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Lighting retrofit and maintenance models with decay and adaptive control

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Abstract: In lighting retrofit projects, a lamp population is subject to decay, which results in significantly deteriorated energy efficiency (EE) and reduced cost saving. Incremental retrofit and maintenance are studied to overcome the decay in the population, so that EE performance can be sustained. Current models of natural decay cannot reflect the interactive dynamics of incremental retrofit and maintenance, so a new decay model is proposed for these interventions. Using a control approach, a multiple-input and multiple-output state equation is formulated. Adaptive control laws are designed to cope with unknown parameters of the proposed model, and to achieve stable performance improvement. This new model is verified, based on empirical data, and the results of adaptive control indicate that the number of working lamps can be maintained as a required value.

1 Introduction

The deployment of energy efficiency (EE) programmes, as a kind of demand-side management (DSM), is one of the most useful alternative solutions for reducing power demand and greenhouse gas emissions [1–3]. With around 40% of the total demand, the building sector will have a great potential to reduce total demand, therefore improving the building EE becomes urgent [3, 4]. Since the start of this century, many policies and projects concerning building retrofit (also referred to as innovation or refurbishment in the literature) have been initiated all over the world to improve building EE, as building retrofit is currently the most feasible and practical way to reduce the demand of the building sector.

Many building retrofit projects are relevant to lighting retrofit [5–7]. Due to easy accessibility and energy saving, light retrofit projects are promoted in various EE incentive programmes, such as clean development mechanisms (CDM) [8], white tradable certificate schemes [9], DSM, and performance contracting [10]. In lighting retrofit projects, energy-efficient lamps, such as compact fluorescent light (CFL) and light-emitting diodes (LED), are used to retrofit less efficient incandescent lights. In general, building EE retrofit (BEER) refers to changing out-of-date facilities in existing buildings through innovative and efficient technologies for lighting, water heating, ventilation/cooling/heating, building envelope, and other energy-consuming systems [11, 12]. There are a large number of these energy systems, and their EE performance has highly complicated correlations. Therefore, designing an optimal retrofit strategy for minimal building energy consumption is a difficult task of BEER, especially BEER on a large scale.

In a recent study [13], the large-scale BEER was defined, modelled, and optimised in a time-building-technology framework. The large-scale BEER was unveiled in three dimensions, i.e. time, building, and technology. In the building dimension, different types of building, such as, office, commercial/residential/industrial buildings, school, and hospital, will be assigned different priorities for retrofit in a large-scale BEER project. In the technology dimension, different types of technology will have different priorities for retrofit. In the time dimension, incremental retrofit can be done every year, and investment is also assigned a different amount each year. In this framework of large-scale BEER, there remain several open issues that require further study, such as decay and maintenance.

In this paper, decay and maintenance in lighting retrofit projects will be studied, because lighting projects are representative and relatively simple to model. Lighting projects involve large populations that are suitable for statistical models. For example, energy-saving control strategies are proposed to minimise the energy consumption of multi-group lighting sources [14, 15]. For the LED lighting systems, lumen depreciation is studied by diagnosing individual LED failures using a photosensor system [16].

For lighting retrofit projects, one practical issue in the time dimension is to model the decay of the population with multiple interventions, such as incremental retrofit and maintenance [17]. The decay model of once-off retrofit has been studied in lighting retrofit projects, in which accurate decay models provide basis knowledge to design retrofit plans, and cost-effective metering plans [4, 18, 19]. In fact, the population of installed efficient lights are subject to deterioration due to certain factors, such as flickering, lamp burnout, and ballast failure, so the number of working lamps is dynamically decreasing. Consequently, the performance of energy saving, financial payback, and carbon emission deteriorates over time. In many projects, maintenance is essentially required in the contract, so that the failed facilities can be repaired or replaced to overcome the deterioration in EE performance. In case of both incremental retrofit and maintenance, the population decay model should be re-formulated to solve practical problems in EE applications, such as lighting retrofit, measurement and verification [20–22], energy reliability [23–25], and distributed generation [26]. Such kinds of decay become more complicated than natural decay in the following three respects.

First, there is an aggregate population of installed lights with different working time. For example, the population of installed lights in the first year has a decay curve different from that of the population of installed lights in the second and subsequent years. Second, in the case of both incremental retrofit and maintenance, interaction of multiple variables is involved in the decay model. The new intervention of maintenance has brought new characteristics to the decay model. Maintenance will change the average EE performance and the average working time, and consequently the decay curve will become non-singular. Third, the parameters of the decay model are usually unknown, although they can be estimated through additional tests on a population of similar lamps.

The other issue in the time dimension is making long-term plans for incremental retrofit and maintenance even if the decay model is known. The retrofit plans have been intensively studied by using empirical [27] and multi-criteria methods [28, 29]. In approaches to optimisation, several conflicting objectives, such as EE, financial payback, carbon emission, and other technical, economic, ecological, social, aesthetical concerns, have been optimised in the design of retrofit plans. For stable performance improvement, maintenance plans for lighting and other building facilities have become a recent focus in this research area. For lighting retrofit projects, optimal maintenance planning is proposed to optimise the number of lights to replace the failed lamps, so that the EE lighting project achieves sustainable performance in terms of maximal energy savings and the cost-benefit ratio [30]. For general BEER projects, corrective maintenance planning for building energy systems, such as lighting, monitoring, water heating, and oven, is proposed to design optimal maintenance plans for maximising energy saving and minimising the internal rate of return [31, 32].

However, current planning schemes in the literature cannot be extended in retrofit projects with multiple interventions, as they have neglected the interaction between incremental retrofit and maintenance. To the best of our knowledge, there are few studies of combined planning for incremental retrofit and maintenance, especially in the large-scale BEER projects. As the parameters in the decay model are unknown, the parameters should be estimated at the early stage of implementation process, which brings extra challenges for planning. Therefore, in this paper, adaptive control is studied for the planning problem of lighting projects with unknown parameters. The stability of adaptive control can ensure stable EE performance of these retrofit projects. Due to the closed-loop mechanism, uncertainty about lamp decay or lumen degradation can be attenuated.

The contributions of this paper are three-fold. First, a mathematical model is built for lighting projects with incremental retrofit and maintenance. The decay of the lamp population has a logistic-like curve that is related to the number of retrofitted lamps. The proposed model is verified based on the empirical data and the interactive dynamics can also be observed in the verification. Second, the decay process is studied by using a control approach, in which a multiple-input and multiple-output (MIMO) control system is derived, based on the proposed decay model. Third, to handle the unknown parameters in the system model, adaptive control laws are designed for planning incremental retrofit and maintenance. The stability of the proposed control laws is proven with the Lyapunov theory, which ensures that the EE performance can be sustained at a desired value.

The paper is organised as follows. Several models of natural lighting decay are introduced in Section 2. In Section 3, the decay of an aggregate population with incremental retrofit and maintenance is modelled, and the MIMO state equation is formulated. In Section 4, adaptive control laws are newly designed, and the stability of the adaptive controller is proved. In Section 5, the model is verified, and the adaptive control is tested and analysed in the simulation. Then paper is concluded in Section 6.

2 Models of natural decay

System dynamics in lighting projects, such as the CFL project and the LED project, is caused by the performance decay of the lamps. The performance decay or deterioration of working lamps affects the energy saving, financial profit, and carbon emission over the evaluation period. An exact model of the population decay is necessary to reflect the system dynamics. Irrespective of the types of light used, three kinds of decay model are commonly applied.

Let $N(t)$ denote the population size at the tth year. $N(0)$ is the size of initial population. In many natural phenomena, such as population growth and radioactive decay, quantities grow or decay at a rate proportional to their size. In other words, they satisfy the following differential equation:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = kN(t), \tag{1}$$

where k is the decay rate. Note that (1) is called the law of natural growth if $k > 0$, and it is called the law of natural decay if $k < 0$.

The only solution of (1) is an exponential function

$$N(t) = N(0)e^{kt}. \tag{2}$$

Remark 1: Equation (2) satisfied the law of (1) as

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(0)(e^{kt})' = kN(0)e^{kt} = kN(t). \tag{3}$$

The exponential decay model is commonly used in different areas [33]. Normally a constant decay rate (failure rate) applies to this model, but in certain cases, the decay rate changes over time. In this lighting application, the lamp population exhibits ageing, so that old lamps are more likely to fail at any time than newly installed lamps.

As the second kind of decay model, a linear population decay, suggested in CDM guidelines, is utilised in the lighting projects [18]. In the linear model, the population is linearly decayed over the rated lifetime L as

$$N(t) = \begin{cases} N(0)\left(1 - \frac{t}{L} * \frac{100 - \rho_L}{100}\right), & t \le L \\ 0, & t > L \end{cases}, \tag{4}$$

where ρ_L is the percentage of surviving lamps left at the end of the rated lifetime L ($\rho_L = 50$ is recommended) [18]. When $t > L$, all lamps are deemed to have failed in this model.

In empirical studies on the useful life of facilities in retrofit projects [34], the decay curves are found to have logistic shapes. The Poland efficient lighting project (PELP), conducted by the World Bank through the International Finance Corporation, also indicates a logistic curve for a population of 1.2 million lamps [35]. According to studies in the South Africa context [36, 37], a general form of logistic function is formulated to fit empirical data. As the third kind of decay model, the general form is expressed as

$$N(t) = \frac{N(0)}{\gamma + e^{\beta t - K}} \tag{5}$$

where β and γ are two parameters related to characteristics of the device, and $K = -\ln(1 - r)$. As stated, this general form of the decay model is especially applicable to the engineering context.

According to the three models, the decay dynamics can be obtained for each model as

$$\frac{\mathrm{d}\Phi_1}{\mathrm{d}t} = k\Phi_1 \tag{6}$$

$$\frac{\mathrm{d}\Phi_2}{\mathrm{d}t} = -\frac{100 - \rho_L}{100L}, \quad t < L \tag{7}$$

$$\frac{\mathrm{d}\Phi_3}{\mathrm{d}t} = -\beta\Phi_3(1 - \gamma\Phi_3) \tag{8}$$

where $\Phi = N(t)/N(0)$ is the proportion of working lamps surviving at time t. Φ_1, Φ_2, Φ_3 denote the proportion calculated in each model, respectively. Note that (8) is deduced from the differentiation at both sides of (5). For each model, an example of the decay curve is plotted in Fig. 1.

In this figure, decay parameters are set as $k = -1$, $L = 5$, $K = 3$, $\beta = 0.95$, and $\gamma = 1.05$. Note that these models have been used to approximate the decay of CFL and LED. These models only fit essential factors of natural decay without any intervention, so these decay curves are all non-increasing as shown in the figure.

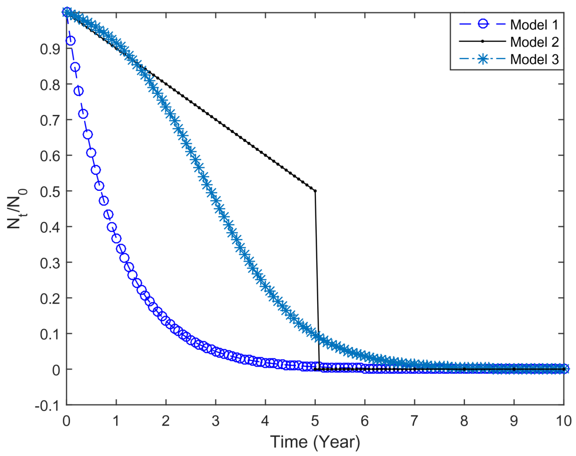

Fig. 1 *Decay curves in the three decay models ($k = -1$, $L = 5$, $K = 3$, $\beta = 0.95$ and $\gamma = 1.05$)*

3 System dynamics with multiple interventions

When only considering natural decay in the lighting retrofit project, system dynamics can be generalised as

$$\dot{\Phi} = f(\Phi), \tag{9}$$

where $f(\Phi)$ is the decay function. With respect to model 3, it follows that $f(\Phi) = -\beta\Phi(1-\gamma\Phi)$.

In the control approach, the number of working lamps is regarded as a state variable, i.e. $x(t) = N(t)$. Based on the logistic decay models (5) and (8), the state equation can be expressed as

$$\dot{x} = -\beta x + \frac{\beta\gamma}{x(0)}x^2, \tag{10}$$

where x is the number of working lamps, and $x(0)$ is the size of initial population. Note that the control system studied is a non-linear system.

Remark 2: The state equation (10) is obtained by the differentiation at both sides of (5) as

$$\begin{aligned}\dot{x} &= x(0)\left(\frac{1}{\gamma + e^{\beta t - K}}\right)' \\ &= -x(0)\frac{\beta e^{\beta t - K}}{(\gamma + e^{\beta t - K})^2}\end{aligned}. \tag{11}$$

By substituting $e^{\beta t - K} = x(0)/x - \gamma$ in the above equation, (10) can be obtained.

The dynamics of natural decay cannot fit the practical decay dynamics well when there are multiple interventions, which include incremental retrofit and maintenance. Incremental retrofit means that retrofit does not only happen at the beginning, but also happens subsequently at multiple times. Compared with once-off retrofit, the number of retrofitted facilities in this case is incremental over time, so we call this incremental retrofit. To ensure stable performance of EE and cost saving, the broken or ill-conditioned facilities should be repaired or replaced. Maintenance means replacement or repair of retrofitted lamps that have deteriorated, and maintenance will be conducted frequently every year. It is obvious that both incremental retrofit and maintenance have certain effects on the decay dynamics (10).

However, (10) cannot be applied, when retrofit and maintenance are interacted during the whole evaluation period. At time t, a new EE facility could be used for maintenance of a retrofitted facility in a poor condition, and it could also be used for retrofitting an existing old facility. In other words, incremental retrofit will increase the population size and the number of working lamps, but maintenance only increases the number of working lamps. When different effects of incremental retrofit and maintenance are included into (10), the number of working lamps with incremental retrofit and maintenance can be expressed as

$$\dot{x} = -\beta x + \frac{\beta\gamma x^2}{x(0) + \int_{\tau=0}^{t} u_1\,\mathrm{d}\tau} + u_1 + u_2, \tag{12}$$

where $u_1(t)$ is the number of retrofitted lamps at time t, and $u_2(t)$ is the number of lamps undergoing maintenance. Note that when there is no intervention, i.e. $u_1(t) = u_2(t) = 0$, (12) is equivalent with (10). When only maintenance is done, i.e. $u_1(t) = 0$, the population size remains the same as $x(0)$, and the number of working lamps increases by $u_2(t)$. When only retrofit is done, i.e. $u_2(t) = 0$, the population size increases by the cumulative number of retrofitted lamps, i.e. $\int_{\tau=0}^{t} u_1\,\mathrm{d}\tau$, and the number of working lamps also increases by $u_1(t)$.

Define $x_1 = x(0) + \int_{\tau=0}^{t} u_1\,\mathrm{d}\tau$ and $x_2 = x$. The state equation 12 can be transformed as

$$\begin{cases} \dot{x}_1 = u_1, \\ \dot{x}_2 = -\beta x_2 + \dfrac{\beta\gamma x_2^2}{x_1} + u_1 + u_2, \end{cases} \tag{13}$$

In the control approach, the system dynamics with incremental retrofit and maintenance is a standard non-linear system. Actually, $x_1(t)$ is the cumulative number of retrofitted lamps at time t, and $x_2(t)$ is the number of working lamps at time t. Note that $x_1(t) \geq x_2(t)$ is a practical constraint.

Given a sampling period t_0, the continuous control system can be written into a discrete form. The discrete system can be formulated as (14), where k represents the index of sample and $x_1(0) \geq x_2(0)$. $u_1(k)$ is the number of retrofitted lamps over the kth interval. $u_2(k)$ is the number of lamps undergoing maintenance over the kth interval. $x_1(k)$ is the cumulative number of retrofitted lamps at the kth interval, and $x_2(k)$ is the number of working lamps at the kth interval

$$\begin{cases} x_1(k+1) = x_1(k) + u_1(k)t_0, \\ x_2(k+1) = (1-\beta t_0)x_2(k) + \beta\gamma t_0\dfrac{x_2(k)^2}{x_1(k)} + u_1(k)t_0 + u_2(k)t_0, \end{cases} \tag{14}$$

The average working time of the population is defined as the working time of all lamps divided by the population size. The average working time is related to the percentage of working lamps and the working hours of each lamp. The average working time can indicate the lumen level, which means that a lamp with more working hours will be subject to more lumen deprecation.

Theorem 1: Given a population of energy efficient lamps, maintenance will result in a shorter average working time than incremental retrofit.

Proof: Assume that the average working hours of the population at time t is $\overline{L}_t$, and the population size of retrofitted lamps is $x_1(t)$. If only n lamps is maintained, the average working hours after maintenance can be calculated as $(\overline{L}_t(x_1(t) - n))/x_1(t)$.

If only n lamps is retrofitted, the average working hours after the incremental retrofit is calculated as $(\overline{L}_t x_1(t))/(x_1(t) + n)$.

It is obvious that

$$\overline{L}_t(x_1^2(t) - n^2) < \overline{L}_t x_1^2(t). \tag{15}$$

Divide both sides with $x_1(x_1(t) + n)$, then

$$\frac{\overline{L}_t(x_1(t) - n)}{x_1(t)} < \frac{\overline{L}_t x_1(t)}{x_1(t) + n}. \tag{16}$$

The proof is completed. □

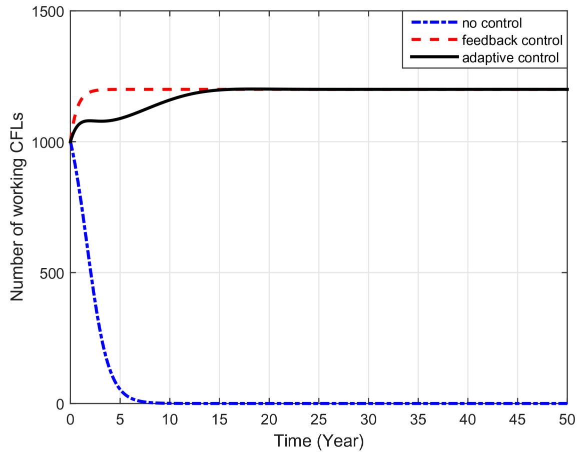

Fig. 2 *Performance of controller design ($k_1 = 1.5$ and $k_2 = 0.5$)*

Table 1 PELP empirical data on surviving rates

Year	1	2	3	4	5	6	7	8	9	10	11
Surviving rate	0.97	0.97	0.91	0.83	0.77	0.4	0.29	0.08	0.02	0.02	0.02

4 Adaptive control

The controller is necessary to keep the number of working lamps in the lighting retrofit, so the performance of EE and cost saving can be stable. As shown in Fig. 2, the number of working lamps will become 0, if there is no controller.

Assume β and γ are known, a feedback controller is required to achieve stable states, so that

$$\lim_{t \to \infty} x_1(t) = r_1, \ \lim_{t \to \infty} x_2(t) = r_2, \tag{17}$$

where r_1 is the reference value of the population size, and r_2 is the reference value of the working lamps. Note that $r_1 \geq r_2$ holds.

Define the tracking error as

$$e_1 = x_1 - r_1, \tag{18}$$

$$e_2 = x_2 - r_2. \tag{19}$$

Take the derivative of e_1 and e_2. It yields

$$\dot{e}_1 = \dot{x}_1 = u_1, \tag{20}$$

$$\begin{aligned} \dot{e}_2 = \dot{x}_2 &= -\beta x_2 + \beta\gamma\frac{x_2^2}{x_1} + u_1 + u_2 \\ &= \phi[-\beta, \beta\gamma]^{\mathrm{T}} + u_1 + u_2, \end{aligned} \tag{21}$$

where $\phi = [x_2, (x_2^2/x_1)]$ is the composite function used for simplicity.

To cancel the non-linear items in (21), a feedback controller is straightforwardly designed as

$$u_1 = -k_1 e_1, \tag{22}$$

$$u_2 = -k_2 e_2 - u_1 - \phi p, \tag{23}$$

where $k_1 > 0$ and $k_2 > 0$ are the control gains. p is the parameter vector to be determined in the controller design. In the assumption of known parameters, $p = [-\beta, \beta\gamma]^{\mathrm{T}}$ can be used to achieve stable control. In other words, the number of working lamps can be kept as the required value with the above scheme according to Theorem 2.

Theorem 2: Assume β and γ are known, the closed-loop system under the feedback controller (22) and (23) with $p = [-\beta, \beta\gamma]^{\mathrm{T}}$ is Lyapunov stable.

The proof has been given in the Appendix. As shown in Fig. 2, the feedback controller can drive the number of working lamps towards the reference value (set as 1200 for illustration). However, the parameters of the model must be known in the feedback control.

If β and γ are unknown in practical applications, it is difficult to determine proper values of p [38]. In this situation, it is necessary to design an adaptive controller for ensuring stable EE performance.

For the adaptive control, the following control scheme is proposed:

$$u_1 = -k_1 e_1 \tag{24}$$

$$u_2 = -k_2 e_2 - u_1 - \phi\hat{p} \tag{25}$$

with the adaptive law for $\hat{p}$ given by

$$\dot{\hat{p}} = \eta\phi^{T} e_2, \tag{26}$$

where $\eta > 0$ is the updating rate, and $\hat{p}$ is the estimate value of p. When the parameters in the decay model are unknown, the proposed adaptive control scheme can also keep the number of working lamps as the required value according to the following theorem.

Theorem 3: If the adaptive controller (24) and (25) with the adaptive law (27) is used, then it is ensured that the tracking error turns to zero as $t \to \infty$

The proof has been given in the Appendix. As shown in Fig. 2, the adaptive controller can also drive the number of working lamps to the reference value, although the parameters of the model are unknown.

For a discrete form, set $\hat{p}(0) = 0$, and the adaptive law for $\hat{p}$ can be expressed as

$$\hat{p}(t+1) = \hat{p}(t) + \eta\phi^{\mathrm{T}}(t)e_2(t)t_0. \tag{27}$$

The adaptive controller can be expressed as

$$u_1(t) = -k_1 e_1(t), \tag{28}$$

$$u_2(t) = -k_2 e_2(t) - u_1(t) - \phi(t)\hat{p}(t). \tag{29}$$

5 Simulation verification

A lighting retrofit project for retrofitting 1500 incandescent lamps is evaluated. After initial retrofit, the population size of retrofitted CFLs is 1000. For each incandescent lamp, the rated power is 60 W. For each CFL, the rated power is 14 W. Based on empirical CFL data in Table 1, parameters in the proposed model are assumed known as $\beta = 0.921$ and $\gamma = 0.986$ (reported in [19]) in the first two case studies. In real applications, these two parameters are usually unknown. In case 3, β and γ are unknown constants, and the performance of adaptive control will be analysed.

As reported in [30], LED decay also follows the logistic curve like that of CFL decay, so observations on the CFL project could be expected to apply to the LED project too. For simplicity, simulation verification on a LED project is omitted here.

5.1 Case 1: Comparison of different models

The proposed model is compared with existing models referred to in Section 2. As known, model 1 is the natural decay model; model 2 is the linear decay model; and model 3 is the logistic decay model. The main characteristic of the proposed model is its ability to reflect different effects of incremental retrofit and maintenance. However, models 1, 2, and 3 cannot be directly applied to describe decay dynamics in such interventions. For fair comparison, the CFL population of incremental retrofit or maintenance (at fourth year) is regarded as an independent population in the three existing models. Then decay curves of three existing models can be plotted as shown in Fig. 3. In the first two tests, 200 CFLs are used for

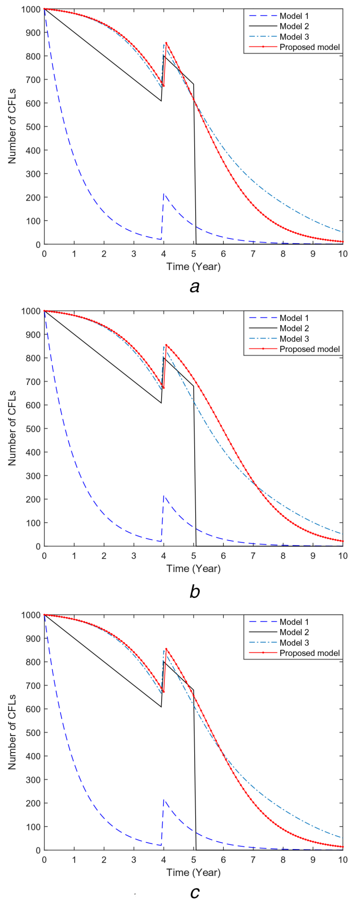

Fig. 3 *Model comparisons*
(a) Decay curves after the fourth-year retrofit, ***(b)*** Decay curves after the fourth-year maintenance, ***(c)*** Decay curves after combined retrofit and maintenance

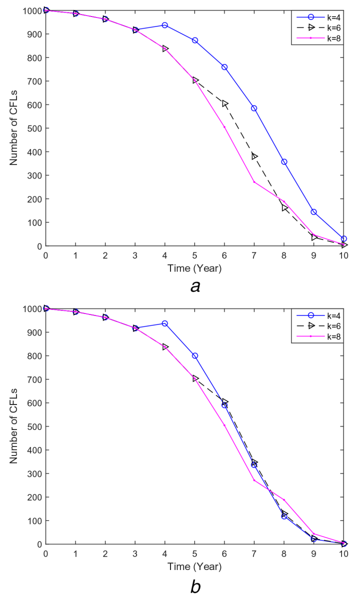

Fig. 4 *Decay curves with intervention at the fourth, sixth, and eighth years, respectively*
(a) Maintenance, ***(b)*** Incremental retrofit

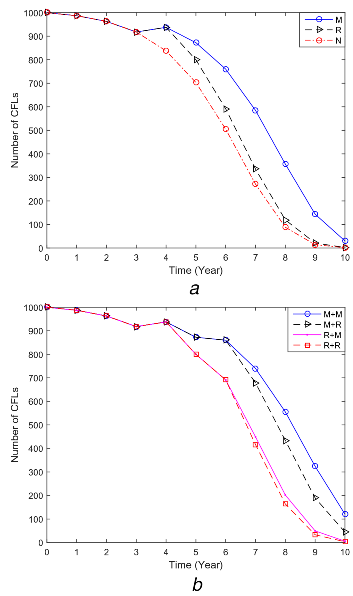

Fig. 5 *Decay curves of different interventions*
(a) Separate intervention, ***(b)*** Multiple interventions

retrofit and maintenance, respectively. The decay curves are plotted in Figs. 3*a* and *b*.

In comparison, another test is conducted where 100 CFLs are used for retrofit and 100 CFLs are used for maintenance, as shown in Fig. 3*c*. It can be noticed that decay curves of models 1, 2, and 3 are the same in the three tests, but decay curves of the proposed model are different. The reason is the fact that the decay dynamics change as incremental retrofit and maintenance are done. These changes are not considered in existing models, but considered in the proposed model. The detailed effects of incremental retrofit and maintenance will be analysed in the following case studies.

5.2 Case 2: Comparison of intervention factors

In the second case, the effects of the intervention time are first evaluated. Hundred new CFLs are used to replace the broken CFLs for maintenance at the fourth, sixth, and eighth year, respectively. The decay curves are plotted in Fig. 4*a*. It can be noticed that the maintenance in the early years shows better results than in the subsequent years. The maintenance at the fourth year results in the slowest decay.

In comparison, 100 CFLs are used to replace another 100 incandescent lamps for incremental retrofit at the fourth, sixth, and eighth year, respectively. As shown in Fig. 4*b*, the same conclusion can be drawn that the incremental retrofit in the early years shows better performance than in the latter years. However, the time effects in the incremental retrofit are not as significant as those in the maintenance. In the case of incremental retrofit, the decay curves of the fourth and sixth years are overlapped after the sixth year. One possible reason is the fact that the population of the incremental retrofit is larger, so that the average effect is less.

Two kinds of interventions, i.e. maintenance and incremental retrofit, are evaluated in the proposed model. In the fourth year, 100 CFLs are used to replace the broken CFLs for maintenance. The decay curve is shown as ‘M’ in Fig. 5*a*. In comparison, 100 new CFLs are used to replace another 100 incandescent lamps for incremental retrofit at the same time. The decay curve is shown as ‘R’ in the figure. The natural decay without any intervention is shown as ‘N’ in the figure. It can be noticed that interventions can postpone the decay process, as the post-intervention population of CFLs has a slower decay rate than the pre-intervention population. It can also be observed that the decay rate of maintenance is slower than that of incremental retrofit as shown in the figure, and that the overall population after maintenance has fewer average working hours than the population after incremental retrofit, which matches the statement of Theorem 1.

Furthermore, multiple interventions are also evaluated in this case study. At the fourth and seventh years, maintenance and incremental retrofit could be chosen by decision makers. The decay curves of different combinations are plotted in Fig. 5*b*. In the figure, ‘ $M+M$’ means that maintenance is conducted at the fourth and seventh year, respectively; ‘ $M+R$’ means maintenance is conducted at the fourth year and incremental retrofit is conducted at the seventh year; ‘ $R+M$’ means that incremental retrofit is conducted at the fourth year and maintenance is conducted at the seventh year; ‘ $R+R$’ means that incremental retrofit is conducted

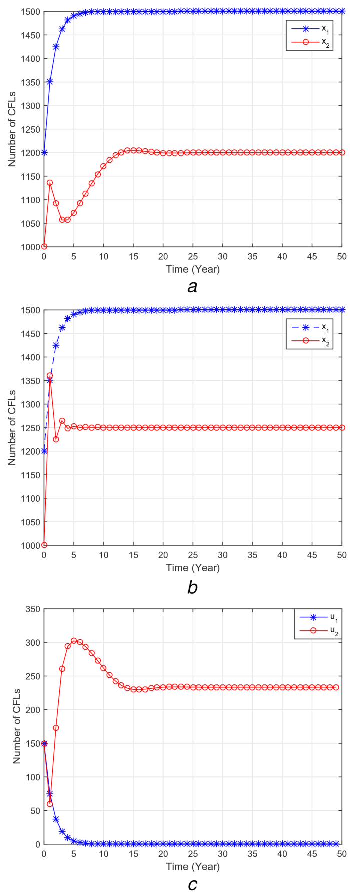

Fig. 6 *Comparison of adaptive control and feedback control*
(a) State profiles in the adaptive control, ***(b)*** State profiles in the feedback control, ***(c)*** Input variables in the adaptive control

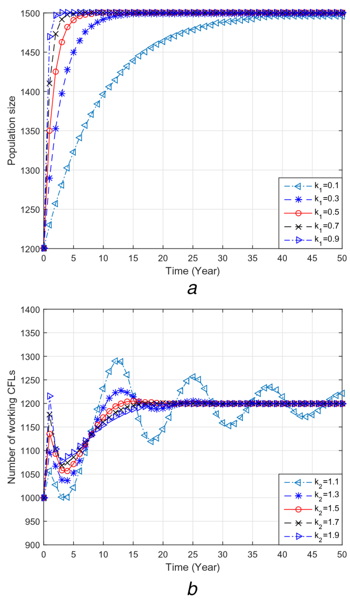

Fig. 7 *Effects of the control gains*
(a) Population size under different k_1, ***(b)*** Number of working CFLs under different k_2

at the fourth and seventh years, respectively. According to Theorem 1, the same observation can be made that the decay of '$M + M$' has the slowest rate, and that the decay of '$R + R$' has the fastest rate.

5.3 Case 3: Performance of adaptive control

In the adaptive controller, the control gains are set as $k_1 = 0.5$ and $k_2 = 1.5$, and the updating rate is $\eta = 1 * 10^{-7}$. The reference values

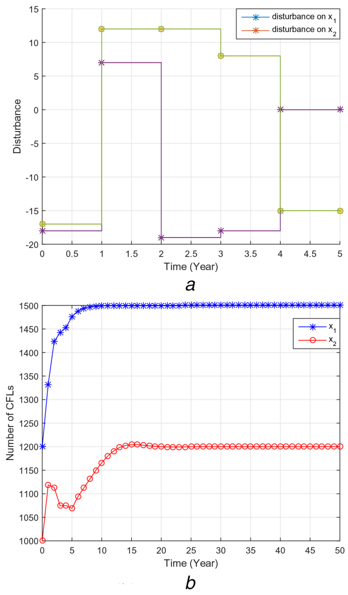

Fig. 8 *Effects of the state uncertainty*
(a) Profiles of the state disturbance, ***(b)*** Profiles of the state variables

of x_1 and x_2 are $r_1 = 1500$ and $r_2 = 1200$, respectively. In other words, the population size of CFLs is expected to be 1500, and the number of working CFLs is expected to be 1200. The control laws $u_1(t)$ and $u_2(t)$, i.e. retrofit and maintenance plans, follow (24) and (25) designed in the adaptive control.

For the adaptive control, the profiles of state variables x_1 and x_2 are plotted in Fig. 6*a*. In the adaptive control, the steady-state errors converge to 0 at finite time. As shown in the figure, it is indicated that $x_1(9) = 1500$ and $x_2(20) = 1200$. In comparison, the state profiles of feedback control, in which $p = [-0.9, 0.81]$, are also given in Fig. 6*b*. It can be noticed that the steady-state error is present in the feedback control. For the adaptive control, the profiles of input variables are plotted in Fig. 6*c*. It can be observed that the maintenance has a constant value and no retrofit is required when $t \geq 20$.

The parameters in the controller are converging to $p_1 = -0.1210$ and $p_2 = -0.0919$. With respect to EE, energy saving is related with the number of working CFLs and daily burning hours. If the average daily burning hour is 5 h, energy saving in the first year is 83,950 kWh. Energy saving in the first 5, 10, and 20 years is 448,380, 915,730, and 1,920,700 kWh, respectively. After 20 years, annual energy saving is constant at 100,740 kWh.

The robustness of control gains is also evaluated in this study. When k_2 and η are kept unchanged, k_1 is set at 0.1, 0.3, 0.5, 0.7, and 0.9, respectively. With these different settings, the profiles of the CFL population size are plotted in Fig. 7*a*. It can be observed that all profiles converge to the reference value, which indicates the robustness of k_1. A large value of k_1 causes x_1 to converge rapidly. When k_1 and η are kept unchanged, k_2 is set at 1.1, 1.3, 1.5, 1.7, and 1.9, respectively. With these different settings, the profiles of working CFLs have been plotted in Fig. 7*b*. It can be observed that the profiles converge to the reference value, which indicates the robustness of k_2. However, a small k_2 (e.g. $k_2 < 1.1$) causes x_2 to converge slowly with some oscillation.

The adaptive control is also tested in a case with state uncertainty. Assume that state variables experience disturbance during the first 5 years, and the disturbance values are random numbers on the scale $[-20, 20]$, as shown in Fig. 8*a*. As a result, state profiles can also converge to reference ones, as shown in Fig. 8*b*. Therefore, it can be concluded that the designed adaptive controller is stable to reject some uncertainty.

6 Conclusion

As an example of BEER, the lighting retrofit project is studied. In consideration of incremental retrofit and maintenance, a new decay model is proposed for the lighting retrofit project. Based on the characteristics of natural decay, the population decay with multiple interventions is formulated in the proposed model. In the control approach, a MIMO state equation is formulated to express the interactive dynamics based on the proposed decay model. Retrofit and maintenance plans are studied to stabilise the number of working lamps and the size of the overall population. To cope with unknown parameters of the system, an adaptive control approach is proposed to design stable plans. The stability is proven theoretically, and is tested in simulations.

Several observations were made in this study. First, maintenance could contribute more to conquer performance decay than incremental retrofit. Second, the early intervention (maintenance or retrofit) was preferred to postpone performance decay. Third, the adaptive control was robust to deliver stable EE performance. This work is challenging and important in the field of energy system and reliability. In future, stochastic models could be studied for the economic analysis, efforts on novel LED models, and control methods could also be made with regard to emergence of LED.

7 References

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8 Appendix

8.1 Appendix 1: Proof of Theorem 2

Denote the Lyapunov function as $V(x_1, x_2)$. A Lyapunov function candidate is defined as

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2. \tag{30}$$

For $\forall e_1 \neq 0, \forall e_2 \neq 0$, it is obvious that $V > 0$. The derivative function can be deduced as

$$\dot{V} = \dot{e}_1 e_1 + \dot{e}_2 e_2. \tag{31}$$

Substituting (20) and (23) into the above equation, the derivative can be transformed as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 < 0. \tag{32}$$

According to Lyapunov stability theory, the feedback controller is stable. The proof is completed.

8.2 Appendix 2: Proof of Theorem 3

Choosing the Lyapunov function candidate as

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2\eta}\tilde{p}^T\tilde{p} \tag{33}$$

where $\tilde{p} = p - \hat{p}$. The derivative of V w.r.t. time is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 - \frac{1}{\eta}\tilde{p}^T\dot{\hat{p}} \\ &= e_1 u_1 + e_2(\phi p + u_1 + u_2) - \frac{1}{\eta}\tilde{p}^T\dot{\hat{p}} \end{aligned} \tag{34}$$

Substituting the control laws (24) and (25) into (34), it can be deduced that

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 + e_2 \phi \tilde{p} - \frac{1}{\eta} \tilde{p}^T \dot{\hat{p}} \tag{35}$$

Inserting the adaptive law (27) into (35), it can be deduced that

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 \le 0 \tag{36}$$

which shows that $V(t)$ is globally uniformly ultimately bounded (i.e. $V(t) \in L_\infty$), which implies that $e_1 \in L_\infty$, $e_2 \in L_\infty$, and $\tilde{p} \in L_\infty$, which further implies that $x_1 \in L_\infty$, $x_2 \in L_\infty$, and $\hat{p} \in L_\infty$. From the definition of ϕ, we have $\phi \in L_\infty$. Then from (24) and (25) and (27), it follows that $u_1 \in L_\infty$, $u_2 \in L_\infty$, and $\dot{\hat{p}} \in L_\infty$. From (20) and (21) it is seen that $\dot{e}_1 \in L_\infty$ and $\dot{e}_2 \in L_\infty$. From (36) we have

$$k_1 \int_0^t e_1^2(\tau)\,\mathrm{d}\tau + k_2 \int_0^t e_2^2(\tau)\,\mathrm{d}\tau + V(t) = V(0) \tag{37}$$

which implies that $e_1 \in L_2$ and $e_2 \in L_2$. According to Barbalat Lemma, it shows that $\lim_{t \to \infty} e_1(t) \to 0$ and $\lim_{t \to \infty} e_2(t) \to 0$ as $t \to \infty$. The proof is completed.