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Game-theoretic demand-side management and closed-loop control for a class of networked smart grid

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Abstract: A new systematic scheme is proposed for demand-side management and control of a class of networked smart grid. The networked smart grid is modelled into an evolutionary game, and the theory of semi-tensor product is applied to analyse its dynamic behaviours. By using a newly defined stacked form of structural matrix, a closed-loop feedback control is designed to shape the structural matrix of the evolutionary game, such that the Nash equilibrium (with respect to minimum total cost) can be reached and maintained. Based on the proposed feedback control, a robust control can be designed for the smart grid to reach the Nash equilibrium in case of model uncertainties resulted from fake behaviours of some users. Examples are provided to illustrate the theoretical results.

1 Introduction

Demand-side management has been proved to be a fairly effective way to achieve energy-efficiency of large-scale energy systems. The core issue of demand-side management is to design strategies to optimise user behaviours, such that the provided energy can be consumed with superior efficiency. Some typical results of demand-side management for energy systems include multi-objective optimisation [1], model predictive control [2] and smart pricing [3, 4] etc. Demand-side management for large-scaled energy systems (smart grid, for example) can be designed and implemented in either centralised scheme or distributed scheme. In centralised scheme, the strategy is proposed specifically for each components of the energy system to optimise energy consumption [5, 6]. In distributed scheme, each user has the authorisation of making own decisions within the scope of some common regulations [7]. Distributed demand-side management is of particular interest in recent years due to its simple structure and energy-efficiency. Smart pricing is a representative strategy of distributed demand-side management [3], where pricing policy is designed by the grid provider, and users communicate with each other and make decisions accordingly. Some further results include adaptive pricing [8] and predictive pricing [9].

In researches on distributed demand-side management of smart grid [10], game theory is widely applied to design the optimal strategy with respect to energy-efficiency. The existence of Nash equilibrium (NE) in a smart grid indicates that users are possible to reach and maintain some types of consensus [11]. Strategies are often designed by grid providers, and the game is played among providers and users under the proposed strategies, such that the users can be induced as expected by grid providers [12]. In reality, the games are often played repeatedly, and can be modelled into evolutionary (dynamic) games [13]. The evolutionary games can be described by dynamic equations, where control techniques (model predictive control [14, 15], for example) can be applied to search for the NE. Another construction of the evolutionary game is to set and subsidize some cooperative communities, such that decisions of other communities in the networked grid can be influenced indirectly. In this type of evolutionary game, the fundamental game is played only among users (excluding the grid provider), and it is named as control evolutionary game [13].

During recent years, the theory of semi-tensor product [16] has been applied to solve evolutionary game-theoretic problems. The semi-tensor product aims to extend traditional matrix products to a more general and compatible form, such that multiplication can be processed with matrices of any dimensions. Within the framework of semi-tensor product, a dynamic logic system (evolutionary game, for example) can be expressed in a unique linear form [16]. By using semi-tensor product, properties such as controllability, observability and stability of the dynamic logic system can be evaluated [17–19]. Linear open-loop or closed-loop control can be designed in logic form for the controlled logic system [18]. With coordinate transformations, controllers in logic forms can be designed for disturbance decoupling [20, 21]. Other results in applications of semi-tensor product include optimal control [22, 23], receding horizon control [24] and fault-detection [25, 26]. Properties of the evolutionary game (existence of the NE, for example) can be analysed by using the linear form of dynamic logic systems [13]. Until now, results in evolutionary game with model uncertainties are fairly limited.

In this paper, we are aiming to propose a closed-loop strategy for demand-side management and control of the smart grid in [27], where the objective is to actively assign the behaviours of the cooperative communities (controllers), such that other communities can be induced to minimise the total cost of the entire networked system. Meanwhile, uncertainties due to incorrect information should be addressed by some robust strategies. Theoretical results are derived and proved in the framework of evolutionary game theory and semi-tensor product. The main *contributions* (different from the previous study in [27]) include that (i) a hybrid modelling strategy is given in the framework of logic calculations and diagrams of true values, such that the dynamic equation is in a unified form (instead of switching forms); (ii) a closed-loop feedback control in linear form (instead of logic form) is proposed for the networked smart grid, such that the system can be induced to reach and maintain an NE, where the total cost is minimised; (iii) a robust feedback control in linear form is designed, such that some model uncertainties can be decoupled. Several examples are provided throughout this paper to illustrate the theoretical results.

The layout of this paper is arranged as follows. In Section 2, some preliminaries are given, and the objective of this paper is stated explicitly. In Section 3, the detailed strategy of modelling and control for the specific class of smart grid is given. In Section

4, a robust control strategy is proposed to decouple model uncertainties. In Section 5, a simulation case is provided to illustrate the theoretical results. This paper is concluded in the final section.

2 Problem formulation

2.1 Preliminaries on game theory

Information interchange within networked system can be described by a directed graph $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$, where $\mathscr{V} = \{\pi_1, \pi_2, \dots, \pi_n\}$ is a set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of edges that depict information flow between nodes. An edge (π_i, π_j) in $\mathscr{G}$ denotes that the information of node π_i is available to π_j, and π_i is defined as a neighbour of π_j. The index set of all neighbours of node π_j is denoted by $\mathscr{N}_j = \{i: (\pi_i, \pi_j) \in \mathscr{E}\}$. In an undirected graph, $(\pi_i, \pi_j) \in \mathscr{E} \Leftrightarrow (\pi_j, \pi_i) \in \mathscr{E}$. The adjacent matrix $\mathscr{A} \triangleq [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} = 1$ if $(\pi_j, \pi_i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. It is assumed that $a_{ii} = 0$. More details on network system can be found in [28].

Definition 1: A normal finite game $\mathscr{H}$ can be formulated by (i) the set of players: $\mathscr{V} = \{\pi_1, \pi_2, \dots, \pi_n\}$, where π_i denotes the ith player; (ii) the strategy set for each player: $\mathscr{X}_i = \{x_{i1}, x_{i2}, \dots, x_{ik}\}$, where $i = 1, \dots, n$ and (iii) the cost function: $c_i(x_i, x_{-i})$, where $x_i \in \mathscr{X}_i$ denotes the strategy selected by player i, and x_{-i} denotes strategies of other players excluding player i. Each player chooses strategies from $\mathscr{X}_i$ to minimise its individual cost function.

Definition 2: Nash equilibrium (NE), denoted by $(x_1^*, x_2^*, \dots, x_n^*)$, is a local optimal response for a normal finite game, where no individual would gain by unilaterally changing its own strategy: $c_i(x_i^*, x_{-i}^*) \le c_i(x_i, x_{-i}^*)$.

If a game can be played repeatedly with an updating law:

$$\Pi: \; x_i(t+1) = f(x_i(t), x_{-i}(t), c_i(t)), \tag{1}$$

where $t \ge 0$ denotes the discrete sampling time, then it is named evolutionary game (dynamic game).

In an evolutionary game played by multiple players, a typical updating law can be given by Unconditional imitation with fixed priority [13]:

$$x_i(t+1) = x_{j^*}(t), \quad j^* = \arg\min_{j \in \mathscr{N}_i} c_j(x_j(t), x_{-j}(t)). \tag{2}$$

If j^* is non-unique, then select the minimal j^* as priority.

The following definitions and theorems are cited from [13].

Definition 3: The networked evolutionary game is composed by

i. a networked graph $\mathscr{G}$;
ii. a normal finite game $\mathscr{H}$ that can be played repeatedly;
iii. an updating law Π.

Remark 1: The above definition of the networked evolutionary game is slightly different from that of [13], where fundamental networked game (FNG) is required. In this paper, the normal finite game is used in Definition 3.

Definition 4: The control networked evolutionary game $(\mathscr{G}_c, \mathscr{H}, \Pi)$, where $\{\mathscr{X}, \mathscr{U}\}$ is a partition of $\mathscr{V}$ ($\mathscr{X} \cup \mathscr{U} = \mathscr{V}$ and $\mathscr{X} \cap \mathscr{U} = \varnothing$), is composed by (i) a normal finite game $\mathscr{H}$ that is played repeatedly; (ii) a networked graph $\mathscr{G}_c = (\mathscr{X} \cup \mathscr{U}, \mathscr{E})$, where $\{\mathscr{X}, \mathscr{U}\}$ is a partition of $\mathscr{V}$ ($\mathscr{X} \cup \mathscr{U} = \mathscr{V}$ and $\mathscr{X} \cap \mathscr{U} = \varnothing$), and strategies of $\mathscr{U}$ can be actively assigned and (iii) an updating law Π.

2.2 Preliminaries on semi-tensor product

Contents in this section are cited from [13, 16–18]

Definition 5: The semi-tensor product of two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{p \times q}$ can be defined by

$$A \ltimes B \triangleq (A \otimes I_{o/n})(B \otimes I_{o/p}) \in \mathbb{R}^{(mo/n) \times (qo/p)}, \tag{3}$$

where $o = \mathrm{lcm}(n, p)$ denotes the least common multiple of n and p and $\otimes$ denotes the Kronecker product.

Definition 6: The fundamental vector $\delta_n^i \in \mathscr{D}_n = \{\delta_n^1, \dots, \delta_n^n\}$ is defined as the ith column of the identity matrix $I_{n \times n}$. It can be further defined that

$$\delta_n[i, j, \dots, k] \triangleq [\delta_n^i, \delta_n^j, \dots, \delta_n^k] \tag{4}$$

for more compact expressions.

Lemma 1: With equivalence $i \sim \delta_n^i$, $i = 1, 2, \dots, n$, a logic function $f: \mathscr{D}_n^k \to \mathscr{D}_n$ can be rewritten by $f(x_1, x_2, \dots, x_k) = M_f \ltimes_{i=1}^k x_i$, where M_f is the structure matrix of logic function f.

Lemma 2: For a logic dynamic system

$$x_i(t+1) = f_i(x_i(t), x_{-i}(t)) = M_{fi} \ltimes_{i=1}^n x_i, \quad i = 1, \dots, n,$$

it can be rewritten in the form of

$$x(t+1) = M_f x(t), \tag{5}$$

where $x(t) \triangleq \ltimes_{i=1}^n x_i$ and

$$M_f \triangleq M_{f1} * M_{f2} * \cdots * M_{fn}. \tag{6}$$

Here, $*$ denotes the Khatri–Rao product:

$$M * N \triangleq [\mathrm{col}_1(M) \ltimes \mathrm{col}_1(N), \dots, \mathrm{col}_s(M) \ltimes \mathrm{col}_s(N)],$$

where $M \in \mathbb{R}^{p \times s}$ and $N \in \mathbb{R}^{q \times s}$ and $\mathrm{col}_i(M)$ denotes the ith column of matrix M. Moreover, the controlled logic dynamic system

$$x_i(t+1) = f_i(x_i(t), x_{-i}(t), u(t)), \quad i = 1, \dots, n,$$

can be rewritten in the form of

$$x(t+1) = M_f u(t) x(t). \tag{7}$$

Lemma 3: For a logic dynamic system given by (5), δ_n^i is its fixed point, if and only if the diagonal element m_{ii} of M_f equals 1.

Lemma 4: For logic variable $x \in \mathscr{D}_k$, it satisfies

$$x^2 = x \ltimes x = \Phi_k x, \tag{8}$$

where

$$\Phi_k = \ltimes_{j=1}^k \left(I_{2^{j-1}} \otimes \left(I_2 \otimes W_{[2, 2^{k-j}]} M_r\right)\right), \tag{9}$$

and $W_{[i, j]}$ is the swap matrix defined in [16].

2.3 Problem statement

In this paper, the problem in [27] is extended to a more general form, which can be described as follows.

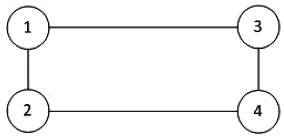

Fig. 1 *Communication graph of communities in Example 1 [27]*

Consider a smart grid with N communities (users) and the network denoted by adjacent matrix $\mathscr{A}$. Let $p_i(t)$ denote the price paid by community i at time t. Each community has the option of using either the grid power ($x_i(t) = \delta_2^1$) or the local diesel power ($x_i(t) = \delta_2^2$), where $x_i(t)$ is defined as the state (or strategy) of community i at time t. The price of the diesel power is fixed at p_d, and the price of the grid power varies according to the number of grid users:

$$p_g(t) = p_g(N_g(t)), \tag{10}$$

where $N_g(t)$ denotes the number of communities using the grid power at time t.

The price paid by community i is given by

$$p_i(t) = \begin{cases} p_g(t), & \text{if } x_i(t) = \delta_2^1, \\ p_d, & \text{if } x_i(t) = \delta_2^2. \end{cases} \tag{11}$$

The cost function of each community is defined by

$$c_i(x_i(t), x_{-i}(t)) = p_i(t) + \alpha\left(p_i(t) - \min_{j \in \mathscr{N}_i} p_j(t)\right), \tag{12}$$

where $\alpha > 0$ is a constant weight coefficient. The cost function (12) indicates that each community, while pursuing the lowest price, feels uncomfortable if it pays a higher price than those of its neighbours. The updating law Π is given by unconditional imitation (2), implying each community would change its strategy to that of the neighbour with the lowest cost. The total cost at time t is defined by

$$C(t) = \sum_{i=1}^{N} p_i(t), \tag{13}$$

which is the sum of prices paid by all communities.

Remark 2: In the problem statement, c_i is the cost function of the individual community i, and it includes its happiness (described by $\alpha\left(p_i(t) - \min_{j \in \mathscr{N}_i} p_j(t)\right)$) when paying the price p_i. The individual community is supposed to select strategies based on the individual cost c_i. However, individual communities do not cooperatively consider the total cost, and it should be considered by the grid provider. It is supposed that the grid provider has no idea on the happiness of individual communities. Consequently, we construct the total cost simply by adding individual prices (as given by (13)), and it is believed that this model can better describe a more realistic situation.

In this networked smart grid, it is supposed that some of the users cooperate with the grid provider, such that other communities can be induced to cooperate, and the minimum total cost can be achieved. The cooperative users are denoted as controllers:

$$u_i(t) = x_i(t), \quad \text{if } i \in \mathscr{U}, \tag{14}$$

and their selection (of using grid or diesel) can be actively assigned. The controllers can be rewritten in a compact form:

$$u = \ltimes_{i \in \mathscr{U}} u_i, \tag{15}$$

if there are more than one controller.

One *objective* of this paper is to design a systematic algorithm to construct a closed-loop feedback control $u = Gx$, such that there exist an optimal NE for the networked grid, and the optimal NE can be reached and maintained.

Remark 3: In [27], an open loop control is proposed to reach and maintain the optimal total cost. This paper differs from [27] that we aim to design a closed-loop feedback control.

In the networked grid, it is possible that some communities may not report their selections honestly, and other users would be misled. This situation is defined as 'model uncertainty' in this paper. In case of model uncertainty, another *objective* is to find a robust control u, such that the networked grid with model uncertainty will still reach and maintain the optimal NE.

Example 1: Take cases 1 and 3 in [27] for example. In case 1, there are four communities either using the grid power or the diesel power. The network is depicted by Fig. 1, and the price policy is given in Table 1. The cost function is given by (12), and the strategy update of each community is given by (2). It can be seen that the optimal total cost occurs at $(x_1, x_2, x_3, x_4) = (1, 1, 1, 2)$, where community 4 uses the diesel power and others use the grid power. However, this is not an NE, and community 4 would switch to the strategy performed by community 2, because community 2 pays less. Case 3 in [27] is the controlled version of case 1. In case 3, community 4 cooperates with the grid provider and performs as the controller. It can stay at using diesel power such that the optimal cost can be maintained.

3 Demand-side management and control of the networked grid

3.1 Unified model of the networked grid

In the networked smart grid, the real-time price relates directly to the number of communities using the grid power, thus it is not a logic function. As a consequence, the dynamic equation of the networked grid cannot be derived directly from logic expressions. In [27], an algorithm was proposed to construct the dynamic equation of the networked grid in a switching form by using diagrams of true values.

In this section, a new hybrid algorithm is proposed by using logic calculations and diagrams of true values. Define the state of the network:

$$x = \ltimes_{i=1}^{N} x_i, \tag{16}$$

where $x_i \in \mathscr{D}_2$ denotes the choice of community i.

Here, based on the cost function $c_i(x)$ given in (12), a comparison matrix can be defined by

$$C(x) \triangleq \left[c_{ij}(x)\right]_{N \times N}, \tag{17}$$

where the comparison functions are defined by

$$c_{ij}(x) = \begin{cases} \delta_2^1, & \text{if } c_i(x) < c_j(x), \text{ or } j \notin \mathscr{N}_i, \\ \delta_2^2, & \text{if } c_i(x) \geq c_j(x). \end{cases} \tag{18}$$

Table 1 Prices of diesel power and grid power

User number	0	1	2	3	4
grid power price	8	7	7	6.5	7.5
diesel power price	7.2	7.2	7.2	7.2	7.2

Values are not absolute prices; they are assigned to reflect differences of prices in various scenarios.

The comparison functions $c_{ij}(x)$ are logic functions, and their dynamic forms $c_{ij}(x) = C_{ij}x$ can be calculated by using diagrams of true values.

The decision of community i with respect to one of its neighbours j can be described by

$$x_i(t+1) = L_{dij}(c_{ij})x_i(t)x_j(t), \tag{19}$$

where

$$L_{dij}(c_{ij}) = \begin{cases} \delta_2[1,1,2,2], & \text{if } c_{ij} = \delta_2^1, \\ \delta_2[1,2,1,2], & \text{if } c_{ij} = \delta_2^2, \end{cases} \tag{20}$$

or equivalently

$$\begin{aligned} L_{dij}(c_{ij}) &= \delta_2[1,1,2,2,1,2,1,2]c_{ij}(x) = L_d c_{ij}(x) \\ &= L_d C_{ij} x. \end{aligned}$$

The physical implication of (19) is that community i would adopt the strategy of its neighbour j if its cost is larger than or equal to that of j; otherwise, it would keep its strategy.

Define $x_i^+ = x_i(k+1)$ for short. It follows from (19) that the decision of community i with respect to all its neighbours can be calculated by

$$x_i^+ = L_{diN} \cdots L_{dij} \cdots L_{di1} x_i x_1 \cdots x_j \cdots x_N, \tag{21}$$

where $j \neq i$. Then, with the swap matrix, it holds that

$$x_i^+ = \left(\ltimes_{j=0, j \neq N-i}^{N-1} L_{di,N-j} \right)\left(\ltimes_{j=1, j \neq i}^{i-1} I_{2^{j-1}} \otimes W_{[2,2]} \right)x, \tag{22}$$

where

$$\begin{aligned} \ltimes_{j=0, j \neq N-i}^{N-1} L_{di,N-j} &= L_d C_{iN} x L_d C_{iN-1} x \cdots L_d C_{i1} x \\ &= L_d C_{iN}\left(I_{2^N} \otimes L_d C_{iN-1}\right)x^2 L_d C_{iN-2} x \cdots L_d C_{i1} x \\ &= L_d C_{iN}\left(I_{2^N} \otimes L_d C_{iN-1}\Phi_N\right)x L_d C_{iN-2} x \cdots L_d C_{i1} x \\ &= L_d C_{iN} \ltimes_{j=1, j \neq N-i}^{N-1} \left(I_{2^N} \otimes L_d C_{iN-j}\Phi_N\right)x, \end{aligned}$$

and Φ_k is calculated by (9).

It then follows that

$$x_i^+ = M_i x, \tag{23}$$

where $M_i = M_{i1}M_{i2}\Phi_N$ and

$$M_{i1} \triangleq L_d C_{iN} \ltimes_{j=1, j \neq N-i}^{N-1} \left(I_{2^N} \otimes L_d C_{iN-j}\Phi_N\right), \tag{24}$$

$$M_{i2} \triangleq I_{2^N} \otimes \left(\ltimes_{j=1, j \neq i}^{i-1} I_{2^{j-1}} \otimes W_{[2,2]} \right), \tag{25}$$

and Φ_N can be calculated by (9) with $k = N$.

It then follows from Lemma 2 that

$$x^+ = Mx, \tag{26}$$

where $M = M_1 * M_2 * \cdots * M_N$.

Example 2: Take case 1 in [27] for example, where $N = 4$. The true values of C_{ij} can be calculated based on the price policy. With direct calculation proposed in this section, the structural matrix can be calculated by

$$M = \delta_{16}[1,1,1,1,1,1,1,2,1,1,1,1,1,1,9,16]. \tag{27}$$

It can be seen that there is only two NEs δ_{16}^1 and δ_{16}^{16}, where all communities have the same choice of using grid power or diesel power. However, the cost function is not optimal at both NEs. Some other strategies should be proposed to induce the choices of the communities such that optimal cost can be achieved.

Suppose that r of the N states are controllers:

$$x_{i_1} = u_1, \ldots, x_{i_r} = u_r, \tag{28}$$

and

$$x \triangleq \ltimes_{j=1, x_j \neq u_k}^{N-r} x_j, \quad u \triangleq \ltimes_{j=1}^{r} u_j. \tag{29}$$

It follows from (23) that

$$x_i^+ = M_i\left(\ltimes_{j=1}^{r} W_{[2,2^{i_j - j}]} \right)ux, \quad i \neq i_1, \ldots, i_r, \tag{30}$$

and

$$x^+ = Lux, \tag{31}$$

where

$$L = M_1\left(\ltimes_{j=1}^{r} W_{[2,2^{i_j - j}]} \right) * \cdots * M_N\left(\ltimes_{j=1}^{r} W_{[2,2^{i_j - j}]} \right). \tag{32}$$

Example 3: Take case 3 in [27] for example, where $N = 4$ and $r = 1$. The structure matrix was given in a switching form. By using the algorithm in this section, the structure matrix can be calculated by

$$L = \delta_8[1,1,1,1,1,1,1,5,1,1,1,1,1,2,3,8], \tag{33}$$

and the dynamics of the smart grid is given by $x^+ = Lux$.

3.2 Stability of the evolutionary game

Results in this section are derived straightforwardly from Lemma 3 and [13, 29].

Definition 7: In the logic dynamic system (26), the NE x_0 is locally attracting, if there exists at least one other state x_1 such that (26) starting at x_1 would finally converge to x_0 after finite steps. The NE x_0 is globally attracting, if the logic dynamic system (26) starting at any state would finally converge to x_0 after finite steps.

Theorem 1: Denote the structural matrix of (26) by $M = \delta_{2^N}[m_1, m_2, m_3, \ldots, m_{2^N}]$. Suppose that there exist some fixed points such that $m_i = i$ for some integers $i \in [1,2,3,\ldots,2^N]$. The fixed point $\delta_{2^N}^{m_i}$ (or equivalently $\delta_{2^N}^i$) is locally attracting, if and only if there exist some other m_j $(j \neq i)$ such that $m_j = m_i$.

Proof: Sufficiency: if $m_j = m_i$ and $x = \delta_{2^N}^j$, then

$$x^+ = \delta_{2^N}[m_1, \ldots, \underbrace{m_i}_{\text{column } j}, \ldots, m_{2^N}]\delta_{2^N}^j = \delta_{2^N}^{m_i} = \delta_{2^N}^i, \tag{34}$$

indicating that $\delta_{2^N}^{m_i}$ (or equivalently $\delta_{2^N}^i$) is locally attracting.

Necessity: the fixed point $x = \delta_{2^N}^{m_i} = \delta_{2^N}^i$ is locally attracting, implying that there exists at least one other state $x_0 = \delta_{2^N}^j \neq x$ such that $x_0^+ = M\delta_{2^N}^j = \delta_{2^N}^i$.

It should be noted that the calculation of $M\delta_{2^N}^j$ is to pick out the jth column of M; consequently, it holds that $m_j = m_i = i$, and the necessity is proved. □

Corollary 1: Suppose that there are n fixed points $\delta_{2^N}^{i_1}, \ldots, \delta_{2^N}^{i_n}$ in system (26). If there exists a integer $k \in [1, 2, 3, \ldots, 2^N]$ such that the columns of $M^k = \delta_{2^N}[m_1, \ldots, m_{2^N}]$ are all composed by $\delta_{2^N}^{i_1}, \ldots, \delta_{2^N}^{i_n}$, then any state with initial value $x(0) = \delta_{2^N}^j$ finally converges to $x = \delta_{2^N}^{m_j}$ which is a fixed point of (26).

Proof: This result follows directly from Theorem 1. □

Remark 4: See [13] for definitions and properties of limit cycles. In this paper, it is expected that all states would converge to NEs with the proposed feedback control.

Theorem 2: If the structural matrix of (26) is given in the following form:

$$M = \delta_{2^N}[i, i, \ldots, i], \tag{35}$$

with the certain integer $i \in [1, 2, 3, \ldots, 2^N]$, then $\delta_{2^N}^i$ is globally attracting, and it is an NE of the system (26).

Proof: According to Lemma 3, $\delta_{2^N}^i$ is a fixed point, since the ith element of ith column of M is 1.
With direct calculation, it can be obtained that any initial states would reach $\delta_{2^N}^i$ after one step, indicating that $\delta_{2^N}^i$ is attracting globally. Consequently, the state $\delta_{2^N}^i$ is an NE. □

Remark 5: This theorem is a direct result given in [29].

Corollary 2: Suppose that $\delta_{2^N}^i$ is a fixed point of (26) with the certain integer i satisfying $0 < i \leq 2^N$. If the structural matrix of (26) satisfies

$$M^k = \delta_{2^N}[i, i, \ldots, i], \tag{36}$$

for a certain integer $k \in [1, 2, 3, \ldots, 2^N]$, then $\delta_{2^N}^i$ is globally attracting and it is an NE of the system (26).

Proof: With direct calculation, it can be obtained that any initial states would reach $\delta_{2^N}^i$ after at most k steps, indicating that $\delta_{2^N}^i$ is attracting globally. Consequently, the state $\delta_{2^N}^i$ is an NE. □

3.3 Feedback control via structural shaping

The behaviour of the networked smart grid is uniquely determined by its structural matrix. In this section, a strategy is proposed to shape its structural matrix into the desired form by using the feedback control:

$$u = Gx, \tag{37}$$

where the logic matrix $G \in \mathscr{L}_{2^r \times 2^{N-r}}$ is the control gain to be designed.

Substituting (37) into (31) yields

$$x^+ = LGx^2 = LG\Phi_{N-r}x \tag{38}$$

implying that the aim is to design G, such that

$$LG\Phi_{N-r} = M_{\rm d}, \tag{39}$$

where $M_{\rm d} = \delta_{2^{N-r}}[m_1, \ldots, m_{2^{N-r}}] \in \mathscr{L}_{2^{N-r} \times 2^{N-r}}$ is the desired structural matrix of the networked smart grid.

Denote $G = \delta_{2^r}[g_1, g_2, \ldots, g_{2^{N-r}}]$, where $g_i \in [1, 2, 3, \ldots, 2^r]$. It can be calculated directly that

$$\begin{aligned} G\Phi_{N-r} &= (G \otimes I_{2^{N-r}})\Phi_{N-r} \\ &= [\delta_{2^r}^{g_1} \otimes I_{2^{N-r}}, \ldots, \delta_{2^r}^{g_{2^{N-r}}} \otimes I_{2^{N-r}}]\Phi_{N-r}, \end{aligned} \tag{40}$$

where Φ_{N-r} can be calculated from (9), and it is in the following form:

$$\Phi_{N-r} = \delta_{2^{2(N-r)}}[1, 1 + \Delta, 1 + 2\Delta, \ldots, 2^{2(N-r)}], \tag{41}$$

where $\Delta = (2^{2(N-r)} - 1)/(2^{N-r} - 1)$ and $2^{2(N-r)} = 1 + (2^{N-r} - 1)\Delta$.

It is indicated from (40) and (41) that $G\Phi_{N-r}$ is actually the first, $(1 + \Delta)$th, $(1 + 2\Delta)$th, … columns of $G \otimes I_{2^{N-r}}$, and it can be written by

$$G\Phi_{N-r} = \delta_{2^N}\left[(g_1 - 1)2^{N-r} + 1, \ldots, (g_{2^{N-r}} - 1)2^{N-r} + N - r\right] \tag{42}$$

Denote $L = \delta_{2^{N-r}}\left[l_1, l_2, \ldots, l_{2^N}\right]$, where $l_i \in [1, \ldots, 2^{N-r}]$. It can be re-arranged into a stacked form:

$$\mathbf{L} = \delta_{2^{N-r}}\begin{bmatrix} l_1 & l_2 & l_3 & \cdots & l_{2^{N-r}} \\ l_{2^{N-r}+1} & l_{2^{N-r}+2} & & & l_{2 \cdot 2^{N-r}} \\ l_{2 \cdot 2^{N-r}+1} & & & & l_{3 \cdot 2^{N-r}} \\ \vdots & & & \ddots & \vdots \\ l_{(r-1)2^{N-r}+1} & & \cdots & & l_{2^N} \end{bmatrix}. \tag{43}$$

Each $\delta_{2^{N-r}}^{l_i}$ is defined as 'block element' of the stacked matrix $\mathbf{L}$. It can be calculated straightforwardly that the ith column of $LG\Phi_{N-r}$ is the g_ith element of the ith column of $\mathbf{L}$; or equivalently, it can be claimed that the structural matrix $LG\Phi_{N-r}$ of the closed-loop system can be shaped by assigning proper g_i.

Example 4: Consider the structure matrix L of the controlled logic system in Example 3. Its stacked form is given by

$$\mathbf{L} = \delta_8\begin{bmatrix} 1, 1, 1, 1, 1, 1, 1, 5 \\ 1, 1, 1, 1, 1, 2, 3, 8 \end{bmatrix}. \tag{44}$$

If the feedback gain is designed by $G = \delta_2[1, 2, 1, 2, 1, 1, 2, 2]$, then

$$LG\Phi_3 = \delta_8[1, 1, 1, 1, 1, 1, 3, 8]. \tag{45}$$

It can then be claimed that, with the feedback control, there is one stable (attracting) fixed point δ_8^1 and one unstable fixed point δ_8^8. The stable fixed point δ_8^1 is an NE, according to Definition 2.

The above derivations can be summarised in the following theorem.

Theorem 3: The structural matrix of logic dynamic system (31) can be shaped by feedback control (37) into a desired structural matrix, if the columns of the desired structural matrix appear in the corresponding columns of the stacked matrix $\mathbf{L}$ given by (43).

According to the above theoretical results, the design procedure of G can be summarised as follows:

1. Rewrite the structural matrix L into its stacked form $\mathbf{L}$;
2. Find the state (the block element) with the lowest cost in $\mathbf{L}$ (if such block element exists);
3. Find the possible transient states in $\mathbf{L}$ to satisfy the performances specified by theorems or corollaries in Section

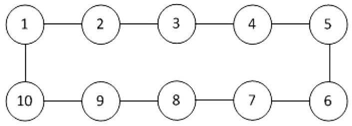

Fig. 2 *Communication structure of the grid with ten communities [27]*

3.2, such that the state with the lowest cost can be reached and maintained.

4. Design G to pick out the desired block elements in steps 2 and 3, such that the desired closed-loop structural matrix M_{d} can be achieved, and the closed-loop system is capable of reaching and maintaining the NE where the total cost is minimised.

Example 5: Consider the control logic system in Example 3. It can be shaped into the system with only one stable NE. The desired structure matrix can be given by

$$M_{\mathrm{d}} = \delta_8[1, 1, 1, 1, 1, 2, 3, 5], \tag{46}$$

and the control gain can be correspondingly designed by

$$G = \delta_2[1, 1, 1, 1, 1, 2, 2, 1]. \tag{47}$$

In fact, the unstable fixed point can be eliminated, if the last column of G is designed by δ_2^1.

4 Robust control for evolutionary game with model uncertainties

In this section, it is supposed that some of the columns in the structural matrix L are uncertain.

According to the theoretical result in Section 3.3, the effect of G is to select one element from each column in $\mathbf{L}$. The following theorem can be obtained straightforwardly.

Theorem 4: Consider the controlled logic system (31) with uncertainties in its structural matrix L. The uncertainties can be decoupled by the feedback control $u = Gx$, if there exists at least one block element that is not uncertain in each column of $\mathbf{L}$.

Proof: If there exists at least one element that is not uncertain in each column of $\mathbf{L}$, then G can be designed such that the determined block element in each column can be selected, and $LG\Phi_{N-r}$ is without uncertainties. □

Example 6: Consider the system in Example 3, where the community 1 sometimes reports fake information of its choice to its neighbours. The fake information is $\hat{x}_1 = \delta_2[2, 1]x_1$. It can be assumed that the prices paid by communities depend on actual information, and the controller knows the actual information. Then the structural matrix can be calculated by

$$L = \delta_8[*, 1, 1, 1, 1, 1, 1, 5, 1, *, *, *, 1, *, *, 8], \tag{48}$$

where $*$ denotes the columns that are uncertain due to the fake information. It follows that

$$\mathbf{L} = \delta_8\begin{bmatrix} *, & 1, & 1, & 1, & 1, & 1, & 1, & 5 \\ 1, & *, & *, & *, & 1, & *, & *, & 8 \end{bmatrix}. \tag{49}$$

where there exists at least one determined element in each column. The control gain can be designed by

$$G = \delta_2[2, 1, 1, 1, 2, 1, 1, 1], \tag{50}$$

such that the structural matrix of the closed-loop system is given by

$$LG\Phi_3 = \delta_8[1, 1, 1, 1, 1, 1, 1, 5], \tag{51}$$

and δ_8^1 is still an NE.

Remark 6: It should be noted that uncertainty decoupling does not necessarily indicate stabilisation. For stabilisation, it requires that the determined elements in $\mathbf{L}$ are capable of constructing a corresponding desired structural matrix.

5 Simulation and discussion

Consider the grid of ten communities, where the communication structure is described by a undirected circular graph, as displayed by Fig. 2. In this grid, the price of diesel power is given by $p_{\mathrm{d}} = 7$, and the price of grid power is given by

$$p_{\mathrm{g}}(n) = \frac{(n - 4.8)^2}{25} + 6.6, \tag{52}$$

where n is the number of communities using the grid power. In this example, communities 7 and 10 are subsidised by the grid provider, and are cooperative controllers. The updating strategies of other communities are given by unconditional imitation (2), where the cost function is given by (12) with the coefficient $\alpha = 0.8$. The objective is to design closed-loop control strategy (for communities 7 and 10), such that the total price $\sum_{i=1}^{10} p_i$ is the lowest. In the simulation, we suppose that the initial state (the choice of the eight communities other than the controllers) is given by $x(0) = \delta_{256}^{240}$.

With the modelling and control strategy presented in Section 3, it can be found in the stacked matrix $\mathbf{L}$ that the desired NE (the state with the lowest cost) is δ_{256}^4, and the feedback control gain in (37) can be designed (not uniquely) by

$$\begin{aligned} G_{4\times 256} = \delta_4[&3, 1, 1, 4, 4, 2, 1, 4, 4, 2, 2, 4, 2, 1, 2, 2, \\ &4, 2, 2, 4, 2, 1, 2, 4, 3, 1, 2, 4, 1, 3, 3, 1, \\ &4, 2, 3, 4, 2, 1, 2, 4, 2, 1, 2, 4, 1, 2, 3, 2, \\ &2, 1, 2, 4, 1, 2, 3, 4, 1, 2, 3, 1, 2, 3, 4, 1, \\ &4, 2, 3, 4, 2, 1, 2, 4, 2, 1, 2, 4, 1, 2, 3, 4, \\ &2, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, \\ &4, 1, 2, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, \\ &1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 1, \\ &4, 2, 3, 4, 2, 1, 3, 4, 2, 1, 3, 4, 1, 2, 3, 4, \\ &2, 1, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, \\ &2, 1, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, \\ &1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 4, 3, \\ &3, 1, 3, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 1, 1, 1, \\ &1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 2, 3, 3, \\ &1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 4, 3, \\ &1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 3, 1] \end{aligned} \tag{53}$$

to reach and maintain the NE. Simulation results are displayed in Figs. 3–5.

As can be seen from Fig. 3, with the proposed closed-loop control strategy of the cooperative communities, other communities are induced to use the grid power, and there are six communities using the grid power finally. The real-time price of the grid power is illustrated by Fig. 4, where the price varies with the number of communities using the grid power. The total price paid by all communities is shown in Fig. 5, where it is capable of reaching and maintaining a lowest value with the proposed closed-loop control.

It can be observed from the simulation results that, starting from $x = \delta_{256}^{240}$, it takes six steps to reach the NE. It implies that although the proposed feedback control guarantees the stability and robustness (as expected theoretically), but not necessarily the transient performance. Some future works on optimal feedback control of the controlled networked evolutionary game can be conducted to improve the transient performance.

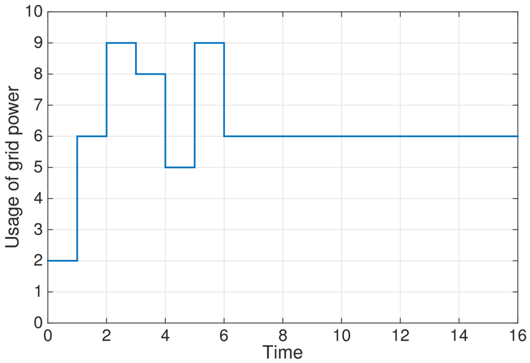

Fig. 3 *Number of communities using the grid power*

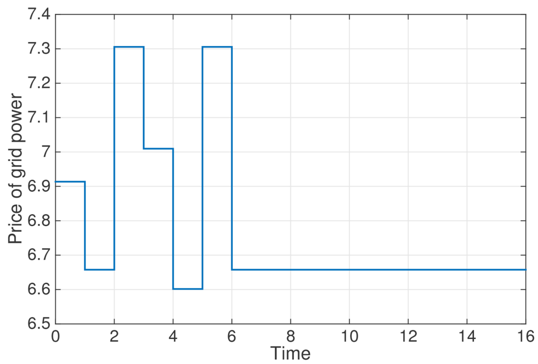

Fig. 4 *Variation of prices of the grid power*

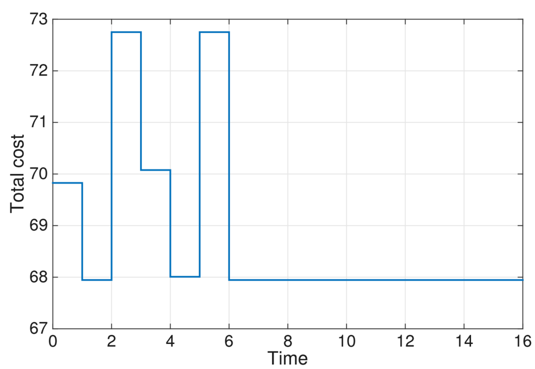

Fig. 5 *The total price paid by all communities*

Remark 7: In this paper, the NE is selected as the state with lowest cost in the stacked matrix **L**, and the proposed closed-loop control is designed to reach and maintain the NE according to theorems and corollaries in Section 3.2. It should be noted that, however, the proposed control does not necessarily guarantee the existence of the NE. For more information on existence of NE, see the theoretical results on controllability of evolutionary games [13] and logic systems [17].

6 Conclusion

In this paper, a closed-loop feedback control is constructed for management and control of a class of networked smart grid, where behaviours of the users are affected by their neighbours, and some users are cooperative to induce the behaviours of others. Dynamic modelling of the networked smart grid can be obtained in a standard form by using logic calculations and diagrams of true values. The feedback control is designed by using a newly proposed stacked form of the structural matrix. With the proposed feedback control, the structural matrix of the closed-loop system can be shaped into some desired forms, such that corresponding performances can be achieved. Moreover, based on the proposed structure shaping technique, a robust control is proposed such that some model uncertainties can be decoupled from the closed-loop system. Theoretical results are proved, and are illustrated by some examples.

7 References

[1] Nwulu, N.I., Xia, X.: 'Multi-objective dynamic economic emission dispatch of electric power generation integrated with game theory based demand response programs', *Energy Convers. Manage.*, 2015, **89**, pp. 963–974

[2] Zhang, J., Xia, X.: 'A model predictive control approach to the periodic implementation of the solutions of the optimal dynamic resource allocation problem', *Automatica*, 2011, **47**, (2), pp. 358–362

[3] Jiang, T., Cao, Y., Yu, L., *et al.*: 'Load shaping strategy based on energy storage and dynamic pricing in smart grid', *IEEE Trans. Smart Grid*, 2014, **5**, (6), pp. 2868–2876

[4] Liang, X., Li, X., Lu, R., *et al.*: 'Udp: usage-based dynamic pricing with privacy preservation for smart grid', *IEEE Trans. Smart Grid*, 2013, **4**, (1), pp. 141–150

[5] Zhu, B., Tazvinga, H., Xia, X.: 'Switched model predictive control for energy dispatching of a photovoltaic-diesel-battery hybrid power system', *IEEE Trans. Control Syst. Technol.*, 2015, **23**, (3), pp. 1229–1236

[6] Samadi, P., Mohsenianrad, H., Schober, R., *et al.*: 'Advanced demand side management for the future smart grid using mechanism design', *IEEE Trans. Smart Grid*, 2012, **3**, (3), pp. 1170–1180

[7] Atzeni, I., Ordonez, L.G., Scutari, G., *et al.*: 'Demand-side management via distributed energy generation and storage optimization', *IEEE Trans. Smart Grid*, 2013, **4**, (2), pp. 866–876

[8] Haider, H.T., See, O.H., Elmenreich, W.: 'Dynamic residential load scheduling based on adaptive consumption level pricing scheme', *Electr. Power Syst. Res.*, 2016, **133**, pp. 27–35

[9] Real, A.J.D., Arce, A., Bordons, C.: 'Combined environmental and economic dispatch of smart grids using distributed model predictive control', *Int. J. Electr. Power Energy Syst.*, 2014, **54**, pp. 65–76

[10] Du, L., Grijalva, S., Harley, R.G.: 'Game-theoretic formulation of power dispatch with guaranteed convergence and prioritized bestresponse', *IEEE Trans. Sustain. Energy*, 2015, **6**, (1), pp. 51–59

[11] Mohsenianrad, A., Wong, V.W.S., Jatskevich, J., *et al.*: 'Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid', *IEEE Trans. Smart Grid*, 2010, **1**, (3), pp. 320–331

[12] Fadlullah, Z.M., Quan, D.M., Kato, N., *et al.*: 'Gtes: an optimized game-theoretic demand-side management scheme for smart grid', *IEEE Syst. J.*, 2014, **8**, (2), pp. 588–597

[13] Cheng, D., He, F., Qi, H., *et al.*: 'Modeling, analysis and control of networked evolutionary games', *IEEE Trans. Autom. Control*, 2015, **60**, (9), pp. 2402–2415

[14] Stephens, E.R., Smith, D.B., Mahanti, A.: 'Game theoretic model predictive control for distributed energy demand-side management', *IEEE Trans. Smart Grid*, 2015, **6**, (3), pp. 1394–1402

[15] Zhu, B., Xia, X.: 'Lyapunov-based adaptive model predictive control for unconstrained non-linear systems with parametric uncertainties', *IET Control Theory Applic.*, 2016, **10**, (15), pp. 1937–1943

[16] Cheng, D., Qi, H.: 'A linear representation of dynamics of boolean networks', *IEEE Trans. Autom. Control*, 2010, **55**, (10), pp. 2251–2258

[17] Cheng, D., Qi, H.: 'Controllability and observability of boolean control networks', *Automatica*, 2009, **45**, (7), pp. 1659–1667

[18] Cheng, D., Qi, H., Li, Z., *et al.*: 'Stability and stabilization of boolean networks', *Int. J. Robust Nonlinear Control*, 2011, **21**, (2), pp. 134–156

[19] Li, H., Wang, Y.: 'On reachability and controllability of switched boolean control networks', *Automatica*, 2012, **48**, (11), pp. 2917–2922

[20] Cheng, D.: 'Disturbance decoupling of boolean control networks', *IEEE Trans. Autom. Control*, 2011, **56**, (1), pp. 2–10

[21] Yang, M., Li, R., Chu, T.: 'Controller design for disturbance decoupling of boolean control networks', *Automatica*, 2013, **49**, (1), pp. 273–277

[22] Fornasini, E., Valcher, M.E.: 'Optimal control of boolean control networks', *IEEE Trans. Autom. Control*, 2014, **59**, (5), pp. 1258–1270

[23] Zhao, Y., Li, Z., Cheng, D.: 'Optimal control of logical control networks', *IEEE Trans. Autom. Control*, 2011, **56**, (8), pp. 1766–1776

[24] Cheng, D., Zhao, Y., Xu, T.: 'Receding horizon based feedback optimization for mix-valued logical networks', *IEEE Trans. Autom. Control*, 2015, **60**, (12), pp. 3362–3366

[25] Fornasini, E., Valcher, M.E.: 'Fault detection analysis of boolean control networks', *IEEE Trans. Autom. Control*, 2015, **60**, (10), pp. 2734–2739

[26] Li, H., Wang, Y.: 'Boolean derivative calculation with application to fault detection of combinational circuits via the semi-tensor product method', *Automatica*, 2012, **48**, (4), pp. 688–693

[27] Zhu, B., Xia, X., Wu, Z.: 'Evolutionary game theoretic demand-side management and control for a class of networked smart grid', *Automatica*, 2016, **70**, pp. 94–100

[28] Ren, W.: 'Consensus tracking under directed interaction topologies: algorithms and experiments', *IEEE Trans. Control Syst. Technol.*, 2010, **18**, (1), pp. 230–237

[29] Qi, H., Cheng, D.: 'Analysis and control of boolean networks:a semi-tensor product approach', *Acta Autom. Sin.*, 2011, **37**, (5), pp. 1352–1356