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Optimal motion planning for overhead cranes

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Abstract: Overhead cranes are widely used in industrial applications for material displacing. Many linear or non-linear control schemes have been proposed for overhead cranes and implemented on electronic systems, but energy efficiency of transportation has seldom been considered in motion planning. This study aims at finding an optimal solution of motion planning in terms of energy efficiency for overhead cranes. Using the optimal control method an optimal trajectory is obtained with less energy consumption than the compared trajectories and is also satisfying physical and practical constraints such as swing, acceleration and jerk. Besides the energy optimal model, the authors also propose two other models to optimise time efficiency and safety during transportation. The results obtained have been compared with some existing motion trajectories, and have been shown to be superior to these benchmarks in terms of energy efficiency, time efficiency and safety, respectively.

1 Introduction

In some industrial fields, such as sea ports, manufacturing plants, factories and construction sites, overhead cranes are indispensable for handling materials in the process of transportation and production [1–3]. Owing to high payload capacity, good operational flexibility and efficient energy consumption, overhead cranes are playing significant roles in many industrial applications. Regardless of the type of overhead crane, overhead cranes always have a similar fundamental structure that can be described as a trolley-pendulum system. The overhead crane is a typical non-linear underactuated mechanical system. The system, which consists of a trolley, a supporting frame and a rope connecting the trolley with the payload, has fewer independent control inputs than degrees of freedom. Compared with fully actuated systems, control of underactuated systems is much challenging. Therefore underactuated overhead cranes have attracted much interest from researchers in areas of mechanics and control.

One complete crane motion process is composed of three phases: payload hoisting, horizontal transportation and payload lowering, among which the control of horizontal transportation is the most difficult problem. These three phases can be integrated in one manoeuvre with overlap [4–7] or be decomposed sequentially without overlap [8, 9]. Phase overlap means that the payload can be horizontally transported when it is hoisted and that the payload can be lowered during the horizontal transportation. In this paper, phase overlap is not considered as the scope is specifically modelling motion planning problems confronted in the most complicated horizontal phase. Time efficiency and safety of the horizontal transportation are two main objectives of motion planning that are usually considered in previous work [9, 10]. For time efficiency, the transportation time is preferred to be as short as possible while other safety and mechanical constraints

must be satisfied. For safety, the requirement is to suppress or eliminate payload swing for some fragile or harmful payloads. To meet the safety requirement, both transient and residual swing must be controlled in a small tolerance range.

Many control strategies have been proposed to improve their performance in terms of time efficiency and transportation safety of overhead cranes. Owing to the non-linear nature of the crane system [11], non-linear control methods, such as sliding-mode control [12], flatness-based inverse methods [4, 5, 7] and energy-based control scheme [13], have been applied to control overhead cranes. For simplicity, the original complicated crane systems are usually linearised for using linear control methods. Under the assumption of small payload swing, the non-linear crane model can be linearised around its equilibrium points, and then linear control approaches can be used on the simplified linear system. Many linear control methods have been applied to overhead cranes, including feedback control [14–16], input shaping (feedforward control) [17–19] and optimal control [20–22]. Feedback control methods utilise measurements and estimates of the system states to reduce the payload's swing, whereas feedforward approaches alter the actuator commands so that system oscillations are reduced. Optimal control methods look for optimal solutions to minimise (or maximise) the defined objectives of crane problems subject to certain problem-based constraints.

In some control strategies, motion planning is a preliminary step to generate operation commands (profiles of acceleration or velocity) before online control of overhead cranes [23, 24]. Here motion planning means the trajectory design of trolley velocity and acceleration, which are regarded as control inputs (or states) of the crane system with velocity-driven or force-driven trolley. Unlike the inverse methods to control the stability of tracking the reference of outputs [4, 5], optimal control is a useful method to design the profile of control inputs with certain

optimal performance index, such as time efficiency and swing angle. Optimal design of the transportation trajectory not only improves the control performance in terms of efficiency or safety, but also releases some pressure on control for the sequential strategy. Researchers have focused their efforts on motion planning of overhead cranes. The most difficult task in motion planning is to analyse the kinematic coupling behaviour between the underactuated payload's swing and the actuated trolley's motion. In [20, 21], optimal control theory was effective to generate optimal transportation trajectories that could minimise steady-state error or transportation time. A motion planning-based adaptive control strategy [25] was proposed to make the trolley track the planned S-shaped trajectory, where an online updating mechanism was employed to enhance the control stability under different working conditions. In [8], an anti-swing motion planning scheme with swing suppression was designed based on the anti-swing control law and typical crane operations in practice. In [9], three analytical trajectories were designed to address the coupling behaviour and consider practical constraints such as swing amplitude, residual swing and physical constraints.

Among existing literature on motion planning, some trajectories are designed according to the empirical experiences; some trajectories are designed for satisfying practical constraints; and the other trajectories are designed in consideration of time efficiency of motion planning. In [20], time optimal control theory has been evaluated on the bang-bang control system of cranes, but their time-optimal solution has not considered the continuous system and the constraint of swing amplitude. In [21, 22], optimal trajectories have been designed for continuous systems of cranes subject to the swing constraint. It may be noted that swing-optimal control is seldom considered to minimise the swing angle separately for safety in transportation. Another issue that has been neglected in the crane control is energy efficiency. Nowadays, the energy issue turns out to be significantly urgent as power demand keeps increasing when a large number of machines, such as belt conveyors and cranes, have been equipped in industry [26]. In the crane control, the profile of power, as well as the total energy consumption, is closely related to the transportation trajectory, so energy efficiency may be optimised by choosing a proper motion trajectory. These two issues will be analysed and solved in our proposed strategies for overhead cranes.

In this paper, two objectives, safety and energy efficiency, are studied under the optimal control theory. First, swing-optimal model is proposed to minimise the swing amplitude and residual oscillation, to provide the highest degree of safety during horizontal transportation. Second, energy-optimal model is proposed to minimise the energy consumption in the transportation period. The power profile obtained is smoothed, and its peak is reduced as well. For the purpose of better comparison, we will revisit the time-optimal motion planning problem, but we include more practical constraints, such as maximum swing amplitude, and maximum jerk of motion. These three proposed motion planning strategies have covered the demanding requirements of crane control systems, and results obtained in the simulation section indicate their superiority in terms of time efficiency, safety and energy efficiency.

This paper is organised as follows. Section 2 describes the dynamic model of the overhead crane system. Then literature review on motion planning of overhead cranes is given in Section 3. In Section 4, the discrete-time dynamic equation is deduced and three optimal strategies in terms of the safety,

energy efficiency and time efficiency are proposed. The comparison study is performed in Section 5. The last section concludes this paper.

2 Description of the overhead crane system

An overhead crane can be sketched as shown in Fig. 1, where the trolley moves on the horizontal bridge and the payload is connected with a constant-length rope. $x(t)$, $\theta(t)$ and $F(t)$ denote the trolley's position, the payload's swing angle and overall force on the trolley, respectively. In this paper, stiffness and mass of the rope, as well as air resistance, are neglected and the load is considered as a point mass. Moreover, as this study only focuses on the open-loop optimal planning of horizontal transportation, hoisting and the effects of wind disturbance are not considered. Then the overhead crane system with constant rope length can be described as follows

$$(M + m)\ddot{x} + ml \cos \theta \ddot{\theta} - ml \sin \theta \dot{\theta}^2 = F \tag{1}$$

$$ml^2 \ddot{\theta} + ml \cos \theta \ddot{x} + mgl \sin \theta = 0 \tag{2}$$

where M and m denote masses of the trolley and the payload, respectively. l is the length of the rope; g the gravitational acceleration. The overall force F is composed of the actuating force F_a and the friction F_r as

$$F = F_a - F_r \tag{3}$$

Motivated by the friction models in [27, 28], this paper employs the same non-linear friction model as

$$F_r = f_{r0} \tanh \dot{x}/\xi - k_r |\dot{x}| \dot{x} \tag{4}$$

where f_{r0} , $k_r \in \mathbb{R}$ are friction-related parameters and $\xi \in \mathbb{R}$ is a static friction coefficient, which can be obtained from off-line experimental analysis. Note that the small friction caused by the payload's swing is neglected in the above model.

The payload-swing dynamics (2) defines the kinematic relationship between the trolley acceleration and the load swing. For this reason, the swing dynamics can be considered as a kinematic equation for motion planning and can be rewritten as

$$l\ddot{\theta} + \cos \theta \ddot{x} + g \sin \theta = 0 \tag{5}$$

When the swing angle is small enough ($\theta < 5^\circ$), the kinematic equation can be linearised with the approximations of

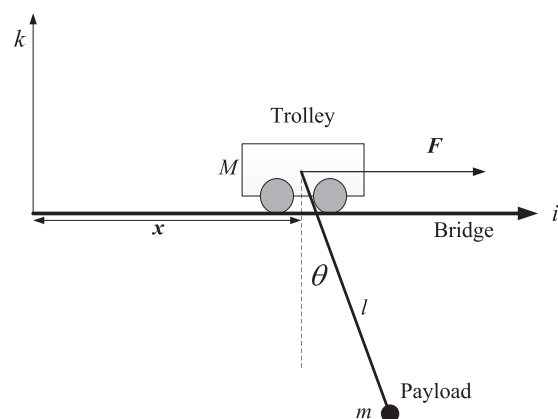


Fig. 1 Two-dimensional overhead crane system

$\cos \theta \simeq 1$ and $\sin \theta \simeq \theta$. The approximated linear kinematics can be obtained as

$$l\ddot{\theta} + \ddot{x} + g\theta = 0 \quad (6)$$

In the evaluated planning period T , a successful trajectory must ensure the crane to arrive at the destination. For a successful trajectory planning, several constraints must be satisfied according to practical constraints.

(1) The trolley reaches the desired location p_d at the end of the period. The final states of the payload must be static with no swing so that it can be lowered immediately as

$$x(T) = p_d, \quad \dot{x}(T) = 0, \quad \theta(T) = 0, \quad \dot{\theta}(T) = 0 \quad (7)$$

(2) During the horizontal transportation, velocity and acceleration of the trolley must be limited in certain ranges as

$$\begin{cases} 0 \leq \dot{x}(t) \leq v_m \\ |\ddot{x}(t)| \leq a_m \end{cases}, \quad t \in (0, T] \quad (8)$$

where v_m and a_m are the permitted maximum of velocity and acceleration, respectively.

(3) The payload swing during the transportation must be limited within a safe range as

$$|\theta(t)| \leq \theta_m, \quad t \in (0, T] \quad (9)$$

where θ_m is the permitted maximum of swing amplitude.

(4) The jerk (defined as the time derivative of acceleration $j(t) = \dot{\ddot{x}}(t)$) must be limited to a reasonable range to satisfy the mechanical constraint and to prolong the motor's lifetime

$$|j(t)| \leq j_m, \quad t \in (0, T] \quad (10)$$

where j_m is the permitted maximal jerk in the horizontal transportation.

3 Literature review

Different models, as well as control methods of overhead cranes, have been proposed in the past decades. These have been well reviewed in [29]. As the scope of this paper focuses on the motion planning methods, five trajectory references mentioned in [8, 9, 25] are analysed in this section. For the same specific overhead system, we have plotted these five trajectories in Fig. 2. For each trajectory, the profiles of velocity, acceleration and swing angle are shown in the figure.

3.1 Trajectory 1

Lee [8] proposes a smooth velocity profile for a smooth low-jerk motion. Lee's trajectory has been named Trajectory 1 as shown in Fig. 2. When the trolley reaches the destination, the velocity and acceleration become 0. As the

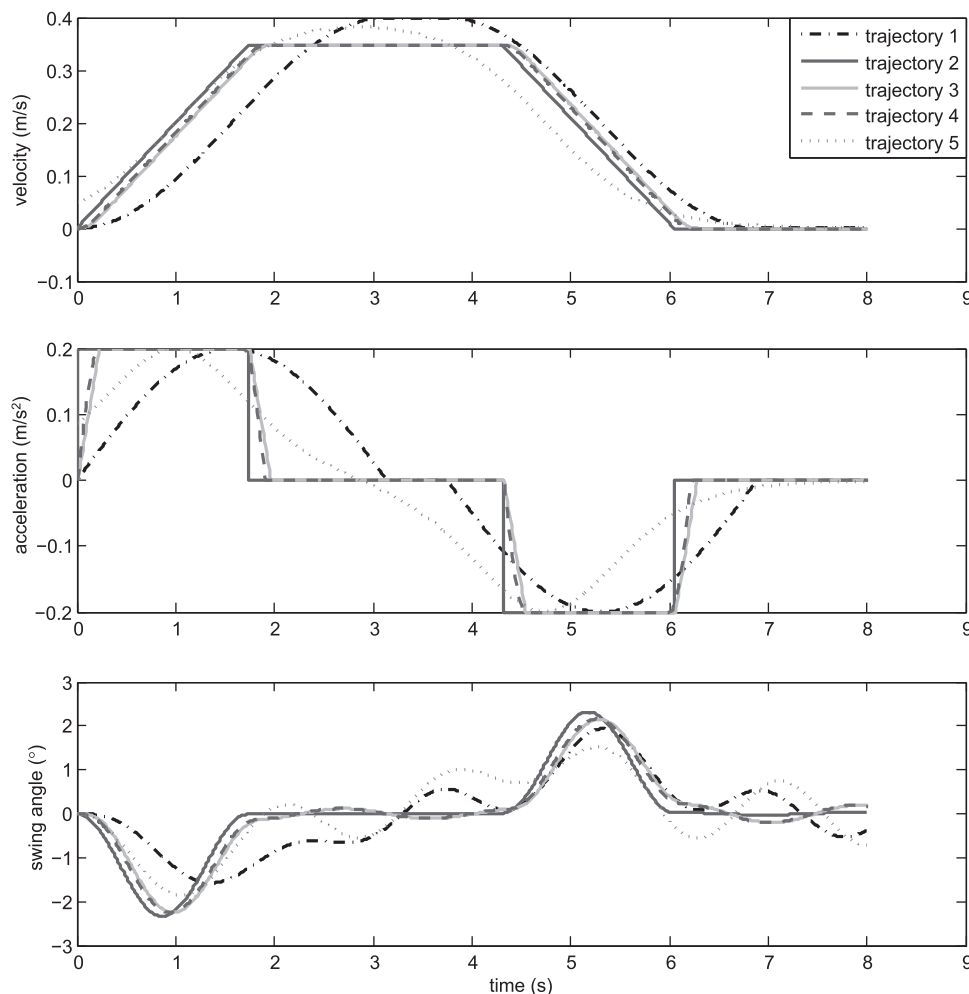


Fig. 2 Trajectory references in terms of velocity, acceleration and swing angle

swing suppression has not been considered in Trajectory 1, the residual swing is unacceptably large.

3.2 Trajectory 2

In [9, 30], a symmetric three-segment trajectory is proposed to eliminate the residual swing. From Fig. 2, it can be noticed that Trajectory 2 has shorter arrival time than the other four trajectories. It has zero residual swing, and the jerk at the time t_{2a} , $t_{2a} + t_{2c}$ and $2t_{2a} + t_{2c}$ is infinite, which implies a discontinuous control signal and hence potential damage to the actuating motor.

3.3 Trajectory 3

In Trajectory 3 [9], some linear ramp functions at the transition stage are inserted into the three-segment trajectory to ensure smooth motion. It can be noticed from Fig. 2 that the residual swing of Trajectory 3 is zero when the trolley stops. Unlike Trajectory 2 with infinite jerk, Trajectory 3 has jerk not larger than a predefined value during transportation.

3.4 Trajectory 4

Like in Trajectory 3, ramp functions in Trajectory 4 [9] have also been inserted into the transient stages. The main difference is that the ramp functions are not linear but S-shaped functions. For Trajectory 4, the profiles of velocity, acceleration and swing angle are given in Fig. 2. They are almost the same as the profiles of Trajectory 3 because of similar structures.

3.5 Trajectory 5

In [25], another S-curve trajectory is used as the reference for non-linear tracking. Trajectory 5 satisfies the physical constraints of velocity and acceleration. The obtained trajectory always has the positive velocity bounded to the permitted maximum of velocity; its acceleration is also bounded to the permitted maximum of acceleration. It can be noticed from Fig. 2 that the residual swing exists in Trajectory 5, and that the arriving time is longest among these referred trajectories. Another observation is that the initial velocity is not zero in Trajectory 5, so the acceleration profile of Trajectory 5 cannot be used in open-loop controllers for the static cranes. Only closed-loop controllers (such as PID or fuzzy controllers) can use this profile of acceleration (or velocity) as the reference for tracking. As this paper only considers motion planning when the initial velocity of crane is zero, Trajectory 5 is not included in the simulation section.

4 Optimal control models

The five trajectories mentioned above have analytical formulations and convenient setting of parameters, which are easily applied to some real-time implementations. Although some of them have the advantage of zero residual swing, their swing amplitudes have not been optimised. Among these five trajectories, Trajectory 2 has the shortest transportation time, which is still not the minimal one. Another important issue is the fact that the energy consumption has not been considered yet in these trajectories. Therefore it is necessary and meaningful to propose such trajectories that can minimise the transportation time, swing angle and

energy consumption. In this section, three optimal control strategies are proposed to provide optimal solutions in terms of transportation safety, energy and time efficiency. Each proposed model is a discrete model with the input of acceleration. Therefore the continuous system is first discretised by a sampling period t_0 . Over the evaluated planning period $T = N \cdot t_0$, the discrete system of the overhead crane can be formulated as (32) and (33)

$$(M + m)\Delta^2x(n) + ml \cos \theta(n)\Delta^2\theta(n) - ml \sin \theta(n)\Delta\theta(n)^2 = F(n) \tag{11}$$

$$l\Delta^2\theta(n) + \Delta^2x(n) + g\theta(n) = 0 \tag{12}$$

where $n = 1, \dots, N$; $x(n)$, $\theta(n)$ and $F(n)$ represent the displacement, swing angle and overall force at the n th sample, respectively. Δ and Δ^2 represent the first and second-order difference as

$$\begin{cases} \Delta q(n) = (q(n+1) - q(n))/t_0 \\ \Delta^2 q(n) = (\Delta q(n+1) - \Delta q(n))/t_0, \\ q(n) = [x(n), \theta(n)]^T \end{cases} \tag{13}$$

In the description of proposed models, we denote the input vector (acceleration) as \mathbf{a} ($a(n) = \Delta^2x(n)$), and denote the state vector of velocity as \mathbf{v} ($v(n) = \Delta x(n)$). When the initial state of the crane is static with zero swing, the velocity \mathbf{v} , the displacement \mathbf{x} and the swing angle θ can be expressed by the acceleration \mathbf{a} as

$$\begin{cases} \mathbf{a} = [\ddot{x}(1), \dots, \ddot{x}(N)]^T \\ \mathbf{v} = A_v \mathbf{a} t_0 \\ \mathbf{x} = A_x \mathbf{a} t_0^2 \\ \theta = A_\theta \mathbf{a} t_0^2 \end{cases} \tag{14}$$

where

$$A_v = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ 1 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} \tag{15}$$

$$A_x = \begin{bmatrix} 0.5 & 0 & 0 & \dots & 0 \\ 1.5 & 0.5 & 0 & \dots & 0 \\ 2.5 & 1.5 & 0.5 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N - 0.5 & N - 1.5 & N - 2.5 & \dots & 0.5 \end{bmatrix} \tag{16}$$

$$A_\theta = -\frac{1}{l}B^{-1} = -\frac{1}{l} \begin{bmatrix} 1 + \frac{gt_0}{l} & 0 & \dots & 0 & 0 & 0 \\ -2 & 1 + \frac{gt_0}{l} & \dots & 0 & 0 & 0 \\ 1 & -2 & \ddots & 0 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \ddots & 1 & -2 & 1 + \frac{gt_0}{l} \end{bmatrix}^{-1} \tag{17}$$

Note that the matrix B is non-singular. Equation (14) is deduced according to Newton's second law of motion. Based on the discrete system, three optimal control models are then introduced as follows:

(1) Swing-optimal model

The objective function of swing-optimal model is about safety over the horizontal transportation. Safety of an overhead crane can be reflected by two metrics, that is, the maximal swing angle and the residual swing. In the evaluated planning period T , the objective function will integrate both two metrics as

$$J_1(\mathbf{a}) = \alpha \max_{n \in \{1, \dots, N\}} \theta(n) + (1 - \alpha) \sum_{n=N-N_r+1}^N \frac{\theta(n)}{N_r} \quad (18)$$

where N_r is the number of samples considered in the residual swing. The first component of the right-hand side is the maximal swing angle, and the second component of the right-hand side is the residual swing. The coefficient α is used to integrate these two metrics. α is set to 0.5 in this paper. The residual swing is defined as the average swing angle during the final $N_r \cdot t_0$ period.

By substituting $\theta(n)$ with acceleration $a(n)$ using (14), the swing-optimal model (18) can be expressed by the acceleration. In this model, the input vector \mathbf{a} is bounded in the range $[-a_m, a_m]$ as

$$a(n) \in [-a_m, a_m], \quad n = 1, \dots, N \quad (19)$$

The constraints of this model include the equality constraints of final displacement, velocity and swing angle, and also include the inequality constraints of velocity, swing angle and jerk. The set of constraints $\mathcal{C}(N)$ is formulated as

$$\mathcal{C}(N) : \begin{cases} -a_m \leq a(n) \leq a_m, & n = 1, \dots, N \\ x(N) = x_p \\ v(N) = 0 \\ \theta(N) = 0 \\ \theta(N - 1) = 0 \\ 0 \leq v(n) \leq k_v, & n = 1, \dots, N \\ |\theta(n)| \leq \theta_{\max}, & n = 1, \dots, N \\ |a(n + 1) - a(n)| \leq j_m, & n = 1, \dots, N - 1 \\ |a(1)| \leq j_m \\ |a(N)| \leq j_m \end{cases} \quad (20)$$

To minimise the objective function (18) under the constraints, the solution is expected with the highest degree of safety.

(2) Energy-optimal model

The objective function of energy-optimal model is energy consumption over the horizontal transportation. Within the planning period T , energy consumption can be calculated as

$$E = \int_0^T P dt = \int_0^T F_a \dot{x} dt \quad (21)$$

where F_a is the output force of the actuating motor, P is the power of the actuating motor and E is the energy consumption of the motor. For the discrete system, the proposed

energy-optimal model can be formulated as

$$J_2(\mathbf{a}) = \sum_{n=1}^N F_a(n)v(n) = \sum_{n=1}^N [F(n) + F_r(n)]v(n) \quad (22)$$

where the overall force $F(n)$ is computed as (11), and the friction $F_r(n)$ can be formulated similarly with (4) as

$$F_r(n) = f_{r0}(\tanh v(n)/\xi) - k_r|v(n)|v(n) \quad (23)$$

By substituting $v(n)$ and $\theta(n)$ with $a(n)$ using (14), the energy consumption can be expressed by the acceleration \mathbf{a} . The input vector is also bounded in $[-a_m, a_m]$. The energy optimal control model has the same constraints $\mathcal{C}(N)$ as the swing optimal control model. For the energy-optimal model, we aim to minimise the energy consumption in (22) under the practical constraints.

(3) Time-optimal model

The objective function of time-optimal model is equivalent with the transportation time. If the transportation time is denoted as $t_f = N_f \cdot t_0$, then the time-optimal model can be formulated as

$$J_3(\mathbf{a}) = N_f \quad (24)$$

It is difficult to express N_f by the acceleration \mathbf{a} . We can only determine N_f by checking the feasibility of a specific acceleration profile. $\mathcal{C}(i)$ denotes constraints that must be satisfied when the transportation can be completed at $i \cdot t_0$. $\mathcal{C}(i)$ is formulated as

$$\mathcal{C}(i) : \begin{cases} -a_m \leq a(n) \leq a_m, & n = 1, \dots, i \\ x(i) = x_p \\ v(i) = 0 \\ \theta(i) = 0 \\ \theta(i - 1) = 0 \\ 0 \leq v(n) \leq k_v, & n = 1, \dots, i \\ |\theta(n)| \leq \theta_{\max}, & n = 1, \dots, i \\ |a(n + 1) - a(n)| \leq j_m, & n = 1, \dots, i - 1 \\ |a(1)| \leq j_m \\ |a(i)| \leq j_m \end{cases} \quad (25)$$

If the profile satisfies the constraints $\mathcal{C}(i)$, the transportation time is not longer than $i \cdot t_0$. N_f can be determined in a backward way (shown in Fig. 3) or a bisection way (shown in

Algorithm 1

- 1 Check the feasibility on constraints $\mathcal{C}(N)$
- 2 **if** $\mathcal{C}(N)$ is satisfied **then**
- 3 $i \leftarrow N$
- 4 **while** $\mathcal{C}(i)$ is satisfied **do**
- 5 $i \leftarrow i - 1$;
- 6 Check the feasibility on constraints $\mathcal{C}(i)$;
- 7 **end**
- 8 $N_f \leftarrow i + 1$;
- 9 **else**
- 10 N_f is null;
- 11 **end**

Fig. 3 Backward way to determine the transportation time

Algorithm 2

```

1 Check the feasibility on constraints  $\mathcal{C}(N)$ 
2 if  $\mathcal{C}(N)$  is satisfied then
3    $c_1 \leftarrow 0, c_2 \leftarrow N;$ 
4   while  $c_2 - c_1 \leq \epsilon$  do
5      $c \leftarrow \lfloor \frac{c_1+c_2}{2} \rfloor;$ 
6     Check the feasibility on constraints  $\mathcal{C}(c)$ 
7     if  $\mathcal{C}(c)$  is satisfied then
8        $c_2 \leftarrow c$ 
9     else
10       $c_1 \leftarrow c$ 
11    end
12  end
13   $N_f \leftarrow c_2$ 
14 else
15    $N_f$  is null;
16 end

```

Fig. 4 Bisection way to determine the transportation time

Algorithm 3

```

1 Search a feasible solution on constraints  $\mathcal{C}(N)$ 
2 if a feasible solution can be found then
3    $c_1 \leftarrow 0, c_2 \leftarrow N;$ 
4   while  $c_2 - c_1 \leq \epsilon$  do
5      $c \leftarrow \lfloor \frac{c_1+c_2}{2} \rfloor;$ 
6     Search a feasible solution on constraints  $\mathcal{C}(c)$ 
7     if a feasible solution can be found then
8        $c_2 \leftarrow c$ 
9     else
10       $c_1 \leftarrow c$ 
11    end
12  end
13  The minimum of  $N_f$  is  $c_2;$ 
14 else
15   The algorithm cannot obtain any solution in the planning period;
16 end

```

Fig. 5 Bisection searching algorithm

Fig. 4) to repeatedly check feasibility. If the transportation time is close enough to the planning period T , the backward algorithm can find N_f effectively. Otherwise, the bisection algorithm is a better choice than the backward algorithm. Although N_f can be known for any profile of acceleration, it is still difficult to find an efficient direction of decreasing N_f . In this paper, we propose a bisection searching algorithm for the time-optimal profile to search the minimal transportation time (shown in Fig. 5). First, search a feasible solution satisfying constraints $\mathcal{C}(N)$. Note that the interval of searching is $[0, N]$. Second, check whether a feasible solution can be found at the midpoint of interval and update the interval as shown in Fig. 5. Third, repeat the second step iteratively until the tolerance error is satisfied.

5 Numerical simulation

The overhead crane system described in [9] is used to test our proposed models and the reference Trajectories 1–4. The physical parameters of the system are listed as follows

$$m = 1.025 \text{ kg}, \quad M = 7 \text{ kg}, \quad l = 0.75 \text{ m}, \quad g = 9.8 \text{ m/s}^2 \quad (26)$$

The desired trolley location in simulation is set as $p_d = 0.6 \text{ m}$, and the practical constraints are given as

$$v_m = 0.4 \text{ m/s}, \quad a_m = 0.2 \text{ m/s}^2, \quad \theta_m = 2^\circ, \quad j_m = 5 \text{ m/s}^3 \quad (27)$$

The parameters for the friction model (4) are referred from the offline regression results in [28] as

$$f_{r0} = 4.4, \quad k_r = -0.5, \quad \xi = 0.01 \quad (28)$$

The simulation is conducted on a PC computer with Intel Core i7 CPU (3.4 GHz) and 8 GB RAM. On the 64-bit Win-7 system, Matlab 2013 is installed as the software platform of coding and simulation.

Test 1: Comparisons of swing

The evaluated planning period T is 6 s, and the sampling period t_0 is 0.01 s. The duration of residual swing is defined as the last 2 s, that is, $N_r = 200$. The optimisation algorithm for solving the swing-optimal model is chosen as the *fmincon* function in Matlab toolbox. Note that *fmincon* is a simple example of solver in our simulations, other more complicated solvers may also be employed instead of it here. In the *fmincon* function, the algorithm type is set as ‘interior-point’ and the maximum function evaluation times are $50 \cdot N$.

For each compared trajectory, the results of maximum swing angle, residual swing and their average are listed in Table 1. The swing-optimal trajectory has the smallest maximal swing angle 0.5278° . Although the residual swing of the optimal trajectory is larger than Trajectories 2, 3 and 4, but the average value of the maximal swing and the residue swing is the smallest among the compared trajectories. In Fig. 6, the profiles of acceleration and swing angle are given. Owing to the jerk constraint, the optimal trajectory has a smooth acceleration profile. In Fig. 6, the same conclusion can be drawn namely that the maximum and average swing angles are smallest and the residual swing is close to zero. The profiles of acceleration and swing angle when $T = 12 \text{ s}$ are also given. The conclusion is clearer that because of longer planning period the maximum swing angle is much smaller than Trajectories 2, 3 and 4, and that the residual swing is close to zero.

In general, a long duration of transportation, that is, a low average velocity, may reduce the swing of the payload. This can be verified by the following experiment to evaluate different planning periods, that is, 4, 6, 8, 10 and 12 s. The resulting profiles of acceleration and swing angle are given in Fig. 7. The maximal swing angle of 4 s period is the largest at 1.623° , and the maximum swing of 12 s period is the smallest at 0.319° . It can be concluded that the maximal swing is closely related to the evaluated planning period. The maximal swing decreases when the transportation period increases, but the change of maximal swing is small when planning period is larger than 10 s.

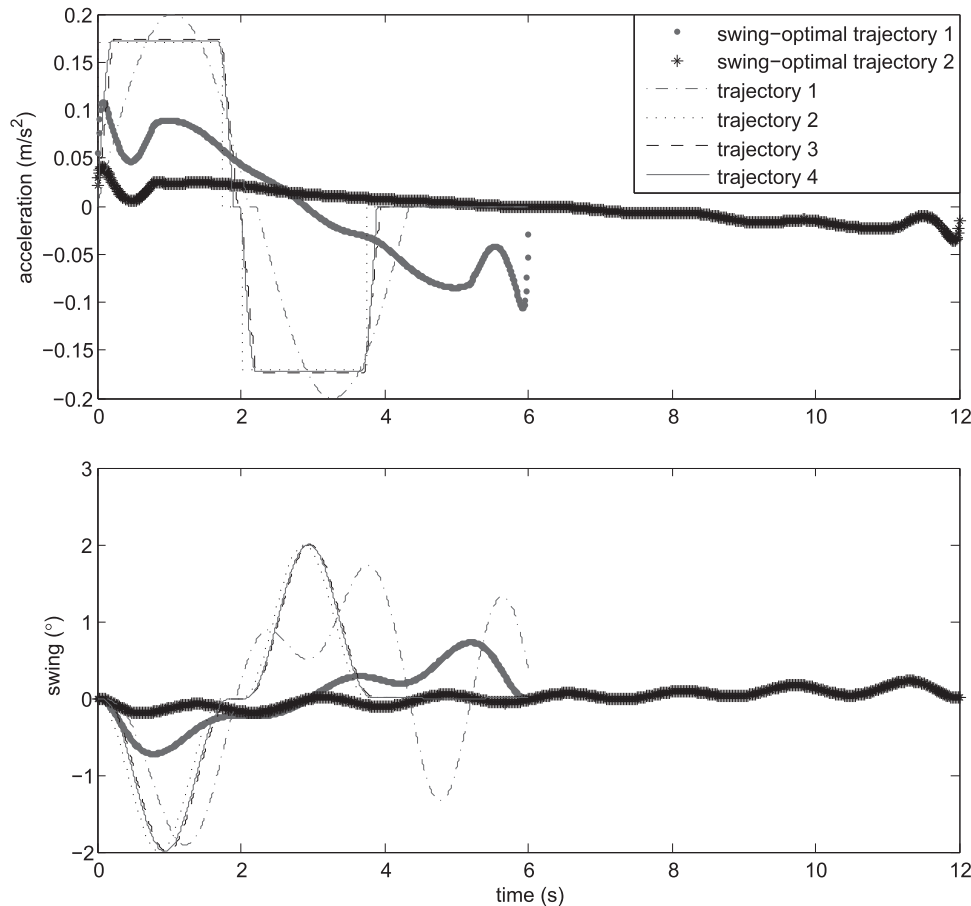
Test 2: Comparisons of energy efficiency

The evaluated planning period T is 6 s, and the sampling period t_0 is 0.01 s. The optimisation algorithm for solving the energy-optimal control model is chosen as the *fmincon* function in the Matlab toolbox. In the *fmincon* function, the algorithm type is set as ‘interior-point’ and the maximum function evaluation times are $50 \cdot N$.

In Table 2, energy consumption and peak power during the entire transportation are listed for each trajectory. For the

Table 1 Comparisons of swing

Trajectory	Swing-optimal	1	2	3	4
maximal swing, deg.	0.5278	1.912	2.000	2.001	2.001
residual swing, deg.	0.3600	0.8634	7.248×10^{-3}	6.622×10^{-3}	6.853×10^{-3}
average, deg.	0.4439	1.388	1.003	1.004	1.004

**Fig. 6** Trolley acceleration profile and the payload swing angle for the swing-optimal model: swing-optimal Trajectory 1 with $T = 6$ s; swing-optimal Trajectory 2 with $T = 12$ s

energy-optimal model, the lowest energy consumption and peak load (as bolded in the table) can be obtained. However, the energy saving in this simulation is not large. One reason found is that the parameters of system referred from [9] are relatively small. When these parameters, such as maximum velocity, acceleration and rope length, turn larger and energy saving becomes more promising. For this energy-optimal trajectory, the maximum swing angle is 0.8948° with the residue swing 0.2994° , and the average is 0.5971° . Profiles of acceleration, swing angle and power when $T = 6$ and 12 s are given as in Fig. 8. Compared with swing angles of Trajectories 1–4, energy-optimal trajectories still have smaller maximum and average swing angle as shown in the figure. As shown in the figure, the maximum swing angle of $T = 12$ s turns to be smaller than $T = 6$ s because of smoother acceleration. It can also be noticed that the optimal trajectory has a smooth power consumption while the other compared trajectories are not smooth. Although the optimal trajectory has a longer transportation time, the sacrifice of time efficiency is paid back with the improvement of energy efficiency.

Table 2 Comparisons of energy efficiency

Trajectory	Energy-optimal	1	2	3	4
energy usage, J	2.626	2.651	2.653	2.652	2.653
peak power, W	0.702	1.323	1.647	1.602	1.589

The relation of energy efficiency and time efficiency has not been clearly stated in the literature. In this section, an experiment with different planning periods has been done to evaluate such relation. Planning periods are set to be 4, 6, 8, 10 and 12 s, respectively. Then after solving the energy-optimal model over each period, energy consumptions can be obtained. When the period is 4 s, energy consumption is the largest at 2.6534 J. When the period is 12 s, energy consumption is the smallest at 2.5777 J. When the planning period obtains larger than 10 s, the change of peak power is

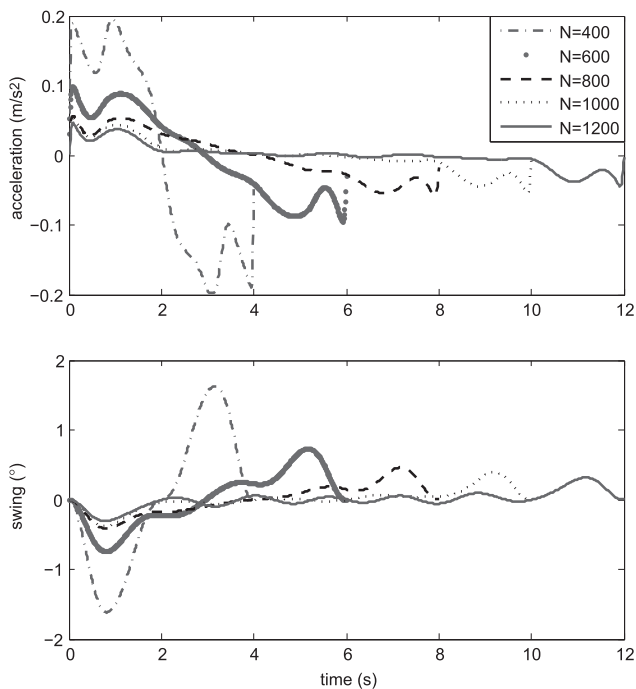


Fig. 7 Profiles with different planning periods for the swing-optimal model

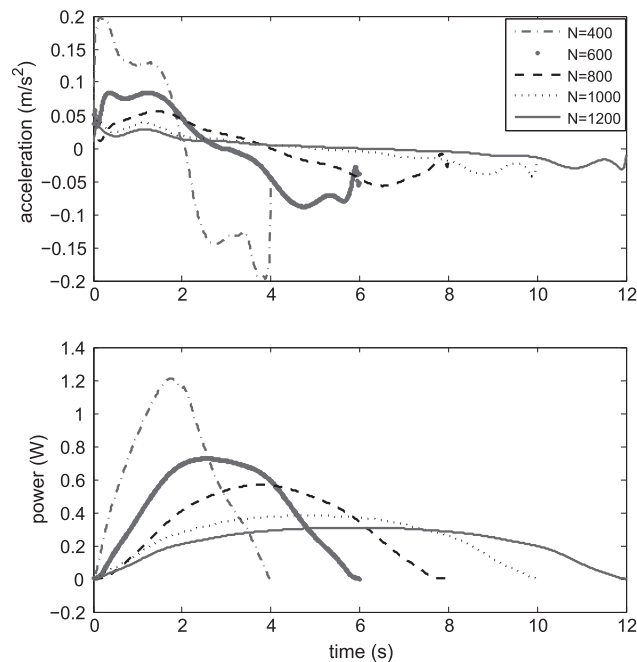


Fig. 9 Profiles with different planning periods for the energy-optimal model

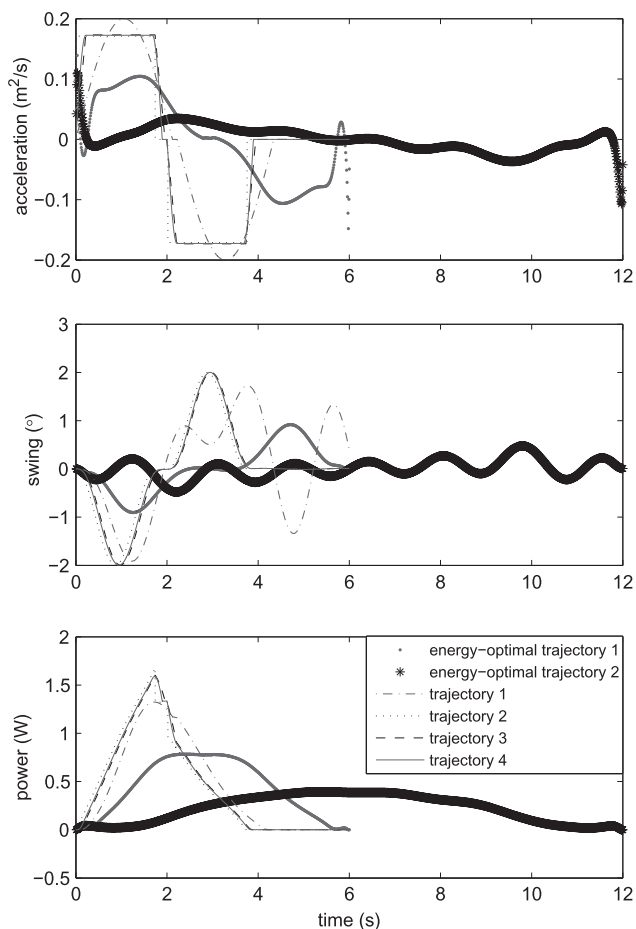


Fig. 8 Profiles of acceleration, swing angle and power for the energy-optimal model: energy-optimal Trajectory 1 with $T = 6$ s; energy-optimal Trajectory 2 with $T = 12$ s

close to zero and the energy consumption has slow or no decrease as shown in Fig. 9.

Test 3: Comparisons of time efficiency

The evaluated transportation period T is 6 s, and the sampling period t_0 is 0.01 s. The optimisation algorithm for solving the time-optimal control model is chosen as the *fmincon* function in the Matlab toolbox. In the *fmincon* function, the maximum function evaluation times are $50 \cdot N$. The algorithm type is set as ‘active-set’, because ‘active-set’ is suitable to find a feasible solution for the time optimal model. The number of iterations in the bisection algorithm is set to 10.

As mentioned in Section 3, Trajectory 2 has the shortest transportation time among the five trajectories reviewed, but the jerk at certain time is infinite. For the tested crane system, the transportation time of Trajectory 2 is 3.66 s. When ignoring the jerk constraint in our time optimal control model, we can obtain an optimal trajectory with

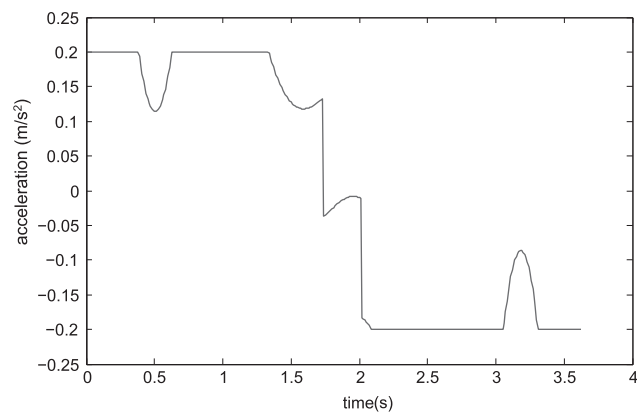


Fig. 10 Profiles of acceleration without the jerk constraint for the time-optimal model

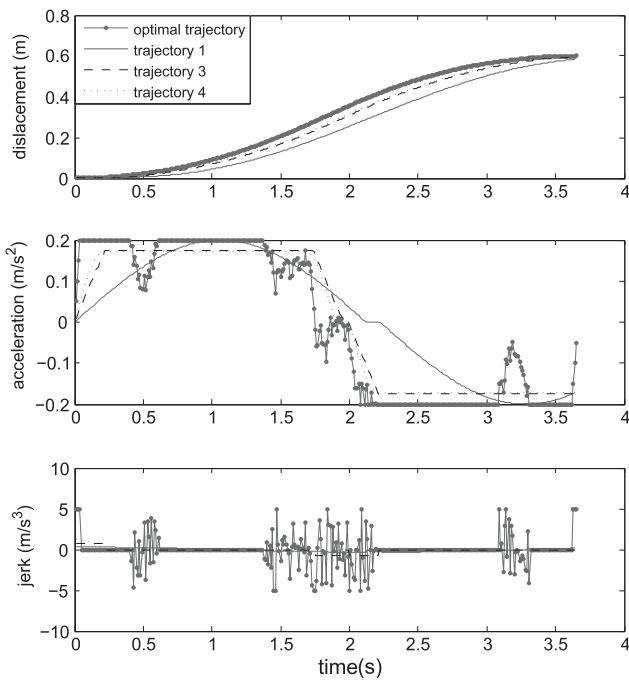


Fig. 11 Profiles of acceleration with jerk constraints for time-optimal model

transportation time of 3.62 s. The profile of acceleration is given in Fig. 10.

The optimal trajectory with the jerk constraint ($j_m = 5$) is given in Fig. 11. The optimal time of transportation is 3.65 s, while the transportation time of Trajectories 1, 3 and 4 is 4.08, 3.76 and 3.73 s, respectively. Fig. 11 shows profiles of displacement, acceleration and jerk. The crane using the optimal trajectory can arrive at the destination earlier than using other trajectories. It can be noticed that the jerk of the optimal trajectory never violates the permitted maximal jerk, which can be set by users according to different requirements. For example, when the actuating motor of the crane system is retrofitted, the maximum of jerk can be changed according to the performance of the new motor.

6 Conclusions

In view of safety, energy efficiency and time efficiency, three optimal motion planning strategies for overhead cranes have been proposed to provide three trajectories with the lowest swing, energy consumption and transportation time, respectively. These optimal planning strategies have considered practical and physical constraints such as maximal swing and jerk. Note that these trajectories obtained by offline computation can be utilised as inputs of open-loop control schemes, or as tracking references of closed-loop control schemes.

In this paper, energy efficiency of crane is newly modelled under practical and physical constraints, including maximal swing and jerk. The result of the energy-optimal model can reduce energy consumption and peak power, which may be useful in the incentive electricity market. The time and swing-optimal strategies can also achieve better performance than existing strategies in terms of time efficiency and safety. In practise, decision makers can choose one optimal strategy among these three proposed ones depending on the requirements of work. As these three objectives

are competing, we have only considered each of them separately in this paper. Methods of integrating three objectives can be reasonably extended, but that is beyond the scope of this paper. In future work, the phases of hoisting and lowering will be combined with the horizontal transportation for complete motion planning using the proposed methods, and the closed-loop methods may be proposed for tracking the planned trajectory and updating the trajectory online. The proposed strategies of motion planning are developed on two-dimensional crane systems, and they are possibly extended on three or more dimensional systems as part of future work.

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