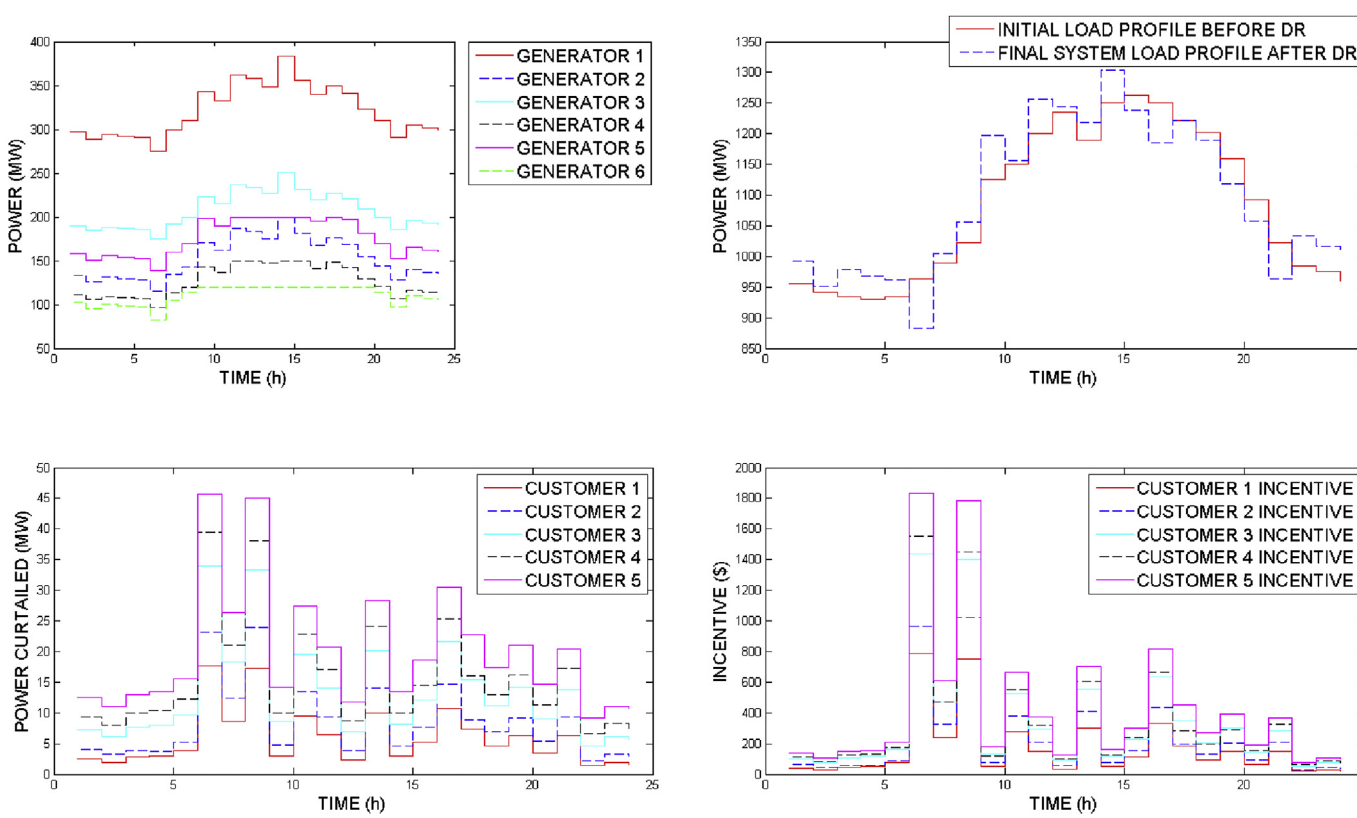


**Fig. 8.** GTDR-PBDEED closed-loop results with disturbance.

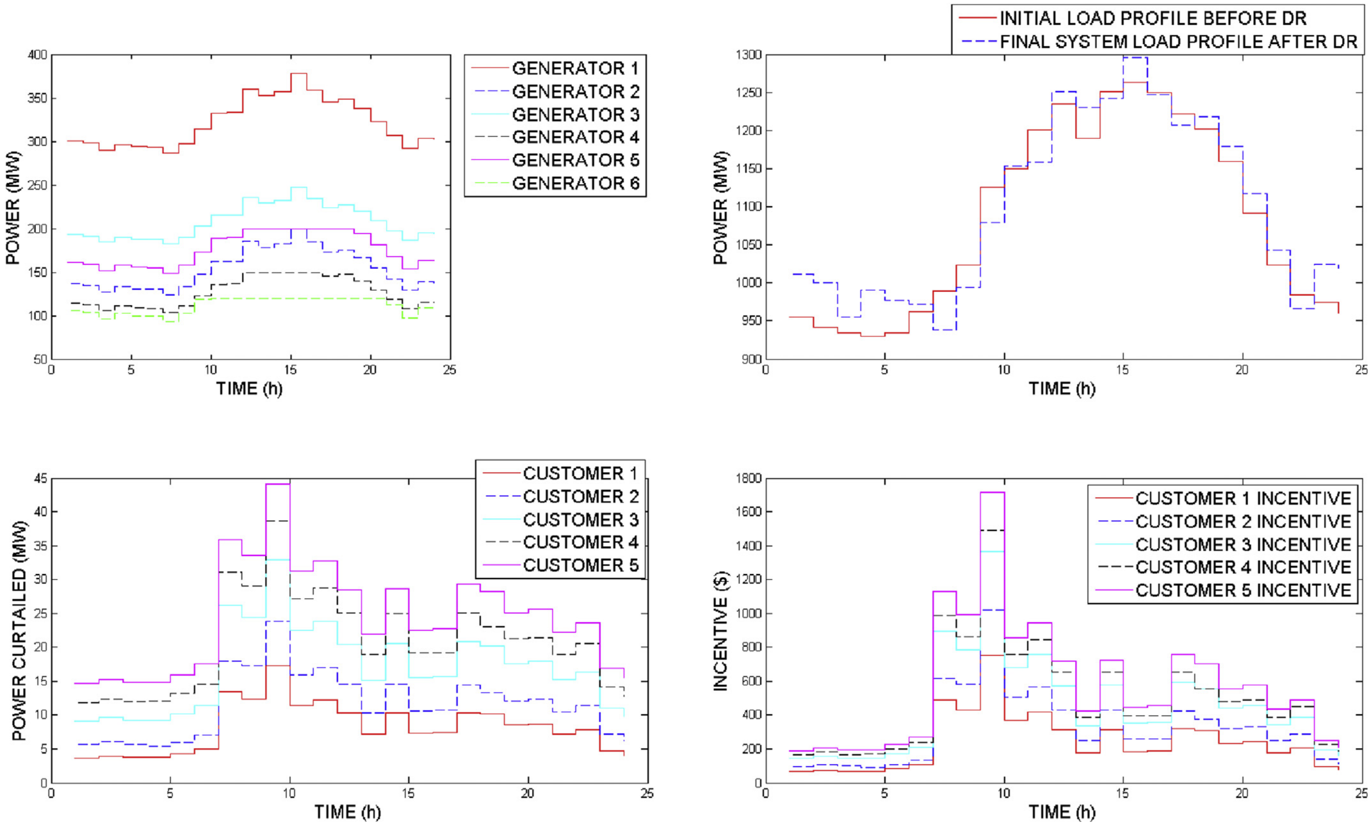


**Fig. 9.** GTDR-DEED open loop results with disturbance.

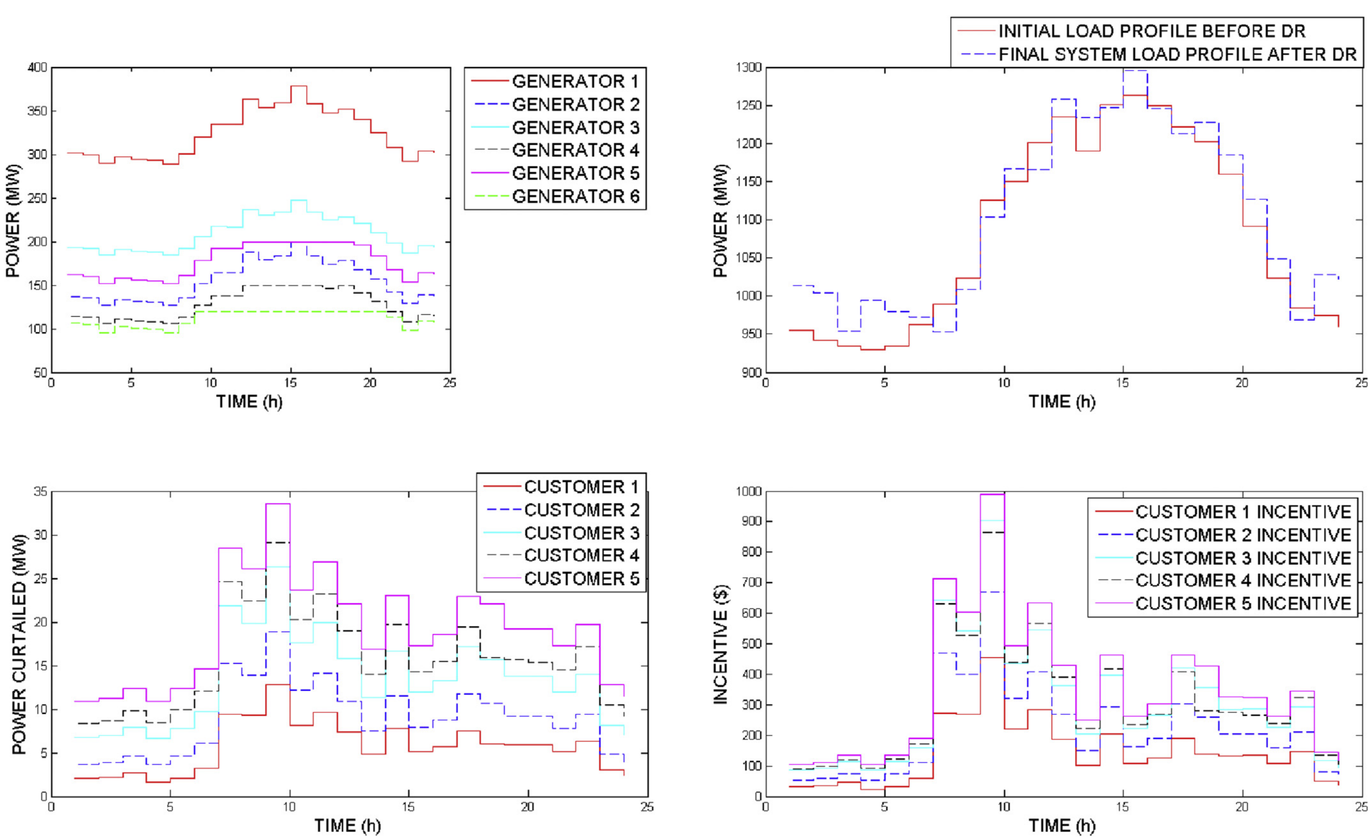


**Fig. 10.** GTDR-PBDEED open loop results with disturbance.

**Table 6**
Results of the open loop and closed-loop approach with disturbance.

| | Open loop | | Closed-loop | |
|---|---|---|---|---|
| | GTDR | GTDR | GTDR | GTDR |
| | DEED | PBDEED | DEED | PBDEED |
| Total fuel cost ($) | 317149.16 | 318937.26 | 314454.33 | 316258.72 |
| Total emissions (lb) | 27,943.02 | 28204.86 | 27,546.48 | 27,865.93 |
| Total customer incentive ($) | 50000 | 31533.21 | 50000 | 35889.02 |
| Total customer energy curtailed (MWh) | 1954.18 | 1504.47 | 1954.05 | 1508.77 |
| Total energy generated (MWh) | 26,273.87 | 26,415.30 | 26,060.27 | 26,200.99 |
| Total energy loss (MWh) | 308.28 | 311.41 | 303.56 | 307.11 |
| Total profit ($) | | 1208383.41 | | 1214384.57 |

From the obtained results in Tables 5 and 6, the GTDR-PBDEED saves less power than GTDR-DEED, therefore more power is generated by GTDR-PBDEED under both open and closed-loop strategies. This means that the emission, cost and losses of GTDR-PBDEED are greater than those of GTDR-DEED. It can also be seen from both tables, that because the utility/ISO in GTDR-PBDEED wants to maximize profit, the total incentive paid to customers never equals the maximum utility budget, unlike in GTDR-DEED where the maximum utility budget is always reached as maximizing profit is not an objective in this case.

In a nutshell, the results show that DR has benefits to the power system either under a regulated or deregulated environment. The results also show the superiority of the closed-loop approach (MPC) over the open loop approach. MPC returns better results than open loop with and without disturbance. Furthermore the convergence ability of the MPC algorithm to the open loop solution is also shown. Looking at Figs. 5 and 6 it shows that closed-loop solutions converge to the open loop solutions. This happens in the fifth hour for the GTDR-DEED and in the fourth hour for the GTDR-PBDEED case. Both cases demonstrate the convergence capability of the MPC algorithm. This means that the MPC algorithm can be restarted at any time instant and would still converge which guarantees optimality at all times which is very important in practical purposes. Again considering Figs. 11 and 12, they show that the total generator output of the closed-loop strategy is in the neighbourhood of the open loop solutions for GTDR-DEED and GTDR-PBDEED respectively. Table 6 shows that the closed-loop approach handles disturbance better and gives better economic and power system parameters. Comparing Table 6 with Table 5, it shows that disturbances actually makes for a more inefficient and expensive system. This is because with disturbances under both open loop and closed-loop control schemes and for both GTDR-DEED and GTDR-PBDEED, the disturbed system actually returns higher fuel costs, emissions, losses and energy generated (see Table 5). However the closed-loop controller still presents better results than the open loop controller.

It is necessary to provide a comparative analysis of obtained results with similar prior works in the literature [8]. The work in Ref. [8] is essentially a GTDR-DEED ″open loop controller without disturbance" problem and the results are in the second column in Table 5. Comparing results with the closed-loop controller (fourth column in Table 5), as has been shown before it is seen that the closed-loop approach returns lower fuel costs ($ 290554.50 to the open loop's $ 291898.16), lower emissions (24332.84 lb to open

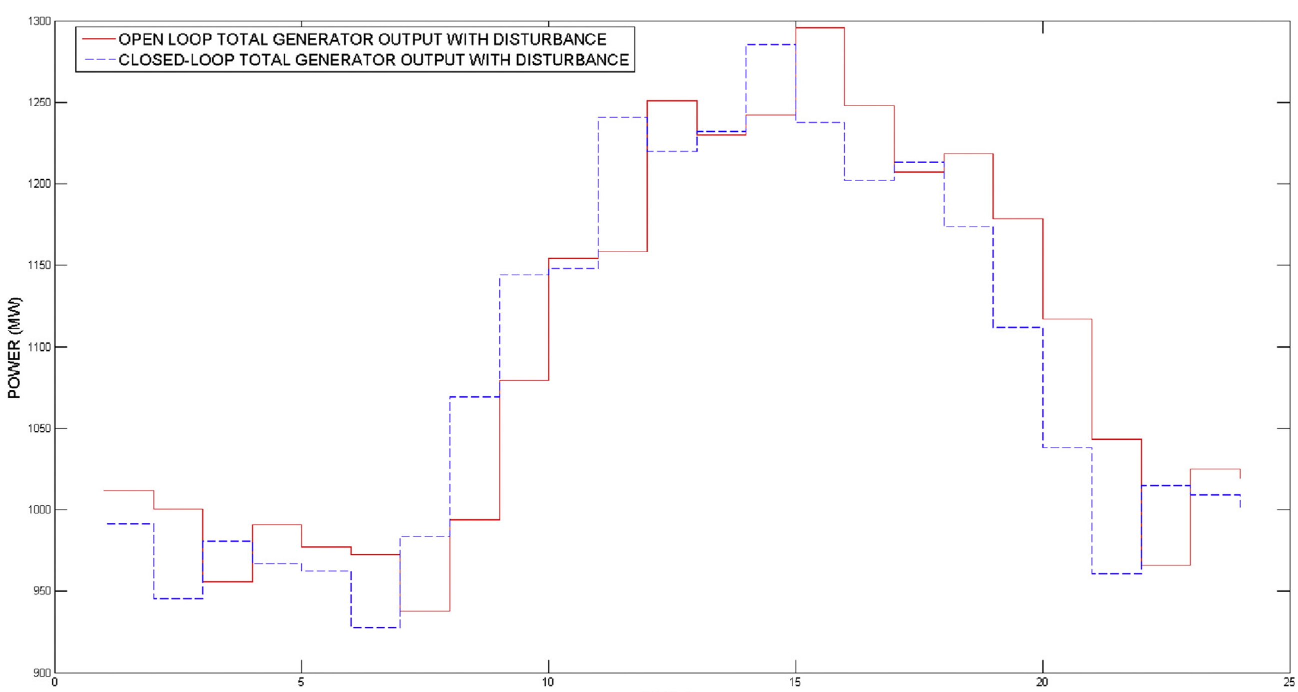


**Fig. 11.** Total generator output of GTDR-DEED using both open loop and closed-loop control with disturbance.

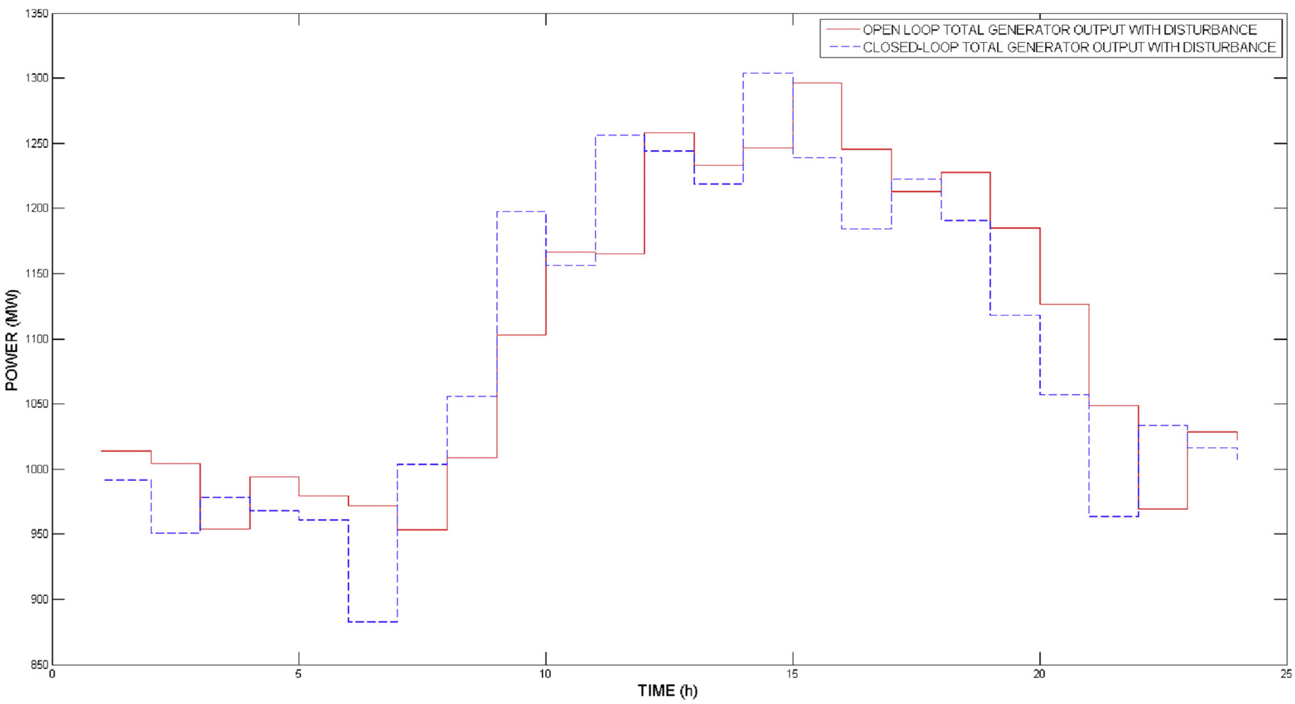


**Fig. 12.** Total generator output of GTDR-PBDEED using both open loop and closed-loop control with disturbance.

loop's 24,474.04 lb) and lower energy loss (264 MWh to open loop's 266 MWh) whilst the closed-loop approach to GTDR-DEED yields better energy curtailment (1957.38 MWh to open loop's 1953.02 MWh).

## 7. Conclusion

In this paper, a game theory based demand response program is integrated into two variants of the economic dispatch problem. Both models determine the optimal generator output, customer power curtailed and customer incentives. Model predictive control which is a closed-loop technique has been applied to solve both models. Obtained results indicate that the closed-loop model generally yields better results using the defined solution parameters than its open loop counterpart. The results are significant in many respects.

Firstly they show that demand response programs when properly conceptualized and incorporating optimal incentives can bring about much needed power system relief. In the two mathematical models developed, demand response programs bought about significant energy curtailment. The closed loop strategy also yields higher optimal energy curtailment for the customers over the open loop strategy ranging from 4.36 MW h to 11.35 MWh for the first and second models respectively. Furthermore, the closed loop strategy is also able to better handle uncertainties due to variations in system data and parameters. The savings in fuel cost due to the adoption of the closed loop strategy ranges from $ 41.28 to $ 1343.66 for the no disturbance case and when disturbance is present, fuel cost savings ranges from $ 2678.54 to $ 2694.83.

Across all simulations performed with disturbance present, the closed loop strategy gave lower fuel costs, emissions, customer incentive and system losses. This shows and validates the superiority of the closed loop approach over the open loop approach. Future work will thus consider the addition of combined heat and power generators and the incorporation of renewable energy sources into the model formulation.

## References

[1] Basu M. Dynamic economic emission dispatch using nondominated sorting genetic algorithm-II. Int J Electr Power Energy Syst 2008;30:140–9.
[2] Basu M. Particle swarm optimization based goal-attainment method for dynamic economic emission dispatch. Electr Power Compon Syst 2006;34:1015–25.
[3] Basu M. Dynamic economic emission dispatch using evolutionary programming and fuzzy satisfied method. Int J Emerg Electr Power Syst 2007;8:1–15.
[4] Xia X, Elaiw AM. Optimal dynamic economic dispatch of generation: a review. Electr Power Syst Res 2010;80:975–86.
[5] Elaiw AM, Xia X, Shehata AM. Application of model predictive control to optimal dynamic dispatch of generation with emission limits. Electr Power Syst Res 2012;84:31–44.
[6] Arif A, Javed F, Arshad N. Integrating renewables economic dispatch with demand side management in micro-grids: a genetic algorithm-based approach. Energy Effic 2014;7:271–84.
[7] Mazidi M, Zakariazadeh A, Jadid S, Siano P. Integrated scheduling of renewable generation and demand response programs in a microgrid. Energy Convers Manag 2014;86:1118–27.
[8] Nwulu NI, Xia X. Multi-objective dynamic economic emission dispatch of electric power generation integrated with game theory based demand response programs. Energy Convers Manag 2015;89:963–74.
[9] Xia X, Zhang J, Elaiw A. An application of model predictive control to the dynamic economic dispatch of power generation. Control Eng Pract 2011;19:638–48.
[10] Ekpenyong UE, Zhang J, Xia X. An improved robust model for generator maintenance scheduling. Electr Power Syst Res 2012;92:29–36.
[11] Tazvinga H, Zhu B, Xia X. Energy dispatch strategy for a photovoltaic-wind-diesel-battery hybrid power system. Sol Energy 2014;108:412–20.
[12] Zhang J, Xia X. A model predictive control approach to the periodic implementation of the solutions of the optimal dynamic resource allocation problem. Automatica 2011;47:358–62.
[13] Ardente F, Beccali M, Cellura M. F.A.L.C.A.D.E.: a fuzzy software for the energy and environmental balances of products. Ecol Model 2004;176:359–79.
[14] Ardente F, Beccali G, Cellura M. Eco-sustainable energy and environmental strategies in design for recycling: the software "ENDLESS". Ecol Model 2003;163:101–18.
[15] Beccali G, Cellura M, Mistretta M. New exergy criterion in the "multi-criteria" context: a life cycle assessment of two plaster products. Energy Convers Manag 2003;44:2821–38.
[16] Cellura M, Guarino F, Longo S, Mistretta M, Orioli A. The role of the building sector for reducing energy consumption and greenhouse gases: an Italian case study. Renew Energy 2013;60:586–97.
[17] Cellura M, Longo S, Mistretta M. The energy and environmental impacts of Italian households consumptions: an input-output approach. Renew Sustain Energy Rev 2011;15:3897–908.

[18] Arul R, Velusami S, Rav G. A new algorithm for combined dynamic economic emission dispatch with security constraints. Energy 2015;79:496–511.
[19] Basu M. Fuel constrained economic emission dispatch using nondominated sorting genetic algorithm-II. Energy 2014;78:649–64.
[20] Younes M, Khodja F, Kherfane RL. Multi-objective economic emission dispatch solution using hybrid FFA (firefly algorithm) and considering wind power penetration. Energy 2014;67:595–606.
[21] Osorio GJ, Lujano-Rojas JM, Matias JCO, Catal JPS. A probabilistic approach to solve the economic dispatch problem with intermittent renewable energy sources. Energy 2015;82:949–59.
[22] Niu Q, Zhang H, Li K, Irwin GW. An efficient harmony search with new pitch adjustment for dynamic economic dispatch. Energy 2014;65:25–43.
[24] Modiri-Delshad M, Rahim NA. Solving non-convex economic dispatch problem via backtracking search algorithm. Energy 2014;77:372–81.
[25] Basu M. Teaching learning based optimization algorithm for multi-area economic dispatch. Energy 2014;68:21–8.
[26] Basu M, Chowdhury A. Cuckoo search algorithm for economic dispatch. Energy 2013;60:99–108.
[27] Ommen T, Markussen WB, Elmegaard B. Comparison of linear, mixed integer and non-linear programming methods in energy system dispatch modelling. Energy 2014;74:109–18.
[28] Kim JS, Edgar TF. Optimal scheduling of combined heat and power plants using mixed-integer nonlinear programming. Energy 2014;77:675–90.
[29] Gaing ZL. Particle swarm optimization to solving the economic dispatch considering the generator constraints. IEEE Trans Power Syst 2003;18:1187–95.
[30] Han XS, Gooi HB. Effective economic dispatch model and algorithm. Int J Electr Power Energy Syst 2007;29:113–20.
[31] Ongsakul W, Tippayachai J. Parallel micro genetic algorithm based on merit order loading solutions for constrained dynamic economic dispatch. Electr Power Syst Res 2002;61:77–88.
[32] Nwulu NI, Xia X, Zhang J. Determining the optimal incentive and number of retrofits for a demand response program in South Africa. In: Proc. of the 5th international conference on applied energy, Pretoria, South Africa; 1–4 July 2013.
[33] Sezgen O, Goldman CA, Krishnarao P. Option value of electricity demand response. Energy 2007;32:108–19.
[34] Khajavi P, Monsef H, Abniki H. Load profile reformation through demand response programs using smart grid. In: Proc. Of the international symposium on modern electric power systems, Wroclaw, Poland; 20-22 September, 2010.
[35] Aalami H, Yousefi GR, Moghadam MP. Demand response modelling considering interruptible/curtailable loads and capacity market programs. Appl Energy 2010;87:243–50.
[36] Nwulu NI, Fahrioglu M. A neural network model for optimal demand management contract design. In: Proc. Of the 10th environmental electrical engineering conference, Rome, Italy; 8–11, May 2011.
[37] Fahrioglu M, Nwulu NI. Investigating a ranking of loads in avoiding potential power system outages. J Electr Rev 2012;88:239–42.
[38] Fahrioglu M, Alvarado FL. Using utility information to calibrate customer demand management behaviour models. IEEE Trans Power Syst 2001;16:317–22.
[39] Fahrioglu M, Alvarado FL. Designing incentive compatible contracts for effective demand management. IEEE Trans Power Syst 2000;15:1255–60.
[40] Nwulu NI, Fahrioglu M. Power system demand management contract design: a comparison between game theory and artificial neural networks. Int Rev Model Simul 2011;4:106–12.
[41] Nwulu NI, Fahrioglu M. A soft computing approach to projecting locational marginal price. Neural Comput Appl 2013;22:1115–24.
[42] Ott AL. PJM locational marginal pricing. EEI transmission pricing school. July 2001.
[43] Bisschop J, Roelofs M. AIMMS language reference, version 3.12, Paragon decision technology, Haarlem. 2011.