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Effects of trends and seasonalities on robustness of the Hurst parameter estimators

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Abstract: Long-range dependence (LRD) is discovered in time series arising from different fields, especially in network traffic and econometrics. Detecting the presence and the intensity of LRD plays a crucial role in time-series analysis and fractional system identification. The existence of LRD is usually indicated by the Hurst parameters. Up to now, many Hurst parameter estimators have been proposed in order to identify the LRD property involved in a time series. Since different estimators have different accuracy and robustness performances, in this study, 13 most popular Hurst parameter estimators are summarised and their estimation performances are investigated. LRD processes with known Hurst parameters are generated as the control data set for the robustness evaluation. In addition, three types of LRD processes are also obtained as the test signals by adding noises in terms of means, trends and seasonalities to the control data set. All 13 Hurst parameter estimators are applied to these LRD processes to estimate the existing Hurst parameters. The estimation results are documented and quantified by the standard errors. Conclusions of the accuracy and robustness performances of the estimators are drawn by comparing the estimation results.

1 Introduction

The study of long-range dependence (LRD) has received considerable attention in diverse research areas, such as agronomy, astronomy, chemistry, economics, engineering, environmental science, geoscience, hydrology, mathematics, physics and statistics [1]. The LRD phenomenon is known as the dependence between observations far away in time. The presence and intensity of LRD are traditionally measured by the Hurst parameters, H, introduced by Hurst [2] during his studies on Nile discharges and problems related to water storage. The H parameter ranges in (0, 1). From a physical point of view, H is a measure of roughness; the roughness or anti-correlation in the signal is maximal when H is close to zero. White noise with zero correlation has $H = 0.5$. Smoother correlated signals have H near 1.0 [3].

The Hurst parameter has a close relationship with power law, long memory, fractal, fractional calculus and chaos theory. Detection of LRD is crucial to time-series analysis, especially to fractional system identification and prediction [4]. Many methods for estimating H are proposed in the literature. For instance, the oldest and most common method is the re-scaled range (R/S) method [5]. The aggregated variance method [6] is based on a dispersional analysis. The periodogram method [7] is the linear regression of the log periodogram. The Whittle estimator [8] is obtained by minimising the objective function based on the periodogram. Abry and Veitch's [9] method is a wavelet-based analysis tool of H. Higuchi's [10] method is based on fractal theory. Most of the above-mentioned estimators are based on linear regression with graphical analysis, except the whittle estimator.

Different estimators have different accuracy and robustness performances. Previous studies [4, 6, 8, 11], have made intensive efforts to compare the accuracy and robustness of the existing Hurst parameter estimators. In [4], 12 Hurst parameter estimators are analysed to compare their robustness against three kinds of noises, namely the 30 dB signal-to-noise ratio (SNR) white Gaussian noise, 30 dB SNR stable noise and fractional autoregressive integrated moving average (FARIMA) with stable innovations. Taqqu *et al.* [6], summarise nine Hurst parameter estimators and compare their performances when they are applied to both the fractional Gaussian noise (fGn) and FARIMA(0, d, 0) processes at some determined Hurst parameter values. Taqqu and Teverovsky [8] concentrate on comparing the robustness of the Whittle-type estimators; both the Gaussian innovations and the infinite-variance symmetric stable innovations are considered. In Ref., [11], the robustness of Hurst parameter estimators for noisy multifractional processes, and multifractional processes with infinite second-order statistics is tested and analysed.

However, none of these studies clearly provides sufficient information for selecting the most suitable Hurst parameter estimators to detect the LRD properties involved in a time series with the presence of means, trends and seasonalities. In this study, four different LRD processes, namely, LRD process with known Hurst parameters, LRD process with non-zero means, LRD process with linear trends and LRD process with seasonalities are generated to evaluate the robustness of existing Hurst parameter estimators. In the following, 13 most popular existing Hurst parameter estimators are documented and investigated: (i) R/S method [5]; (ii) aggregated variance method [12]; (iii) difference variance method [6]; (iv) absolute value method [5]; (v) variance of residuals method [6]; (vi) periodogram method [7]; (vii) modified periodogram method [13]; (viii) Whittle estimator [8]; (ix) diffusion entropy method [14]; (x) Kettani and Gubner's [15] method; (xi) Abry and Veitch's [9] method; (xii) Koutsoyiannis' [16] method; (xiii) Higuchi's [10] method. A brief summary for all the above-mentioned 13 Hurst parameter estimators can be found in [4]. These estimators are applied to the four LRD processes. The estimation results are quantified by the standard errors. The robustness evaluation results show that the presence of trends and seasonalities has an essential influence on most of the 13 Hurst parameter estimators. However, Abry and Veitch's method exhibits strong robustness to the trends, whereas the Whittle estimator is not vulnerable to seasonalities.

This paper is organised as follows. In Section 2, preliminary studies on LRD property and fGn processes are reviewed. In Section 3, the test signals are generated and Hurst parameter estimator evaluation procedures are provided. Subsequently, the performances of the 13 estimators against means, trends and seasonalities are investigated and the results are given in both graphic styles and quantified by the standard errors in Section 4. The performances of the 13 estimators are summarised in Section 5.

2 Preliminaries

Preliminaries on LRD and fGn are essential for this study. Detailed descriptions of these two concepts are provided below.

2.1 Long-range dependence

A stationary process with finite second-order statistics is said to have LRD if its covariance function $C(n)$ decays slowly as $n \to \infty$. That is, there exists an α, $0 < \alpha < 1$, such that

$$\lim_{n\to\infty} \frac{C(n)}{n^{-\alpha}} = c \tag{1}$$

where c is a finite, positive constant. That is to say, for large n, $C(n)$ is similar to c/n^{α} [17]. The parameter α has a relationship to H as $\alpha = 2 - 2H$. The LRD can also be defined by the spectral density. A weak stationary time series X_i is said to be long-range dependent if its spectral density follows

$$f(\lambda) \sim C_f |\lambda|^{-\beta} \tag{2}$$

as $\lambda \to \infty$, for certain $C_f > 0$ and real parameter $\beta \in (0, 1)$. The parameter β is related to the Hurst parameter by $H = (1 + \beta)/2$ [18]. For $0.5 < H < 1$, the process has LRD, for $H = 0.5$ the observations are uncorrelated, and for $0 < H < 0.5$ the process has short-range dependence and the correlations sum up to zero.

2.2 Fractional Gaussian noise

Before testing the Hurst parameter estimators, some control data set with known Hurst parameters are reqired. The control data are better synthesised from the first principle of fractional Brownian motion (fBm), which is a Gaussian process as defined in [19]. Successive increments of an fBm are called fGn [19], which is defined as follows.

Let X_i denote a time series. Then X_i is second-order stationary if its mean value $E(X_i)$ does not depend on i and if the auto-covariance function $E[(X_i - E(X_i))(X_j - E(X_j))]$ depends on i and j only through their difference $k = i–j$, in which case one has

$$\gamma(k) = E[(X_i - E(X_i))(X_{i-k} - E(X_{i-k}))] \tag{3}$$

The variance of the process is $\sigma^2 = \gamma(0) = E[(X_i - E(X_i))^2]$, and the autocorrelation function is $\rho(k) = \gamma(k)/\sigma^2$. A second-order stationary process is said to be exactly second-order self-similar with Hurst exponent $H \in (0, 1)$ if

$$\gamma(k) = (\sigma^2/2)(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}) \tag{4}$$

or equivalently

$$\rho(k) = \frac{1}{2}(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}) \tag{5}$$

If X_i is a Gaussian process, then it is known as an fGn.

3 Test signal generation and Hurst parameter estimator evaluation procedures

3.1 Test signal generation

In order to evaluate the robustness of the 13 Hurst parameter estimators against means, trends and seasonalities, test signals that exhibit the LRD properties with known Hurst parameters are required as control data sets. For this purpose, fGn time series, denoted by $F_k(t, H_j)$ with the standard deviation $\sigma = 1$ and known Hurst parameters H_j are adopted to generate the test signals. Four LRD processes are produced for the robustness evaluation for the 13 estimators. As shown in (6)

$$\tilde{F}_k(t, \hat{H}_j) = F_k(t, H_j) + a + bt + c\sin(2\pi ft) \tag{6}$$

where $\hat{H}_j$ denotes the Hurst parameters that need to be characterised from the following four LRD processes that are produced by adjusting the coefficients a, b and c in (6).

1. *LRD 1:* LRD processes with known Hurst parameters $\tilde{F}_k(t, \hat{H}_j) = F_k(t, H_j)$;
2. *LRD 2:* LRD processes with non-zero means $\tilde{F}_k(t, \hat{H}_j) = F_k(t, H_j) + a$;
3. *LRD 3:* LRD processes with linear trends $\tilde{F}_k(t, \hat{H}_j) = F_k(t, H_j) + bt$;
4. *LRD 4:* LRD processes with seasonalities

$$\tilde{F}_k(t, \hat{H}_j) = F_k(t, H_j) + c\sin(2\pi ft)$$

H_j increases from 0.01 to 0.99 in steps of 0.01, $j = 1, 2, \ldots, 99$; for the estimator evaluation, each of the four LRD

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processes will be generated 100 times for every H_j, where $k = 1, 2, \ldots, 100$ represents the kth generated LRD process; $t = 1, 2, \ldots, 8760$ since the length of an hourly sampled time series for 365 days in a year is $365 \times 24 = 8760$. LRD 1 represents the LRD processes with known Hurst parameters, LRD 2–4 are the LRD processes with additive noises in terms of means, trends and seasonalities, respectively. It is noted that during the evaluation process of the 13 estimators, the mean, trend and seasonality noises will only be added separately.

3.2 Hurst parameter estimator evaluation procedures

In this section, detailed Hurst parameter estimator robustness evaluation procedures are provided as shown in the flowchart in Fig. 1. For LRD 1, the Hurst parameters are estimated by different Hurst parameter estimators by the following steps:

Step 1: Let $j = 1$, $k = 1$ and the temporary variable $H_s = 0$.
Step 2: In the inner loop of the flowchart, 100 replications of each of the four LRD processes are generated. For the kth generated LRD process, $\hat{H}_j$ is estimated by a certain estimator, and then summed up and stored in H_s.
Step 3: The final estimated Hurst parameter $\bar{H}_j$ is the average of the 100 estimates of $\hat{H}_j$.
Step 4: For the outer loop, Steps 2 and 3 will repeat 99 times until all $\bar{H}_j$s are obtained.

During the robust evaluation for each Hurst parameter estimator, $\hat{H}_j$ of LRD 1 is taken as a benchmark for comparison purpose. By following similar evaluation procedures, the final estimated Hurst parameters of LRD 2–4 can also be obtained. The impact of means, trends and seasonalities to the 13 estimators can be characterised by comparing the final Hurst parameter estimates between LRD 1 and LRD 2–4. For LRD 4, daily seasonalities with frequency $f = 1/24$ are generated to add into the LRD 1 processes for the robustness evaluation of the 13 estimators. In order to achieve a fair comparison of the estimation performances for the 13 Hurst parameter estimators, the expectations of a, b and c for LRD 2–4 are calculated, respectively, as provided in Table 1, where N is suggested

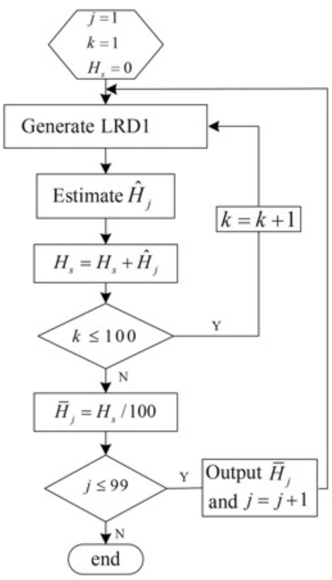

Fig. 1 *Flowchart of the Hurst parameter estimator evaluation*

Table 1 Initial coefficients for the evaluation

LRD processes	Coefficients in (6)
LRD 1	$a = 0$, $b = 0$, $c = 0$
LRD 2	$a = (1/N)\sum_{k=1}^{N}[\max(F_k) - \min(F_k)]$ $b = 0$, $c = 0$
LRD 3	$a = 0$, $c = 0$ $b = (1/(N \times 8760))\sum_{k=1}^{N}[\max(F_k) - \min(F_k)]$
LRD 4	$a = 0$, $b = 0$, $f = 1/24$ $c = (1/N)\sum_{k=1}^{N}[\max(F_k) - \min(F_k)]$

to be a sufficient large number, say 1 000 000; F_k is short for $F_k(t, 0.5)$.

4 Robustness assessment and comparison

In this section, the robustness evaluation results of the 13 Hurst parameter estimators are provided from Figs. 2–14. In these figures, the solid lines (in black) are the known Hurst parameter values H_j as the reference lines. In order to indicate the severity of the estimation bias, two bias threshold $H_j \pm 0.03$, denoted by solid lines (in grey) are also plotted in all these figures. The estimated Hurst

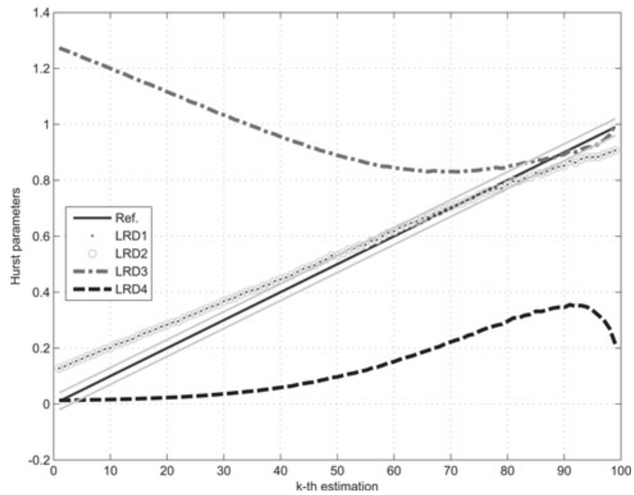

Fig. 2 *Estimates of LRD 1–4 by R/S method*

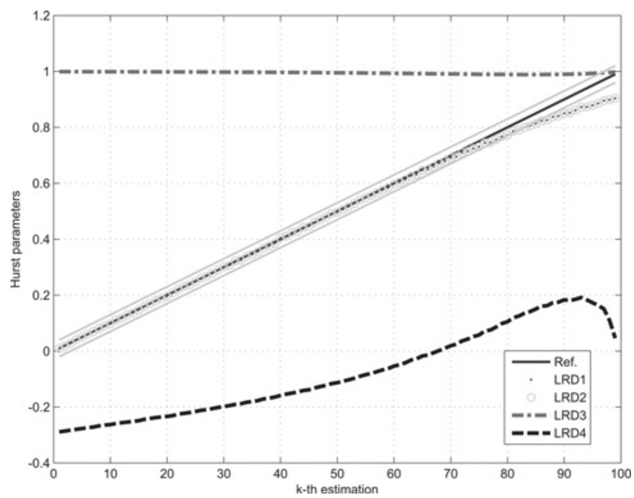

Fig. 3 *Estimates of LRD 1–4 by aggregated variance method*

parameters $\hat{H}_j$ of different LRD processes are denoted by lines with different styles and colours. Specifically, the estimates of LRD 1–4 are represented by a dotted line, a circle line, a dash–dotted line and a dashed line, respectively.

4.1 R/S method

The estimates by the R/S method for the LRD 1–4 processes are presented in Fig. 2. The dotted line and the circle line coincide with each other, which shows that the mean value does not severely affect the R/S estimator. However, the estimates for LRD 1 and 2 are biased for almost all Hurst parameter values ($0 < H < 1$). $\hat{H}$ is apparently overestimated at around $H \in (0, 0.6]$ and underestimated at around $H \in [0.8, 1)$. In addition, the dash–dotted line shows that the estimated Hurst parameters are severely overestimated. Nevertheless, the dashed line is severely underestimated. It indicates that the R/S method is robust neither to the trends nor to the seasonalities.

4.2 Aggregated variance method

The estimates by the aggregated variance method for the LRD 1–4 processes are presented in Fig. 3. The overlap of the dotted line and the circle line shows that the mean value does not influence the aggregated variance estimator. $\hat{H}$ of LRD 1 and 2 is slightly biased at around $H \in (0, 0.7)$. However, an underestimation appears at around $H \in [0.7, 1)$. However, the dash–dotted line is generally close to 1.0 and the dashed line is far underestimated. It shows that this estimator has poor robustness performance to LRD processes with trends and seasonalities.

4.3 Difference variance method

The estimates by the difference-variance method for the LRD 1–4 processes are presented in Fig. 4. The Hurst estimates for LRD 1 and 2 are generally underestimated and overlap with the lower bound of the bias threshold in the range $0 < H < 1$. Therefore the mean value has no influence on this estimator. However, as shown by the dash–dotted line, the estimates are obviously overestimated at around $H \in (0, 0.6) \cup (0.9, 1)$. On the contrary, the estimates are underestimated at around $H \in [0.6, 0.9]$. Moreover, as shown by the dashed line, the seasonalities impose a greater impact on this estimator, and $\hat{H}$ is extremely biased. Thus, the difference-variance method is severely affected by trends and seasonalities.

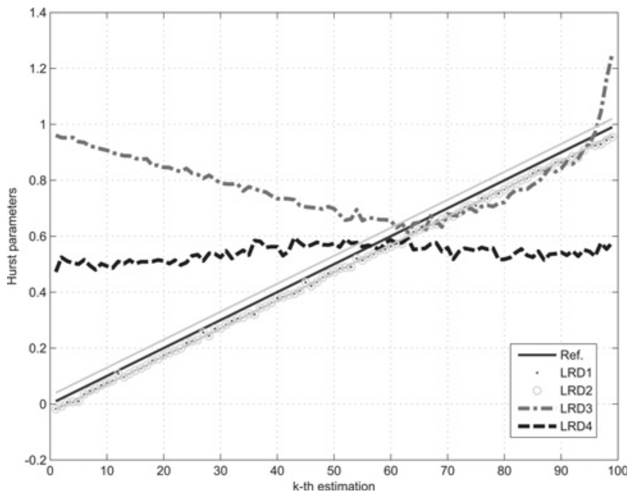

Fig. 4 *Estimates of LRD 1–4 by difference-variance method*

4.4 Absolute value method

The estimates by the absolute value method for the LRD 1–4 are presented in Fig. 5. The estimation performance of the absolute value method is very similar to the aggregated variance method. The estimates are nearly unbiased when H is around (0, 0.7) but underestimated when H is around [0.7, 1) for both LRDs 1 and 2. The dash–dotted line is severely overestimated and the dashed line is always under the reference line. Thus, the absolute value method exhibits poor estimation performance to trends and seasonalities.

4.5 Variance of residuals method

The estimates by the variance of residuals method for the LRD 1–4 are presented in Fig. 6. The dotted line and the circle line show that the variance of residuals method estimates the Hurst parameters for both LRD 1 and 2 accurately given that all estimates lie within the range of the $H \pm 0.03$ bias thresholds (solid line in grey). However, the trends influence this estimator since overestimation occurs at around $H \in (0, 0.8)$ as shown by the dash–dotted line,

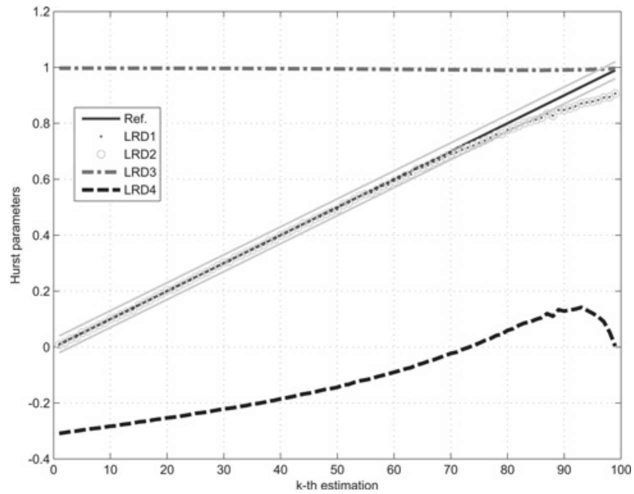

Fig. 5 *Estimates of LRD 1–4 by absolute value method*

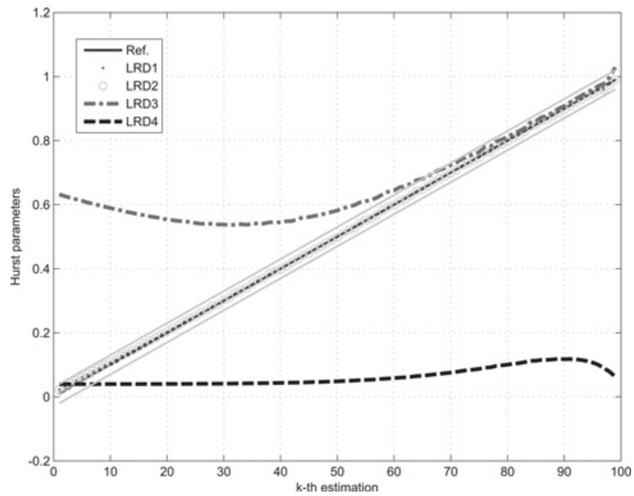

Fig. 6 *Estimates of LRD 1–4 by variance of residuals method*

whereas only slightly biased estimation appears when H is around [0.8, 1). The influence of the seasonalities is more severe, as shown by the dashed line. The estimator can hardly give accurate estimates of LRD 4.

4.6 Periodogram method

The estimates by the periodogram method for the LRDs 1–4 are presented in Fig. 7. The overlap of the dotted line and the circle line shows that the mean value does not affect this estimator. The estimates are slightly biased when H is around (0, 0.2) for LRDs 1 and 2. However, the linear trends make the estimation performance poor. As shown by the dash–dotted line, the Hurst parameters are severely overestimated. In addition, it can be observed that the seasonalities do not affect this estimator. The estimates denoted by the dashed line are slightly biased when H is around (0.2, 1), only a little underestimated when H is around (0, 0.2].

4.7 Modified periodogram method

The estimates by the modified periodogram method for the LRDs 1–4 are presented in Fig. 8. The dotted line and the circle line show that $\hat{H}$ of LRDs 1 and 2 are a little underestimated at around $H \in (0.2, 1)$ since the estimates are very close to the lower bound of the bias threshold. However, the underestimation of $\hat{H}$ is more severe when H is around (0, 0.2] given that the estimates are out of the range of the bias thresholds. The estimates shown by the dash–dotted line is severely overestimated. In addition, the estimates denoted by the dashed line is consistently underestimated when $0 < H < 1$. Generally, the modified periodogram method is severely influenced by the linear trends.

4.8 Whittle estimator

The estimates by the Whittle estimator for the LRDs 1–4 are presented in Fig. 9. From the dotted line and the circle line, it is found that the estimation of H is slightly biased and only a little underestimated when H is close to zero. It indicates that the Whittle estimator is robust to the means. As shown by the dash–dotted line, the trends severely affect the Whittle estimator. The estimated $\hat{H}$ is overestimated and generally greater than 1.0. The dashed line shows that seasonalities have little influence on the Whittle estimator since the estimates are generally within the range of the bias thresholds.

4.9 Diffusion entropy method

The estimates by the diffusion entropy method for the LRDs 1–4 are presented in Fig. 10. The dotted line and the circle line show that the estimates are slightly biased when H is around (0, 0.7), and a little underestimated when H is around [0.7, 1.0). The mean value does not severely influence the diffusion entropy method. However, the estimates $\hat{H}$ are always in range (0.8, 1.0) as provided by the dash–dotted line. Thus, the diffusion entropy estimator is generally overestimated with respect to the impact of linear trends. In addition, the dashed line shows that the estimates for LRD 4 are severely underestimated.

4.10 Kettani and Gubner's method

The estimates by Kettani and Gubner's method for the LRDs 1–4 are presented in Fig. 11. The estimates for LRD 1 are generally accurate when H is around (0, 0.9) and only a little underestimated at around $H \in [0.9, 1)$. The mean value does not affect Kettani and Gubner's method. Thus, the dotted line and the circle line coincide with each other.

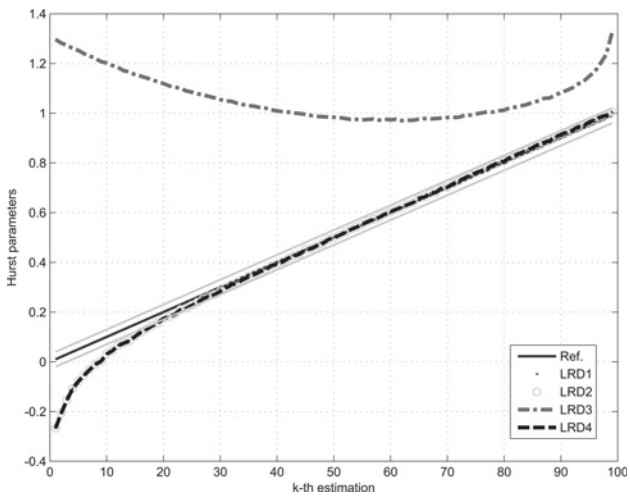

Fig. 7 *Estimates of LRD 1–4 by periodogram method*

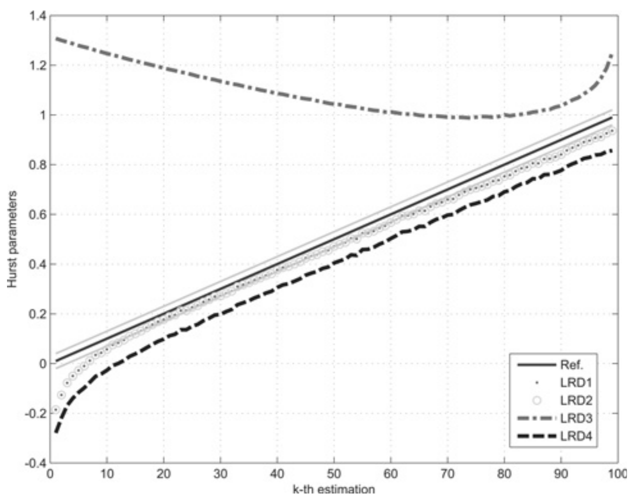

Fig. 8 *Estimates of LRD 1–4 by modified periodogram method*

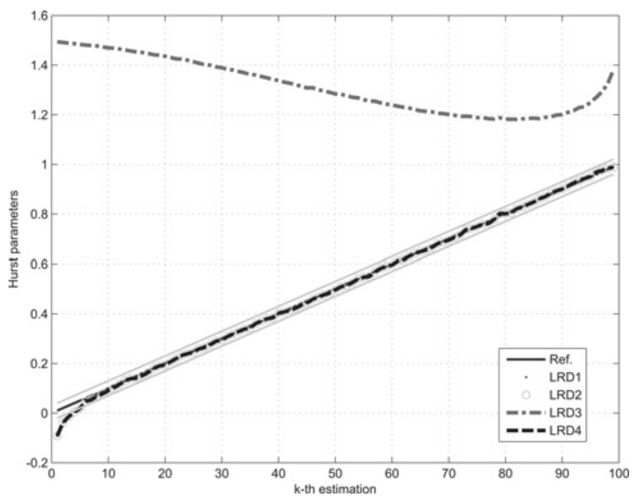

Fig. 9 *Estimates of LRD 1–4 by Whittle estimator*

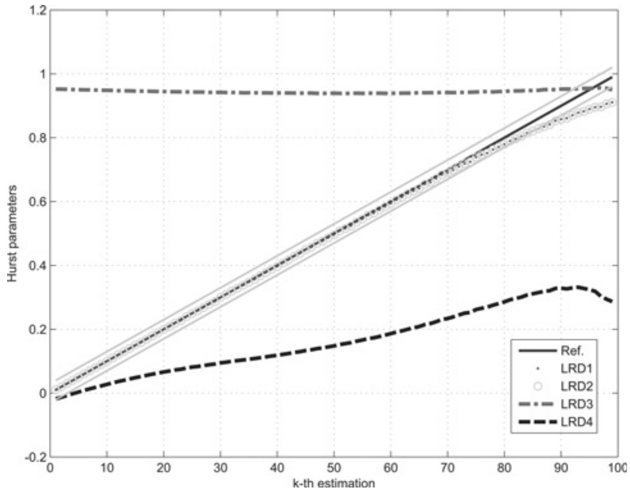

Fig. 10 *Estimates of LRD 1–4 by diffusion entropy method*

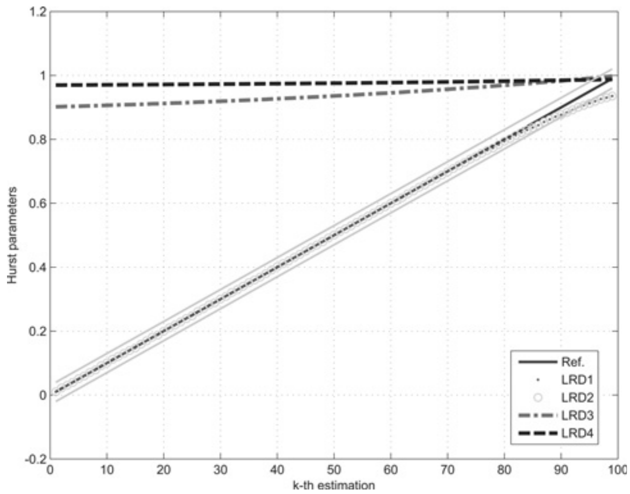

Fig. 11 *Estimates of LRD 1–4 by Kettani and Gubner's method*

However, from the dash–dotted line and the dashed line, it is clear that the estimates of LRDs 3 and 4 are severely overestimated.

4.11 Abry and Veitch's method

The estimates by Abry and Veitch's method for the LRD 1–4 processes are presented in Fig. 12. The Abry and Veitch's method is a wavelet-based Hurst parameter estimator. In this study, the Daubechies wavelet is chosen as the mother wavelet and the number of vanishing moments is three for the estimator evaluation. More detailed explanation can be found in [9]. It is very interesting to see that three of the estimation curves for LRDs 1–3 overlap in Fig. 12. $\hat{H}$ is slightly biased when H is around (0.1, 1.0), and underestimated at around $H \in (0, 0.1]$. The results indicate that this estimator is robust to the means and trends. However, the dashed line is severely underestimated. It is evident that Abry and Veitch's method is poor in estimating the Hurst parameters from time series with seasonalities.

4.12 Koutsoyiannis' method

The estimates by Koutsoyiannis' method for the LRD 1–4 processes are presented in Fig. 13. From the dotted line and the circle line, it is found that the estimates of LRDs 1 and 2 are generally accurate at around $H \in (0.1, 0.9)$. However, this estimator tends to be infinite when H is close to 1.0 as can be found from the dotted line and the circle line. The reason for the infinity estimates are given in [16, 20], where

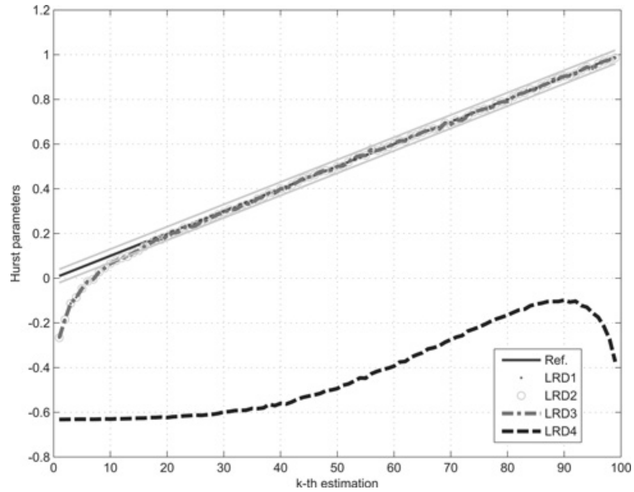

Fig. 12 *Estimates of LRD 1–4 by Abry and Veitch's method*

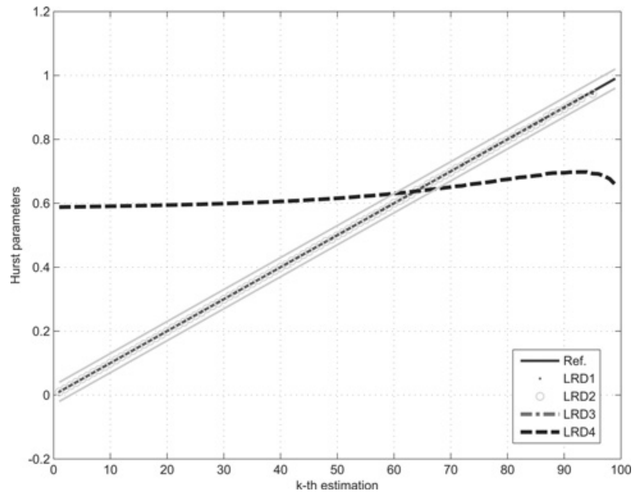

Fig. 13 *Estimates of LRD 1–4 by Koutsoyiannis' method*

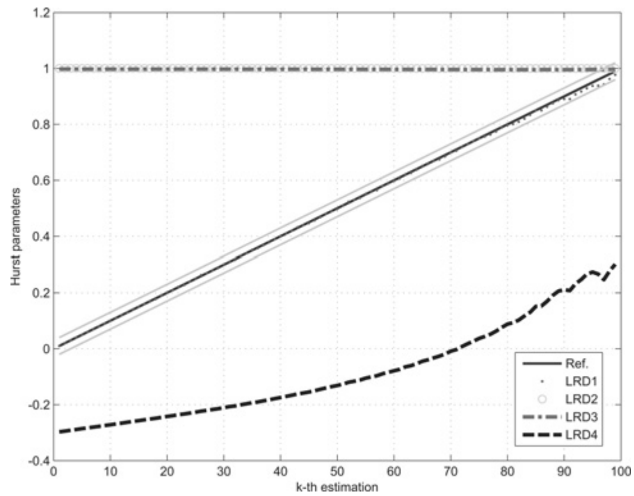

Fig. 14 *Estimates of LRD 1–4 by Higuchi's method*

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Table 2 Standard errors of Hurst parameter estimates

Estimators	LRD 1	LRD 2	LRD 3	LRD 4
R/S	0.0613	0.0613	0.6252	0.4038
aggregated variance	0.0258	0.0258	0.5758	0.5995
difference variance	0.0311	0.0311	0.4395	0.2771
absolute value	0.0261	0.0261	0.5750	0.6347
variance of residuals	0.0045	0.0045	0.2514	0.5124
periodogram	0.0529	0.0529	0.6610	0.0532
modified periodogram	0.0483	0.0483	0.7021	0.1164
whittle estimator	0.0144	0.0144	0.9040	0.0144
diffusion entropy	0.0224	0.0224	0.5307	0.3855
Kettani and Gubner	0.0129	0.0129	0.5131	0.5560
Abry and Veitch	0.0441	0.0441	0.0441	0.9407
Koutsoyiannis	N/A	N/A	N/A	0.2838
Higuchi	0.0062	0.5788	0.5764	0.6006

detailed numerical method in finding $\hat{H}$ is also provided in the two references. In addition, the estimator is very sensitive to the linear trends given that the all estimates for LRD 3 are tends to be infinite. From the dashed line (in black) we find that the estimates are in the range (0.6, 0.7), because of the impact of the seasonalities.

4.13 Higuchi's method

The estimates by Higuchi's method for the LRD 1–4 processes are presented in Fig. 14. The dotted line shows that the estimates are generally accurate for all $H \in (0, 1)$. However this estimator is severely vulnerable to the means, trends and seasonalities. The Hurst estimation is constantly equal to 1.0 for LRD 2 and 3. On the other hand, the estimates are generally underestimated in LRD 4.

4.14 Quantitative comparison of the estimation results

From Figs. 2–14, we can roughly compare the robustness performances of the 13 Hurst parameter estimators. In order to quantify the robustness, the standard errors S of different estimations are calculated. S is defined as

$$S = \sqrt{\frac{\sum_{j=1}^{n} (H_j - \bar{H}_j)^2}{n - 1}} \quad (7)$$

where n is the number of estimated Hurst parameters by each estimator and $n = 99$ in this study. Table 2 gives the standard errors of the estimates for the four different LRDs.

In Table 2, we can find that the standard errors for LRDs 1 and 2 are generally smaller than the standard errors for LRD 3 and 4. It indicates that the linear trends and seasonalities tend to impose worse effects to the Hurst parameter estimators than the mean offset. It is also interesting to find that the standard errors for LRD 1 and 2 are the same except for the Higuchi's method, which indicates that the mean values generally have no influence to 12 of the estimators. In addition, we can also find the best estimator for LRDs 1–4 by looking up the minimum standard errors in each column. For instance, the variance of residuals method is the best one for LRDs 1 and 2, Abry and Veitch's method is the most suitable estimator for LRD 3, whereas the Whittle estimator is recommended to estimate Hurst parameter from LRD 4. However, it is also clear that none of the estimators consistently performed a good estimation performance across four different LRD processes.

5 Conclusion and discussion

In this study, the robustness performances of the 13 Hurst parameter estimators have been evaluated for different LRD processes with the existence of non-zero means, linear trends and seasonalities. Since only a certain level of the mean noise and linear trend are added into LRD 1 and not all frequencies of the seasonalities have been tested in this study, the conclusion might not be decisive. However, the robustness assessment procedures developed in this paper are applicable to practical scenarios, such as the prices in stock markets, residential daily energy usage and so on, where means, trends and daily seasonalities have been exhibited. In addition, the robust evaluation results offer useful information in choosing the most appropriate Hurst parameter estimator for a particular LRD process in order to avoid misusing of these estimators. For instance, for an LRD process with mean offset, the variance of residuals method is highly recommended since the standard error by this estimator is less than 1%. In addition, if an LRD process exhibits linear trends, the Abry and Veitch's method is suggested to be used to estimate the Hurst parameter for this LRD process. Moreover, given an LRD process with seasonalities, the Whittle estimator proves to be the most suitable estimator for the Hurst parameter estimation.

Besides all the above-mentioned contributions of this study, more remarkable and challenging issues related to this study are raised. These issues are listed in the following and they are going to be considered and addressed in our ongoing and future work.

First, one may argue that under the real world scenario, the mean offset, the trends and seasonalities are not confined in isolation, for instance, data sampled from the stock markets, and residential daily energy usage. In this case, an alternative way of estimating the Hurst parameter is to conduct signal decomposition by separating the mean, trend, seasonality and the random component in time-series analysis [21]. The coefficients of mean, trend and seasonality can be obtained by curve fitting and the random component can be modelled as an fBm.

Second, the constant variance of the LRD processes is ideal but not realistic. To further examine the robustness of estimators for real data, it is plausible to model the variance term by the autoregressive conditional heteroskedasticity (ARCH), the generalised ARCH model, the stochastic volatility or diffusion model if it will be used in capturing the Hurst parameter in the real data scenarios such as the above-mentioned data from stock markets or residential daily energy usage.

Thirdy, some real world data that sampled from the stock markets or residential daily energy usage, either in fGn or FARIMA type may be in multifractional or multiscales, see [22, 23]. In these cases, the effects of trends and seasonalities to the *H* estimators are also worth investigation.

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7 References

1 Beran, J.: 'Statistics for long-memory processes' (Chapman & Hall/CRC, New York, 1994, 1st edn.)
2 Hurst, H.E.: 'Long-term storage capacity of reservoirs', *Trans. Am. Soc. Civ. Eng.*, 1951, **116**, (3), pp. 770–799
3 Bassingthwaighte, J.B., Raymond, G.M.: 'Evaluation of the dispersional analysis method for fractal time series', *Ann. Biomed. Eng.*, 1995, **23**, (4), pp. 491–505
4 Sheng, H., Chen, Y.Q., Qiu, T.: 'On the robustness of Hurst estimators', *IET Signal Process.*, 2011, **5**, (2), pp. 209–225
5 Sun, R., Chen, Y.Q., Li, Q.: 'The modeling and prediction of Great Salt Lake elevation time series based on ARFIMA'. Proc. ASME 2007 Int. Design Engineering Technical Conf. and Computers and Information in Engineering Conf. 2007, Las Vegas, Nevada, USA, 2007, pp. 1–11
6 Taqqu, M., Teverovsky, V., Villinger, W.: 'Estimators for long-range dependence: an empirical study', *Fractals*, 1995, **3**, (4), pp. 785–788
7 Geweke, J., Porter-Hudak, S.: 'The estimation and application of long memory time series models', *J. Time Ser. Anal.*, 1983, **4**, pp. 221–238
8 Taqqu, M., Teverovsky, V.: 'Robustness of whittle-type estimators for time series with long-range dependence', *Stoch. Models*, 1997, **13**, (4), pp. 723–757
9 Abry, P., Veitch, D.: 'Wavelet analysis of long-range-dependent traffic', *IEEE Trans. Inf. Theory*, 1998, **44**, (1), pp. 2–15
10 Higuchi, T.: 'Approach to an irregular time series on the basis of the fractal theory', *Phys. D, Nonlinear Phenom.*, 1988, **31**, (2), pp. 277–283
11 Sheng, H., Chen, Y.Q., Qiu, T.: 'Robustness analysis of Hurst estimators for multifractional Gaussian process'. Proc. Fourth IFACWorkshop Fractional Differentiation and its Applications, Badajoz, Spain, 2010, pp. 1–6
12 Blok, H.J.: 'On the nature of the stock market: simulations and experiments'. PhD thesis, Department of Physics and Astronomy, University of British Columbia, 2000
13 Montanari, A., Taqqu, M.S., Teverovsky, V.: 'Estimating long-range dependence in the presence of periodicity: an empirical study', *Math. Comput. Model.*, 1999, **29**, (10-12), pp. 217–228
14 Grigolini, P., Palatella, L., Raffaelli, G.: 'Asymmetric anomalous diffusion: an efficient way to detect memory in time series', *Fractals*, 2001, **9**, (4), pp. 439–449
15 Kettani, H., Gubner, J.: 'A novel approach to the estimation of the long-range dependence parameter', *IEEE Trans. Circuits. Syst.*, 2006, **52**, (6), pp. 463–467
16 Koutsoyiannis, D.: 'Climate change, the Hurst phenomenon, and hydrological statistics', *Hydrol. Sci. J.*, 2003, **48**, (1), pp. 3–24
17 Gubner, J.A.: 'Probability and random processes for electrical and computer engineers' (Cambridge University Press, Cambridge, 2006, 1st edn.)
18 Clegg, R.G.: 'A practical guide to measuring the Hurst parameter', *Int. J. Simul., Syst. Sci. Technol.*, 2006, **7**, (2), pp. 3–14
19 Mandelbrot, B.B., Van Ness, J.W.: 'Fractional Brownian motions, fractional noises and applications', *SIAM Rev.*, 1968, **10**, (4), pp. 422–437
20 Koutsoyiannis, D.: 'Supplementary material for the paper climate change, the Hurst phenomenon, and hydrological statistics'. Technical Report, Department of Water Resources, Faculty of Civil Engineering, National Technical University, Athens, 2003, http://www.itia.ntua.gr/getfile/537/2/2003HSJHurstSuppl.pdf, accessed October 2012
21 Brockwell, P., Davis, R.: 'Introduction to time series and forecasting' (Springer, New York, 2002, 2nd edn.)
22 Li, M., Zhao, W.: 'Quantitatively investigating locally weak stationarity of modified multifractional Gaussian noise', *Phy. A*, 2012, **391**, pp. 6268–6278
23 Li, M., Zhao, W.: 'Representation of a stochastic traffic bound', *IEEE Trans. Parallel Distrib. Syst.*, 2010, **21**, (9), pp. 1368–1372

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