

Continuous output feedback stabilization for nonlinear systems based on sampled and delayed output measurements

Yanjun Shen^{1,*}, Daoyuan Zhang¹ and Xiaohua Xia²

¹College of Electrical Engineering & New Energy, China Three Gorges University, Yichang, Hubei, 443002, China

²Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa

SUMMARY

This paper addresses the problem of output feedback stabilization for nonlinear systems with sampled and delayed output measurements. Firstly, sufficient conditions are proposed to ensure that a class of hybrid systems are globally exponentially stable. Then, based on the sufficient conditions and a dedicated construction continuous observer, an output feedback control law is presented to globally exponentially stabilize the nonlinear systems. The output feedback stabilizer is continuous and hybrid, and can be derived without discretization. The maximum allowable sampling period and the maximum delay are also given. At last, a numerical example is provided to illustrate the design methods. Copyright © 2015 John Wiley & Sons, Ltd.

Received 2 January 2015; Revised 24 September 2015; Accepted 3 November 2015

KEY WORDS: nonlinear systems; output feedback stabilization; hybrid systems; sampled-data; delayed measurements

1. INTRODUCTION

Recently, great progress has been made in the problem of design global asymptotic output feedback control laws for nonlinear systems. For example, the problem of global asymptotic stabilization by output feedback has been studied for a class of nonlinear systems [1]. The nonlinear terms considered in [1] admit an incremental rate that is dependent on the measured output. In [2], the author introduced a technique to stabilize a fully linearizable nonlinear system. The technique was utilized in [3] and [4]. In [5], a linear output feedback controller with dynamic high gain was presented to globally regulate a class of nonlinear systems.

It should be noted that the previous results concerned with output feedback stabilization are based on continuous time analysis. However, for a networked control system, the output is usually transmitted through a shared band-limited digital communication network. It is only available at discrete-time instants. Therefore, it is interesting to study output feedback stabilization for continuous systems with sampled and delayed measurements. More recently, three main approaches are proposed to deal with these problems. The first one is based on discrete time analysis by introducing a consistent approximation of the exact discretized model [6–8]. A quadratic observer incorporated with a quadratic dynamic feedback was proposed to achieve quadratic approximated feedback linearization with stability [7]. In [8], multirate sampled-data output feedback control of a class of nonlinear systems was presented based on high gain observers. It should be noted that in some cases, it will be difficult to obtain an exact discrete time model for a nonlinear system. The second one is based on continuous time analysis followed by discretization [9–11]. For example, sampled-data

*Correspondence to: Yanjun Shen, College of Electrical Engineering & New Energy, China Three Gorges University, Yichang, Hubei, 443002, China.

†E-mail: shenyj@ctgu.edu.cn

output feedback stabilization of nonlinear systems by using high-gain observers has been considered in [9–11]. By carefully choosing the sampling period, the authors proposed a sampled-data output feedback controller such that the closed-loop systems are regionally or globally stable [12]. The results obtained in [12] are based on the assumption that full-state are measurable. The third one is based on a mixed continuous and discrete time analysis without discretization [13–18]. For example, based on the existence of a controller and a special Lyapunov function satisfying certain \mathcal{L}_2 gain conditions, the authors presented a sampled-data output feedback stabilizer to ensure that the closed-loop system is globally stable by using a hybrid system method [13]. It should be noted that the closed-loop system is required to transform into the hybrid system introduced in [19]. In [17, 20], a state feedback law has been constructed to achieve global asymptotic stabilization for nonlinear systems under sampled and delayed measurements, and with inputs subject to delay and zero-order hold. The sampled-data feedback is based on a predictor mapping, which can be constructed inductively. There are few results on sampled-data output feedback stabilization for nonlinear systems in lower-triangular form with delayed output measurements, which motivates the present study.

In this paper, our aim is to design an output feedback stabilizer for a class of nonlinear systems with sampled and delayed measurements. Firstly, sufficient conditions are given to ensure that a class of hybrid systems are globally exponentially stable. The sufficient conditions are derived by constructing an iteration with a parameter. Then, based on the sufficient conditions and a dedicated construction continuous observer, an output feedback control law is presented to globally exponentially stabilize the nonlinear systems. The output feedback stabilizer is continuous and hybrid. It has simple and explicit form and can be derived without discretization. The maximum allowable sampling period and the maximum delay are also given.

This paper is organized as follows. In Section 2, some definitions are presented for hybrid systems. Then, global exponential stable sufficient conditions are given for the hybrid systems. In Section 3, continuous output feedback stabilizer are presented for a class of single-output nonlinear systems with sampled and time delayed measurements. Section 4 provides an example to illustrate the validity of the proposed design methods. Finally, the paper is concluded in Section 5.

2. GLOBAL EXPONENTIAL STABILITY FOR HYBRID SYSTEMS

Let \mathbb{R}^n denote n -dimension real space and \mathbb{R}^+ denote 1-dimension positive real space. For any $x \in \mathbb{R}^n$, let $\|x\| = (x^T x)^{1/2}$. For a continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ and $t \in \mathbb{R}$, let $\lim_{s \rightarrow t^-} f(s) = \lim_{s \rightarrow t, s < t} f(s)$. For a matrix $P \in \mathbb{R}^{n \times n}$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the largest and the smallest eigenvalues of P , respectively.

Consider the following system:

$$\begin{cases} \dot{x}(t) = f(x(t), x(t_k)), \\ t_{k+1} = t_k + T_{k+1}, \\ x(t_{k+1} + \tau_{k+1}) = \lim_{t \rightarrow t_{k+1} + \tau_{k+1}^-} x(t), \\ t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), k \geq 0, \end{cases} \quad (1)$$

where $\{t_k\}_{k \in \mathbb{N}}$ is a sequence of positive numbers defined by $t_{k+1} = t_k + T_{k+1}$, T_{k+1} denotes the $k + 1$ th sampling period, τ_k denotes time delay and has an upper bound τ , that is, $\tau_k \leq \tau$. Let $T_{\max} = \max\{T_k\}$ and $T_{\min} = \min\{T_k\}$. We also assume that $\tau \leq T_{\min}$, which can be taken non-strict with the understanding that by any chance, the update instant $t_k + \tau_k$ meets with the next transmission instant t_{k+1} , the update is implemented before the next sample is taken. The function $f : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is continuous. It should be noted that $x(t)$ is continuous on $t_{k+1} + \tau_{k+1}$, therefore, it is continuous on $[t_0, \infty)$.

Remark 1

Sampling arises simultaneously with input and output delays in many control problems, especially in control over networks. Similar systems to the form (1) arise frequently in certain applications in mathematical control theory and numerical analysis, which have been investigated in [15, 17, 20–24]. For example, the authors proposed a system-theoretic framework to study hybrid uncertain

systems [21]. They also presented characterizations of robust global asymptotic output stability. In [15], observer was designed for certain classes of nonlinear systems with both sampled and delayed measurements by using a small-gain approach. Continuous-discrete observers have also been studied for multi-input and multi-output state affine systems with sampled and delayed output measurements [24].

We next give the following definitions for the hybrid system (1).

Definition 1

Consider the system (1), for each $(t_0, x_0) \in \mathbb{R}^+ \times \mathbb{R}^n$, there exists a piecewise continuous function $t \rightarrow x(t, t_0, x_0)$ with the initial condition $x(t) = x_0$ ($t \in [t_0, t_0 + \tau_0]$) satisfying (1). We call that $x(t, t_0, x_0)$ is a *solution* of the system (1). If $f(\cdot)$ satisfies that $f(0, 0) = 0$, and there exist a non-decreasing function $N : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ and a positive constant λ such that $\|x(t, t_0, x_0)\| < e^{-\lambda(t-t_0)} N(\|x_0\|)$ for any $x_0 \in \mathbb{R}^n$, then, the system (1) is globally exponentially stable.

Now, we give the following sufficient conditions to ensure the hybrid system (1) is globally exponentially stable.

Theorem 1

Suppose there exist three positive constants $\rho_1, \alpha_1, \beta_1$, and a positive definite and radially unbounded function $V(x(t))$ defined on $[t_0, \infty)$, satisfying the following conditions:

$$\left. \frac{dV(x(t))}{dt} \right|_{(1)} \leq -\alpha_1 V(x(t)) + \beta_1 V(x(t_k)), t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), k \geq 0 \tag{2}$$

and

$$\begin{aligned} & \max_{k \geq 0} \left\{ \rho_1 e^{-\alpha_1(T_{k+1}-\tau_k)} + e^{-\alpha_1(T_{k+1}+\tau_{k+1}-\tau_k)} \right\} < 1, \\ & \max_{k \geq 0} \left\{ \left(\frac{\beta_1}{\alpha_1} + \rho_1 \frac{\beta_1}{\alpha_1} - \frac{\beta_1}{\alpha_1} e^{-\alpha_1(T_{k+1}-\tau_k)} (e^{-\alpha_1 \tau_{k+1}} + \rho_1) \right) / \rho_1 \right\} < 1. \end{aligned} \tag{3}$$

Then, the system (1) is globally exponentially stable.

Proof

The main point of the proof is to construct an iteration with a parameter. Then, a convergent sequence with the parameter can be obtained based on the iteration. Therefore, the exponential stability of the system (1) can be derived.

Multiplying the both sides of (2) by $e^{\alpha_1 t}$ yields

$$e^{\alpha_1 t} \left. \frac{d}{dt} V(x(t)) \right|_{(1)} + e^{\alpha_1 t} \alpha_1 V(x(t)) \leq e^{\alpha_1 t} \beta_1 V(x(t_k)), t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}).$$

Integrating the above differential inequality from $t_k + \tau_k$ to t , we have

$$\begin{aligned} V(x(t)) & \leq e^{-\alpha_1(t-t_k-\tau_k)} V(x(t_k + \tau_k)) + \frac{\beta_1}{\alpha_1} V(x(t_k)) \\ & \quad - \frac{\beta_1}{\alpha_1} e^{-\alpha_1(t-t_k-\tau_k)} V(x(t_k)), t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}). \end{aligned} \tag{4}$$

Note that $x(t)$ is continuous on $[t_0, \infty)$. Let $t = t_{k+1} + \tau_{k+1}$ and $t = t_{k+1}$, respectively, then

$$\begin{aligned} V(x(t_{k+1} + \tau_{k+1})) & \leq e^{-\alpha_1(T_{k+1}+\tau_{k+1}-\tau_k)} V(x(t_k + \tau_k)) + \frac{\beta_1}{\alpha_1} V(x(t_k)) \\ & \quad - \frac{\beta_1}{\alpha_1} e^{-\alpha_1(T_{k+1}+\tau_{k+1}-\tau_k)} V(x(t_k)), \end{aligned}$$

and

$$V(x(t_{k+1})) \leq e^{-\alpha_1(T_{k+1}-\tau_k)} V(x(t_k + \tau_k)) + \frac{\beta_1}{\alpha_1} V(x(t_k)) - \frac{\beta_1}{\alpha_1} e^{-\alpha_1(T_{k+1}-\tau_k)} V(x(t_k)).$$

From the previous two inequalities, we have

$$\begin{aligned}
 V(x(t_{k+1} + \tau_{k+1})) + \rho_1 V(x(t_{k+1})) &\leq \left(e^{-\alpha_1(T_{k+1} + \tau_{k+1} - \tau_k)} + \rho_1 e^{-\alpha_1(T_{k+1} - \tau_k)} \right) V(x(t_k + \tau_k)) \\
 &+ \left(\frac{\beta_1}{\alpha_1} + \rho_1 \frac{\beta_1}{\alpha_1} - \frac{\beta_1}{\alpha_1} e^{-\alpha_1(T_{k+1} - \tau_k)} (e^{-\alpha_1 \tau_{k+1}} + \rho_1) \right) V(x(t_k)), \quad k \geq 0.
 \end{aligned}
 \tag{5}$$

Let $\eta_1 = \max \{ \max_{k \geq 0} \{ \rho_1 e^{-\alpha_1(T_{k+1} - \tau_k)} + e^{-\alpha_1(T_{k+1} + \tau_{k+1} - \tau_k)} \}, \max_{k \geq 0} \left\{ \left(\frac{\beta_1}{\alpha_1} + \rho_1 \frac{\beta_1}{\alpha_1} - \frac{\beta_1}{\alpha_1} e^{-\alpha_1(T_{k+1} - \tau_k)} (e^{-\alpha_1 \tau_{k+1}} + \rho_1) \right) / \rho_1 \right\} \}$. From (3), we have $0 < \eta_1 < 1$. Therefore, it follows from (5) that

$$V(x(t_{k+1} + \tau_{k+1})) + \rho_1 V(x(t_{k+1})) \leq \eta_1 [V(x(t_k + \tau_k)) + \rho_1 V(x(t_k))], \quad k \geq 0. \tag{6}$$

Applying iteratively (6) with the parameter ρ_1 , we have

$$V(x(t_k + \tau_k)) + \rho_1 V(x(t_k)) \leq \eta_1^k [V(x(t_0 + \tau_0)) + \rho_1 V(x(t_0))], \quad k \geq 0, \tag{7}$$

that is, the sequence $\{V(x(t_k + \tau_k)) + \rho_1 V(x(t_k))\}$ is convergent. It follows from (4) and (7) that

$$\begin{aligned}
 V(x(t)) &\leq V(x(t_k + \tau_k)) + \frac{\beta_1}{\alpha_1} V(x(t_k)) \leq \eta_1^k \left(1 + \frac{\beta_1}{\alpha_1 \rho_1} \right) [V(x(t_0 + \tau_0)) + \rho_1 V(x(t_0))], \\
 t &\in [t_k + \tau_k, t_{k+1} + \tau_{k+1}).
 \end{aligned}$$

For any $t > t_0 + \tau_0$, there exists $k \geq 0$ such that $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$. Then, $\frac{t - t_0 - \tau_0}{T_{\max} + \tau} - 1 \leq k$. Therefore,

$$\begin{aligned}
 V(x(t)) &\leq \eta_1^{\frac{t - t_0 - \tau_0}{T_{\max} + \tau} - 1} \left(1 + \frac{\beta_1}{\alpha_1 \rho_1} \right) [V(x(t_0 + \tau_0)) + \rho_1 V(x(t_0))] \\
 &= e^{\frac{t}{T_{\max} + \tau} \ln \eta_1} \eta_1^{\frac{-t_0 - \tau_0}{T_{\max} + \tau} - 1} \left(1 + \frac{\beta_1}{\alpha_1 \rho_1} \right) [V(x(t_0 + \tau_0)) + \rho_1 V(x(t_0))],
 \end{aligned}$$

which implies that the system (1) is globally exponentially stable. □

For Theorem 1, we have the following corollaries.

Corollary 1

If there exist three positive constants $\rho_1, \alpha_1, \beta_1$ and a positive definite and radially unbounded function $V(x(t))$ defined on $[t_0, \infty)$, such that the condition (2) and

$$e^{-\alpha_1(T_{\min} - \tau)}(1 + \rho_1) < 1, \quad \frac{\beta_1(1 + \rho_1)}{\alpha_1 \rho_1} < 1, \tag{8}$$

hold, then, the system (1) is globally exponentially stable.

Corollary 2

If there exist two positive constants α_1, β_1 and a positive definite and radially unbounded function $V(x(t))$ defined on $[t_0, \infty)$, satisfying the condition (2) and

$$\alpha_1 > \beta_1, \quad T_{\min} - \tau > \frac{\beta_1}{\alpha_1(\alpha_1 - \beta_1)}, \tag{9}$$

then, the system (1) is globally exponentially stable.

Proof

The condition (9) implies that there exists $\rho > 0$ such that $e^{\alpha_1(T_{\min} - \tau)} - 1 > \alpha_1(T_{\min} - \tau) > \rho > \frac{\beta_1}{\alpha_1 - \beta_1}$. Therefore, the conditions (8) hold. The proof is completed. □

Remark 2

The condition (9) can hold for sufficiently large value of T_{\min} , that is, there is no upper bound of $T_k + \tau_k$. In fact, the inequality (4) holds on $[t_k + \tau_k, t_{k+1} + \tau_{k+1}]$. If T_k is sufficiently large, we have

$$V(x(t_{k+1} + \tau_{k+1})) < \frac{\beta_1}{\alpha_1} V(x(t_k + \tau_k)).$$

Then,

$$V(x(t_k + \tau_k)) < \left(\frac{\beta_1}{\alpha_1}\right)^k V(x(t_0 + \tau_0)) < \left(\frac{\rho_1}{1+\rho_1}\right)^k V(x(t_0 + \tau_0)),$$

which implies that the system (1) is globally exponential stable.

3. CONTINUOUS OUTPUT FEEDBACK STABILIZATION FOR A CLASS OF NONLINEAR SYSTEMS WITH SAMPLED AND DELAYED MEASUREMENTS

In this section, our aim is to propose an output feedback stabilization for the following system

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(x_1(t)), \\ \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(x_1(t), x_2(t), \dots, x_{n-1}(t)), \\ \dot{x}_n(t) = f_n(x_1(t), x_2(t), \dots, x_n(t)) + u(t), \\ y(t) = x_1(t), \end{cases} \quad (10)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}$ is the input. We make the following assumptions. The output $y(t)$ is sampled at instants t_k and is available at instants $t_k + \tau_k$, where $\{t_k\}$ is a strictly increasing sequence and satisfies $\lim_{k \rightarrow \infty} t_k = \infty$, and $\tau_k > 0$ represents the transmission delay. The sampling interval $T_{k+1} = t_{k+1} - t_k$ satisfies $0 < T_{\min} \leq t_{k+1} - t_k \leq T_{\max}$ for two positive real numbers T_{\min} , T_{\max} and for all $k = 0, 1, \dots, \infty$. The transmission delays τ_k are unknown, but have an upper bound τ . We also assume that $\tau \leq T_{\min}$, that is, the measures sampled at instants t_k are available for the observer before the next measures sampled at instants t_{k+1} . We assume that $f_i(x_1, \dots, x_i)$ ($i = 1 \dots, n$) are unknown and satisfy the following conditions

$$|f_i(x_1, \dots, x_i)| \leq l_1(|x_1| + \dots + |x_i|), \quad i = 1, \dots, n, \quad (11)$$

where l_1 is a positive real number.

Remark 3

The system (10) is firstly introduced in [25], where a linear state feedback control law has been proposed to achieve global exponential stability without considering sampled measurements. Under the same condition, the authors in [26] have presented a linear dynamic output compensator to globally exponentially stabilize the system (10). In [27], an exponential observer has been built for a biological system. Global asymptotic stabilization via output feedback has also been studied for nonlinear systems similar to (10) in [1]. The dynamics considered in [1] are in a feedback form and the nonlinear terms have an incremental rate that depends on the measured output. In [17, 20], a state feedback law has been constructed to achieve global asymptotic stabilization for the nonlinear system (10) under sampled and delayed measurements, and with inputs subject to delay and zero-order hold.

Next, we construct the following output feedback stabilizer

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + La_1e_1(t_k), \\ \vdots \\ \dot{\hat{x}}_{n-1}(t) = \hat{x}_n(t) + L^{n-1}a_{n-1}e_1(t_k), \\ \dot{\hat{x}}_n(t) = L^na_n e_1(t_k) + u(t), \\ \hat{x}_i(t_{k+1} + \tau_{k+1}) = \lim_{t \rightarrow (t_{k+1} + \tau_{k+1})^-} \hat{x}_i(t), \\ i = 1, \dots, n, \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), \quad k \geq 0, \end{cases} \tag{12}$$

$$u(t) = -[L^nk_1\hat{x}_1(t) + L^{n-1}k_2\hat{x}_2(t) + \dots + Lk_n\hat{x}_n(t)], \tag{13}$$

where $\hat{x}(t) = \hat{x}(t_0) = \hat{x}_0$ for $t \in [t_0 - T_{\max} - \tau, t_0 + \tau_0]$ and $L \geq 1$, $e_1(t_k) = x_1(t_k) - \hat{x}_1(t_k)$, $a_i > 0$ and $k_i > 0, i = 1, 2, \dots, n$ are the coefficients of the Hurwitz polynomial $s^n + w_1s^{n-1} + \dots + w_{n-1}s + w_n$, with $w_i = a_i$ or $w_i = k_{n-i+1}$. Now, we give the definition of output feedback stabilization for the system (10).

Definition 2

We call that the n -dimensional system (10) is globally exponentially stabilizable under the condition (11) and $x(t) = x_0$ for $t \in [t_0 - T_{\max} - \tau, t_0]$, if there exists an n -dimensional dynamical system (12) such that the $2n$ -dimensional subsystem (10)–(12) satisfies $\|\hat{x}(t)\| \leq e^{-\lambda(t-t_0)}N(\|\hat{x}_0\|, \|x_0\|)$ and $\|\hat{x}(t) - x(t)\| \leq e^{-\lambda(t-t_0)}N(\|\hat{x}_0\|, \|x_0\|)$ for any $x_0 \in \mathbb{R}^n$ and $\hat{x}_0 \in \mathbb{R}^n$, where $\lambda > 0$ and $N : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a non-decreasing function. Or, we call that n -dimensional dynamical system (12) globally exponentially stabilizes the n -dimensional system (10) under the condition (11).

The following lemma is also useful for our main results.

Lemma 1 ([28])

For any positive definite matrix $U \in \mathbb{R}^{n \times n}$, scalar $\gamma > 0$, vector function $w : [0, \gamma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds

$$\left[\int_0^\gamma w(s)ds \right]^T U \left[\int_0^\gamma w(s)ds \right] \leq \gamma \left[\int_0^\gamma w(s)^T U w(s)ds \right].$$

Let $e_i(t) = x_i(t) - \hat{x}_i(t)$ denote the estimation error of the high-gain observer (12). Then, the error dynamics is given by

$$\begin{cases} \dot{e}_1(t) = e_2(t) - La_1e_1(t_k) + f_1(x_1(t)), \\ \vdots \\ \dot{e}_{n-1}(t) = e_n(t) - L^{n-1}a_{n-1}e_1(t_k) \\ \quad + f_{n-1}(x_1(t), x_2(t), \dots, x_{n-1}(t)), \\ \dot{e}_n(t) = -L^na_n e_1(t_k) + f_n(x_1(t), x_2(t), \dots, x_n(t)) \\ e_i(t_{k+1} + \tau_{k+1}) = \lim_{t \rightarrow (t_{k+1} + \tau_{k+1})^-} e(t), \\ i = 1, \dots, n, \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), \quad k \geq 0. \end{cases} \tag{14}$$

In order to simplify the analysis, we consider the following coordinate transformation

$$\varepsilon_i(t) = \frac{e_i(t)}{L^i}, \quad z_i(t) = \frac{\hat{x}_i(t)}{L^i}, \quad i = 1, 2, \dots, n.$$

The closed-loop system (10) and (14) can be expressed as

$$\begin{cases} \dot{\varepsilon}_1(t) = L\varepsilon_2(t) - La_1\varepsilon_1(t) + La_1(\varepsilon_1(t) - \varepsilon_1(t_k)) + \frac{f_1}{L}, \\ \vdots \\ \dot{\varepsilon}_{n-1}(t) = L\varepsilon_n(t) - La_{n-1}\varepsilon_1(t) + La_{n-1}(\varepsilon_1(t) - \varepsilon_1(t_k)) + \frac{f_{n-1}}{L^{n-1}}, \\ \dot{\varepsilon}_n(t) = -La_n\varepsilon_1(t) + La_n(\varepsilon_1(t) - \varepsilon_1(t_k)) + \frac{f_n}{L^n}, \\ \varepsilon_i(t_{k+1} + \tau_{k+1}) = \lim_{t \rightarrow (t_{k+1} + \tau_{k+1})^-} \varepsilon_i(t), \quad i = 1, \dots, n, \end{cases} \quad (15)$$

and

$$\begin{cases} \dot{z}_1(t) = Lz_2(t) + La_1\varepsilon_1(t) - La_1(\varepsilon_1(t) - \varepsilon_1(t_k)), \\ \vdots \\ \dot{z}_{n-1}(t) = Lz_n(t) + La_{n-1}\varepsilon_1(t) - La_{n-1}(\varepsilon_1(t) - \varepsilon_1(t_k)), \\ \dot{z}_n(t) = -L(k_1z_1(t) + \dots + k_nz_n(t)) + La_n\varepsilon_1(t) - La_n(\varepsilon_1(t) - \varepsilon_1(t_k)), \\ z_i(t_{k+1} + \tau_{k+1}) = \lim_{t \rightarrow (t_{k+1} + \tau_{k+1})^-} z_i(t), \\ i = 1, \dots, n, \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), \quad k \geq 0. \end{cases} \quad (16)$$

Now, we give the following results.

Theorem 2

There exists an output feedback control law in the form of (12), which globally exponentially stabilizes the system (10) with the condition (11), if $a_i > 0$ and $k_i > 0$ ($i = 1, \dots, n$) are selected such that there exist two symmetric positive definite matrices P and Q such that

$$A^T P + PA \leq -I, \quad (17)$$

$$B^T Q + QB \leq -2I, \quad (18)$$

are satisfied, and L satisfies

$$L > \{1, l_1, 6nl_1\lambda_1, 2(\beta_2 + 1)nl_1\lambda_1\}, \quad (19)$$

and

$$T_{\max} + \tau \leq \frac{1}{c_3L}, \quad T_{\min} - \tau > \left(\frac{c_2}{c_1 - c_2}\right) \frac{1}{c_1L}, \quad (20)$$

where $c_1 = \min\left\{\frac{1}{8\lambda_1(\beta_2+1)}, \frac{1}{8\lambda_3}\right\}$, $c_3 > \max\left\{8, 16n\bar{a}_1(\lambda_1^2(\beta_2+1) + \lambda_3^2) + c_1, \frac{\kappa_1}{c_1}, \frac{\kappa_1 + \sqrt{\kappa_1^2 + 4c_1^3\kappa_1}}{2c_1^2}\right\}$,

$c_2 = \frac{\kappa_1}{c_3^2}$, $\kappa_1 = \frac{16n\bar{a}_1^2(\lambda_1^2(\beta_2+1) + \lambda_3^2)}{\lambda_2(\beta_2+1)}$, and $\bar{a}_1 = \max\{a_i^2\}$, $\lambda_1 = \lambda_{\max}(P)$, $\lambda_2 = \lambda_{\min}(P)$, $\lambda_3 =$

$$\lambda_{\max}(Q), \lambda_4 = \lambda_{\min}(Q), \beta_2 = 4n\bar{a}_1\lambda_3^2, A = \begin{bmatrix} -a_1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \dots & 1 \\ -a_n & 0 & \dots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & \dots & -k_n \end{bmatrix}.$$

Proof

Consider the following positive definite function

$$V_1(t) = (\beta_2 + 1)\varepsilon(t)^T P \varepsilon(t), \quad V_2(t) = z(t)^T Q z(t), \quad (21)$$

where $\varepsilon(t) = [\varepsilon_1(t), \dots, \varepsilon_n(t)]^T, z(t) = [z_1(t), \dots, z_n(t)]^T$. Let $F = \left(\frac{f_1}{L}, \frac{f_2}{L^2}, \dots, \frac{f_n}{L^n}\right)^T$. Then,

$$\begin{aligned} \frac{dV_1(t)}{dt} \Big|_{(15)} + \frac{dV_2(t)}{dt} \Big|_{(16)} &= L(\beta_2 + 1)\varepsilon(t)^T(PA + A^T P)\varepsilon(t) + 2(\beta_2 + 1)\varepsilon(t)^T P F \\ &+ 2L(\beta_2 + 1)\varepsilon(t)^T P[a_1, \dots, a_n]^T(\varepsilon_1(t) - \varepsilon_1(t_k)) + Lz(t)^T(QB + B^T Q)z(t) \\ &+ 2Lz(t)^T Q[a_1, \dots, a_n]^T \varepsilon_1(t) - 2Lz(t)^T Q[a_1, \dots, a_n]^T(\varepsilon_1(t) - \varepsilon_1(t_k)) \\ &\leq -L(\beta_2 + 1)\varepsilon(t)^T \varepsilon(t) + \frac{1}{4}L(\beta_2 + 1)\varepsilon(t)^T \varepsilon(t) + 4L(\beta_2 + 1)\lambda_1^2 n \bar{a}_1(\varepsilon_1(t) - \varepsilon_1(t_k))^2 \\ &+ 2(\beta_2 + 1)\varepsilon(t)^T P F - 2Lz(t)^T z(t) + Lz(t)^T z(t) + Ln\lambda_3^2 \bar{a}_1 \varepsilon_1(t)^2 + \frac{1}{4}Lz(t)^T z(t) \\ &+ 4Ln\bar{a}_1 \lambda_3^2(\varepsilon_1(t) - \varepsilon_1(t_k))^2 \\ &\leq -\frac{3}{4}L(\beta_2 + 1)\varepsilon(t)^T \varepsilon(t) + 4L(\beta_2 + 1)\lambda_1^2 n \bar{a}_1[\varepsilon_1(t) - \varepsilon_1(t_k)]^2 + 2(\beta_2 + 1)\varepsilon(t)^T P F \\ &- \frac{3}{4}Lz(t)^T z(t) + Ln\lambda_3^2 \bar{a}_1 \varepsilon_1^2(t) + 4Ln\bar{a}_1 \lambda_3^2[\varepsilon_1(t) - \varepsilon_1(t_k)]^2, \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}). \end{aligned} \tag{22}$$

In addition, from the condition (11), it follows that

$$\begin{aligned} |2\varepsilon(t)^T P F| &\leq 2(\varepsilon^T(t) P \varepsilon(t))^{1/2} (F^T P F)^{1/2} \leq 2nl_1 \lambda_1 (\varepsilon^T(t) \varepsilon(t))^{1/2} \\ &\times [(\varepsilon^T(t) \varepsilon(t))^{1/2} + (z(t)^T z(t))^{1/2}] \leq nl_1 \lambda_1 (3\varepsilon^T(t) \varepsilon(t) + z^T(t) z(t)). \end{aligned} \tag{23}$$

By Lemma 1, we have

$$\begin{aligned} |\varepsilon_1(t) - \varepsilon_1(t_k)|^2 &\leq (t - t_k) \int_{t_k}^t |\dot{\varepsilon}_1(s)|^2 ds \\ &\leq 4(t - t_k)L^2 \int_{t_k}^t \left[\varepsilon_2(s)^2 + \frac{l_1^2}{L^2}(z_1(s)^2 + \varepsilon_1(s)^2) + \bar{a}_1 \varepsilon_1(t_k)^2 \right] ds, \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}). \end{aligned} \tag{24}$$

From (22), (23) and (24), it follows that

$$\begin{aligned} \frac{dV_1(t)}{dt} \Big|_{(15)} + \frac{dV_2(t)}{dt} \Big|_{(16)} &\leq -(\beta_2 + 1) \left[\frac{3}{4}L - 3nl_1 \lambda_1 \right] \varepsilon(t)^T \varepsilon(t) + 16(\beta_2 + 1)L^3 \\ &\times n \bar{a}_1^2 \lambda_1^2 (t - t_k)^2 \varepsilon_1(t_k)^2 + 16(\beta_2 + 1)L^3 n \bar{a}_1 \lambda_1^2 (t - t_k) \int_{t_k}^t [\varepsilon_1(s)^2 + \varepsilon_2(s)^2 + z_1(s)^2] ds \\ &- \left(\frac{3}{4}L - (\beta_2 + 1)nl_1 \lambda_1 \right) z(t)^T z(t) + Ln\lambda_3^2 \bar{a}_1 \varepsilon(t)^T \varepsilon(t) + 16L^3 n \bar{a}_1^2 \lambda_3^2 (t - t_k)^2 \varepsilon_1(t_k)^2 \\ &+ 16L^3 n \bar{a}_1 \lambda_3^2 (t - t_k) \int_{t_k}^t [\varepsilon_1(s)^2 + \varepsilon_2(s)^2 + z_1(s)^2] ds, \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}). \end{aligned} \tag{25}$$

Construct the following auxiliary integral function:

$$V_3(t) = \int_{t-T_{\max}-\tau}^t [\varepsilon(s)^T \varepsilon(s) + z(s)^T z(s)] ds, \quad t \in [t_0, \infty).$$

We have

$$\frac{dV_3(t)}{dt} = (T_{\max} + \tau)(\varepsilon(t)^T \varepsilon(t) + z(t)^T z(t)) - \int_{t-T_{\max}-\tau}^t (\varepsilon(s)^T \varepsilon(s) + z(s)^T z(s)) ds, \tag{26}$$

and

$$V_3(t) \leq (T_{\max} + \tau) \int_{t-T_{\max}-\tau}^t [\varepsilon(s)^T \varepsilon(s) + z(s)^T z(s)] ds, \quad t \in [t_0, \infty). \tag{27}$$

Now, consider the following Lyapunov–Krasovskii function

$$V(t) = V_1(t) + V_2(t) + L^2V_3(t). \tag{28}$$

From (19), (25), (26) and (27), we can obtain

$$\begin{aligned} \frac{dV(t)}{dt} \Big|_{(15),(16)} &\leq -[(\beta_2 + 1) \left(\frac{3}{4}L - 3nl_1\lambda_1\right) - n\lambda_3^2\bar{a}_1L] \varepsilon(t)^T \varepsilon(t) - \left[\frac{3}{4}L - (\beta_2 + 1)nl_1\lambda_1\right] z(t)^T z(t) \\ &\quad + (T_{\max} + \tau)L^2[\varepsilon(t)^T \varepsilon(t) + z(t)^T z(t)] \\ &\quad + [16L^3n\bar{a}_1(\lambda_1^2(\beta_2 + 1) + \lambda_3^2)(t - t_k) - L^2] \\ &\quad \int_{t-T_{\max}-\tau}^t [\varepsilon(s)^T \varepsilon(s) + z(s)^T z(s)] ds + 16L^3n\bar{a}_1^2(\lambda_1^2(\beta_2 + 1) + \lambda_3^2)(t - t_k)^2 \varepsilon_1(t_k)^2 \\ &\leq -\frac{1}{\lambda_1} \left[\frac{1}{4} - (T_{\max} + \tau)L\right] LV_1(t) + \left[16n\bar{a}_1(\lambda_1^2(\beta_2 + 1) + \lambda_3^2) - \frac{1}{(T_{\max} + \tau)L}\right] L^3V_3(t) \\ &\quad - \frac{1}{\lambda_3} \left[\left(\frac{1}{4} - (T_{\max} + \tau)L\right) LV_2(t) + \frac{16L^3n\bar{a}_1^2(\lambda_1^2(\beta_2 + 1) + \lambda_3^2)}{\lambda_2(\beta_2 + 1)}(T_{\max} + \tau)^2V_1(t_k)\right]. \end{aligned}$$

Note that $L > \{6nl_1\lambda_1, 2(\beta_2 + 1)nl_1\lambda_1\}$, $\beta_2 = 4n\lambda_3^2\bar{a}_1$, $T_{\max} + \tau < \frac{1}{c_3L}$, then we have

$$\begin{aligned} \frac{dV(t)}{dt} \Big|_{(15),(16)} &\leq -\frac{1}{\lambda_1(\beta_2 + 1)} \left(\frac{1}{4} - \frac{1}{c_3}\right) LV_1(t) + [16n\bar{a}_1(\lambda_1^2(\beta_2 + 1) + \lambda_3^2) - c_3] L^3V_3(t) \\ &\quad - \frac{1}{\lambda_3} \left(\frac{1}{4} - \frac{1}{c_3}\right) LV_2(t) + \frac{16n\bar{a}_1^2(\lambda_1^2(\beta_2 + 1) + \lambda_3^2)}{\lambda_2(\beta_2 + 1)c_3^2} LV_1(t_k). \end{aligned}$$

Because $c_3 > \max\left\{8, 16n\bar{a}_1(\lambda_1^2(\beta_2 + 1) + \lambda_3^2) + c_1, \frac{\kappa_1}{c_1}, \frac{\kappa_1 + \sqrt{\kappa_1^2 + 4c_1^3\kappa_1}}{2c_1^2}\right\}$, $c_1 = \min\left\{\frac{1}{8\lambda_1(\beta_2 + 1)}, \frac{1}{8\lambda_3}\right\}$, $c_2 = \frac{\kappa_1}{c_3^2}$, then

$$\begin{aligned} \frac{dV(t)}{dt} \Big|_{(15),(16)} &\leq -c_1LV_1(t) - c_1L^3V_3(t) - c_1LV_2(t) + c_2LV_1(t_k) \\ &\leq -c_1LV(t) + c_2LV(t_k), t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), k \geq 0. \end{aligned}$$

Let $\alpha_1 = c_1L$, $\beta_1 = c_2L$, and $(\varepsilon(t, t_0, \varepsilon_0, z_0), z(t, t_0, \varepsilon_0, z_0))$ denote the solution of (15)–(16). Then, $T_{\min} - \tau > \frac{c_2}{c_1(c_1 - c_2)L}$ implies that there exists $\rho_1 > 0$ such that $\eta_1 = \max\left\{e^{-\alpha_1(T_{\min} - \tau)}(1 + \rho_1), \frac{\beta_1(1 + \rho_1)}{\alpha_1\rho_1}\right\} < 1$. Using the same methods as in Theorem 1, we obtain that $V(\varepsilon(t)) \leq e^{\frac{t}{T_{\max} + \tau} \ln \eta_1} \eta_1^{\frac{t_0 - \tau_0}{T_{\max} + \tau} - 1} [V(\varepsilon(t_0 + \tau_0, t_0, \varepsilon_0, z_0) + \rho_1 V(\varepsilon(t_0, t_0, \varepsilon_0, z_0)))]$, $t \geq t_0$. Then, $\|\hat{x}(t) - x(t)\| \leq e^{\frac{t}{T_{\max} + \tau} \ln \eta_1} N_1$ and $\|\hat{x}(t)\| \leq e^{\frac{t}{T_{\max} + \tau} \ln \eta_1} N_2$, $t \geq t_0$, where $N_1 = \frac{L^{2(n-1)}}{(\beta_2 + 1)\lambda_2} \eta_1^{\frac{t_0 - \tau_0}{T_{\max} + \tau} - 1} [V(\varepsilon(t_0 + \tau_0, t_0, \varepsilon_0, z_0) + \rho_1 V(\varepsilon(t_0, t_0, \varepsilon_0, z_0)))]$ and $N_2 = \frac{L^{2(n-1)}}{\lambda_4} \eta_1^{\frac{t_0 - \tau_0}{T_{\max} + \tau} - 1} [V(\varepsilon(t_0 + \tau_0, t_0, \varepsilon_0, z_0) + \rho_1 V(\varepsilon(t_0, t_0, \varepsilon_0, z_0)))]$.

Next, we will prove that any solutions of (10) don't finite-time escape on the interval $[t_0, t_0 + \tau_0]$. In fact, from (13), we have $u(t) = u_0 = -[L^n k_1 \hat{x}_1(t_0) + L^{n-1} k_2 \hat{x}_2(t_0) + \dots + L k_n \hat{x}_n(t_0)]$ for $t \in [t_0, t_0 + \tau_0]$. Let $\tilde{f}(x(t)) = [x_2(t) + f_1(x_1(t)), \dots, x_n(t) + f_{n-1}(x_1(t), \dots, x_{n-1}(t)), f_n(x_1(t), \dots, x_n(t))]^T + \tilde{u}$, where $\tilde{u} = [0, \dots, 0, u_0]^T$. Then, the system (10) with the control $u(t) = u_0$ on $[t_0, t_0 + \tau_0]$ can be expressed as

$$\dot{x}(t) = \tilde{f}(x(t)) + \tilde{u}, t \in [t_0, t_0 + \tau_0]. \tag{29}$$

Integrating the differential equation (29) from t_0 to t yields,

$$x(t) = x_0 + \int_{t_0}^t (\tilde{f}(x(s)) + \tilde{u})ds, t \in [t_0, t_0 + \tau_0].$$

Moreover, it follows from (11) that there exists a constant $l_2 > 0$ such that $\|\tilde{f}(x(t))\| \leq l_2\|x(t)\|$. Therefore,

$$\|x(t)\| \leq \|x_0\| + \int_{t_0}^t (\|\tilde{f}(x(s))\| + |u_0|)ds \leq \|x_0\| + \int_{t_0}^t (l_2\|x(s)\| + |u_0|)ds, t \in [t_0, t_0 + \tau_0].$$

By Gronwell Lemma [29], we have

$$\|x(t)\| \leq (\|x_0\| + \tau_0|u_0|)e^{l_2\tau_0}, t \in [t_0, t_0 + \tau_0].$$

The proof is completed. □

From Theorem 2, we also have the following results.

Corollary 3

There exists an output feedback control law in the form of (12), which globally exponentially stabilizes the system (10) with the condition (11), if $a_i > 0$ and $k_i > 0$ ($i = 1, \dots, n$) are selected such that there exist two symmetric positive definite matrices P, Q and a constant $L \geq 1$ such that (17), (18), (19) and

$$\tau < \frac{1}{\rho_2 L} \left(\frac{1}{c_3} - \frac{1}{c_4} \right), T_{\max} < \frac{1}{c_3 L} - \frac{1}{\rho_2 L} \left(\frac{1}{c_3} - \frac{1}{c_4} \right), T_{\min} > \frac{1}{c_4 L} + \frac{1}{\rho_2 L} \left(\frac{1}{c_3} - \frac{1}{c_4} \right), \quad (30)$$

are satisfied, where $\rho_2 > 2, c_4 = \frac{(c_1 - c_2)c_1}{c_2}, c_1, c_3, c_2$ and κ_1 are given in Theorem 2.

Proof

Note that $c_2 = \frac{\kappa_1}{c_3^2}$. Then $c_3 > \frac{\kappa_1}{c_1}$ implies that $c_1 > \frac{\kappa_1}{c_3} = \frac{\kappa_1}{c_3^2} c_3 = c_2 c_3 > 8c_2$. Therefore, $c_4 > 0$. On the other hand, from $c_3 > \frac{\kappa_1 + \sqrt{\kappa_1^2 + 4c_1^2 \kappa_1}}{2c_1^2}$, we have $c_1^2 c_3^2 - c_3 \kappa_1 - c_1 \kappa_1 > 0$. Then, $c_3 < \frac{c_1(c_1 c_3^2 - \kappa_1)}{\kappa_1} = c_1 \left(\frac{c_1}{c_2} - 1 \right)$. Therefore, $c_3 < \frac{c_1(c_1 - c_2)}{c_2} = c_4$. Thus, $\frac{1}{c_3} - \frac{1}{c_4} > 0$. Moreover, it is easy to check that the conditions (30) imply that the conditions (20) are satisfied. The proof is completed. □

Remark 4

In this paper, T_{\max}, τ and T_{\min} depend on the high-gain L (c_2, c_3, c_4, ρ_2 are constants that are independent on L). If the high-gain L is large, T_{\max} and τ will be small to ensure the convergence. In fact, if the nonlinear terms $f_i(x_1, \dots, x_i)$ change dramatically, which means that l_1 is very large, then L will be given larger to dominate the nonlinear terms. Because $e_1(t_k) = x_1(t_k) - \hat{x}_1(t_k)$ is a constant on the interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$, then T_k and τ will be very small to ensure exponential convergence.

Remark 5

In [8], the authors described a class of continuous nonlinear systems at the sampled instants by a discrete-time equation. Then, they considered multirate sampled-data output feedback control of the nonlinear systems without considering transmission delay. It should be noted that in some cases, it will be difficult to obtain an exact discrete time model for a nonlinear system, not to mention that it is nearly impossible to achieve an accurate approximation when there exist unknown nonlinear functions. In [17, 20], a state feedback law has been constructed to achieve global asymptotic stabilization for the nonlinear system (10) under sampled and delayed measurements, and with inputs subject to delay and zero-order hold. The nonlinear terms $f_i(\cdot)$ considered in [17, 20] are Lipschitz. The sampled-data feedback is based on a predictor mapping, which can be constructed inductively. In [13], a sampled-data output feedback stabilizer was presented to ensure that the closed-loop system is globally stable based on a hybrid system method. However, the closed-loop system is required

to transform into the hybrid system introduced in [19], whereas the systems considered in this paper are in lower-triangular form with sampled and delayed measurements, and the nonlinear functions $f_i(x_1, \dots, x_i)$ are unknown. The proposed output feedback stabilizer is continuous and hybrid. It has a simple and explicit form and can be derived without discretization.

Based on Theorem 2 and Corollary 3, an algorithm sketch to set the design parameters is presented as follows:

Step 1: We set the values of a_i and k_i such that (17) and (18) hold;

Step 2: We select L such that the condition (19) is satisfied;

Step 3: Calculate c_1 , select $\rho_2 > 2$ and c_3 such that $c_3 > \max \left\{ 8, 16n\bar{a}_1(\lambda_1^2(\beta_2 + 1) + \lambda_3^2) + \right.$

$$\left. c_1, \frac{\kappa_1}{c_1}, \frac{\kappa_1 + \sqrt{\kappa_1^2 + 4c_1^3\kappa_1}}{2c_1^2} \right\}, \text{ then calculate } c_2 \text{ and } c_4.$$

4. NUMERICAL SIMULATION

In this section, we use an example to show the effectiveness of our output feedback stabilization for nonlinear systems with sampled and time delay measurements.

Example 1

A single-link robot arm system can be modeled by [30] or [31]

$$\begin{cases} \dot{z}_1(t) = z_2(t), \\ \dot{z}_2(t) = \frac{K}{J_2N}z_3(t) - \frac{F_2(t)}{J_2}z_2(t) - \frac{K}{J_2}z_1(t) - \frac{mgd}{J_2}\cos(z_1(t)), \\ \dot{z}_3(t) = z_4(t), \\ \dot{z}_4(t) = \frac{1}{J_1}u(t) + \frac{K}{J_1N}z_1(t) - \frac{K}{J_2N}z_3(t) - \frac{F_1(t)}{J_1}z_4(t), \\ y(t) = z_1(t), \end{cases} \tag{31}$$

where $J_1, J_2, K, N, m, g,$ and d are known parameters, $F_1(t)$ and $F_2(t)$ are viscous friction coefficients that are not precisely known. Suppose $F_1(t)$ and $F_2(t)$ are bounded by an unknown constant $C > 0$. We introduce the change of coordinates $x_1 = z_1, x_2 = z_2, x_3 = \frac{K}{J_2N}z_3 - \frac{mgd}{J_2}, x_4 = \frac{K}{J_2N}z_4$ and the pre-feedback $v = \frac{K}{J_2N} \left(\frac{1}{J_1}u - \frac{mgd}{J_2} \right)$, which transforms (31) into

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = x_3(t) - \frac{F_2(t)}{J_2}x_2(t) - \frac{K}{J_2}x_1(t) - \frac{mgd}{J_2}(\cos(x_1(t)) - 1), \\ \dot{x}_3(t) = x_4(t), \\ \dot{x}_4(t) = v + \frac{K^2}{J_1J_2N^2}x_1(t) - \frac{K}{J_2N}x_3(t) - \frac{F_1(t)}{J_1}x_4(t), \\ y(t) = x_1(t). \end{cases}$$

Construct the following output feedback controller

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + a_1L(y(t_k) - \hat{x}_1(t_k)), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) - \frac{F_2(t)}{J_2}\hat{x}_2(t) - \frac{K}{J_2}\hat{x}_1(t) - \frac{mgd}{J_2}(\cos(\hat{x}_1(t)) - 1) + a_2L^2(y(t_k) - \hat{x}_1(t_k)), \\ \dot{\hat{x}}_3(t) = \hat{x}_4(t) + a_3L^3(y(t_k) - \hat{x}_1(t_k)), \\ \dot{\hat{x}}_4(t) = v + \frac{K^2}{J_1J_2N^2}\hat{x}_1(t) - \frac{K}{J_2N}\hat{x}_3(t) - \frac{F_1(t)}{J_1}\hat{x}_4(t) + a_4L^4(y(t_k) - \hat{x}_1(t_k)), \\ u(t) = -[k_1L^4\hat{x}_1(t) + k_2L^3\hat{x}_2(t) + k_3L^2\hat{x}_3(t) + k_4L\hat{x}_4(t)], \quad t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), \\ \hat{x}_i(t_{k+1} + \tau_{k+1}) = \lim_{t \rightarrow t_{k+1} + \tau_{k+1}^-} \hat{x}_i(t), \quad i = 1, \dots, 4, k \geq 0. \end{cases}$$

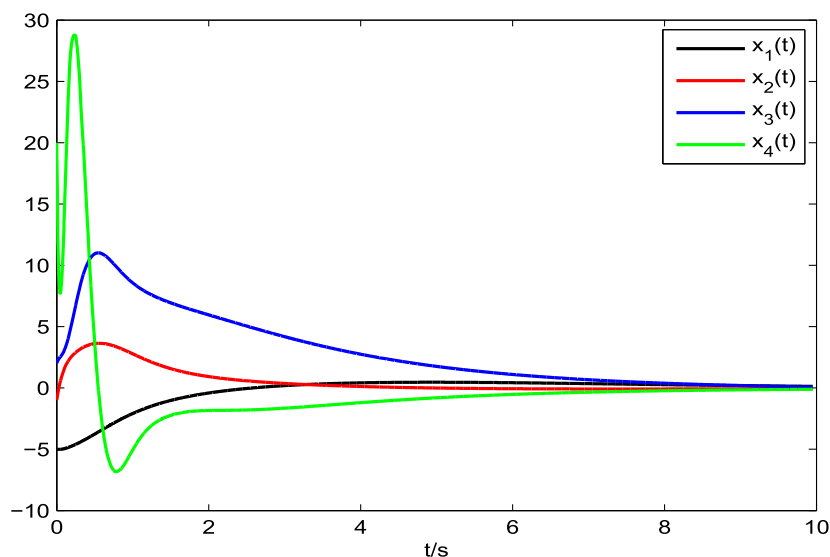


Figure 1. Trajectories of the states $x_i(t)$ ($1 \leq i \leq 4$) with the output feedback stabilizer.

In the following simulation, we apply the system parameters: $K/J_2 = 5$, $mgd/J_2 = 4$, $K^2/(J_1 J_2 N^2) = 2$, $K/(J_2 N) = 3$, $F_1(t)/J_1 = 10$, $F_2(t)/J_2 = 10$ and $L = 2$. The control gains k_i and the observer gains a_i are selected such that (17) and (18) hold. That is, the matrices A and B in (17) and (18) are Hurwitz. The eigenvalues of A and B are in the left half plane. If the control gains and observer gains are selected such that the eigenvalues are in the left half plane but far away from the axis, then the convergence speed of the closed-loop system will be greatly improved. However, large overshoot will arise. By the error-and-trial method, we set the control gains $k_1 = 40$, $k_2 = 78$, $k_3 = 49$, $k_4 = 12$, and the observer gains $a_1 = 4$, $a_2 = 6$, $a_3 = 4$, $a_4 = 1$. The initial conditions of the whole system are $(x_1(0), x_2(0), x_3(0), x_4(0),) = (-5, -1, 4, 20)$ and $(\hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0), \hat{x}_4(0),) = (5, 3, -1, -4)$. The sampling period T_k and the delay τ_k are given as $T_k = 0.1s$ and $\tau_k = 0.05s$, respectively. The simulation results are shown in Figure 1.

5. CONCLUSION

In this paper, we addressed the problem of output feedback stabilization for nonlinear systems with sampled and delayed output measurements. Firstly, sufficient conditions were proposed to ensure that a class of hybrid systems are globally exponentially stable. Then, based on the sufficient conditions and a dedicated construction continuous observer, an output feedback control law was presented to globally exponentially stabilize the nonlinear systems. The output feedback stabilizer was continuous and hybrid, and could be derived without discretization. The maximum allowable sampling period and the maximum delay were also given.

ACKNOWLEDGEMENTS

This work was supported by the National Science Foundation of China (61174216, 61374028, 51177088, 61273183), the Grant National Science Foundation of Hubei Provincial (2013CFA050), the Scientific Innovation Team Project of Hubei Provincial Department of Education (T201504).

REFERENCES

1. Praly L. Asymptotic stabilization via output feedback for lower triangular systems with output dependent incremental rate. *IEEE Transactions on Automatic Control* 2003; **48**(6):1103–1108.
2. Esfandiari F, Khalil HK. Output feedback stabilization of fully linearizable systems. *International Journal of Control* 1992; **56**(5):1007–1037.

3. Lin Z, Saberi A. Robust semi-global stabilization of minimum-phase input-output linearizable systems via partial state and output feedback. *IEEE Transactions on Automatic Control* 1995; **40**(6):1029–1041.
4. Teel AR, Praly L. Tools for semiglobal stabilization by partial state and output feedback. *SIAM Journal on Control and Optimization* 1995; **33**(5):1443–1448.
5. Praly L, Jiang Z. Linear output feedback with dynamic high gain for nonlinear systems. *System & Control Letters* 2004; **53**(2):107–116.
6. Arcak M, Nešić D. A framework for nonlinear sampled-data observer design via approximate discrete-time models and emulation. *Automatica* 2004; **40**(11):1931–1938.
7. Barbot JP, Monaco S, Normand-Cyrot D. Discrete-time approximated linearization of SISO systems under output feedback. *IEEE Transactions on Automatic Control* 1999; **44**(9):1729–1733.
8. Ahrens JH, Tan X, Khalil HK. Multirate sampled-data output feedback control with application to smart material actuated systems. *IEEE Transactions on Automatic Control* 2009; **54**(11):2518–2529.
9. Khalil HK. Performance recovery under output feedback sampled-data stabilization of a class of nonlinear systems. *IEEE Transactions on Automatic Control* 2004; **49**(12):2173–2184.
10. Nešić D, Teel AR. A framework for stabilization of nonlinear sampled-data systems based on their approximate discrete-time model. *IEEE Transactions on Automatic Control* 2004; **49**(7):1103–1122.
11. Nešić D, Teel AR, Kokotović PV. Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations. *System & Control Letters* 1999; **38**(4):259–270.
12. Clarke FH, Ledyev YS, Sontag ED, Subbotin AI. Asymptotic controllability implies feedback stabilization. *IEEE Transactions on Automatic Control* 1997; **54**(11):1394–1407.
13. Nešić D, Teel AR, Carnevale D. Explicit computation of the sampling period in emulation of controllers for nonlinear sampled-data systems. *IEEE Transactions on Automatic Control* 2009; **54**(3):619–624.
14. Nadri H, Hammouri H, Mota R. Observer design for uniformly observable systems with sampled measurements. *IEEE Transactions on Automatic Control* 2013; **58**(3):757–762.
15. Ahmed-Ali T, Karafyllis I, Lamnabhi-Lagarrigue F. Global exponential sampled-data observers for nonlinear systems with delayed measurements. *System & Control Letters* 2013; **62**(7):539–549.
16. Zhang D, Shen Y, Xia X. Continuous observer design for nonlinear systems with sampled and delayed output measurements. *19th IFAC World Congress*, Cape Town, South Africa, 2014; 269–274.
17. Karafyllis I, Krstic M. Nonlinear stabilization under sampled and delayed measurements, and with inputs subject to delay and zero-order hold. *IEEE Transactions on Automatic Control* 2012; **57**(5):1141–1154.
18. Heemels H, Teel AR, Wouw N, Nešić D. Networked control systems with communication constraints: tradeoffs between transmission intervals, delays and performance. *IEEE Transactions on Automatic Control* 2010; **55**(8):1781–1796.
19. Goebel R, Teel AR. Solution to hybrid inclusions via set and graphical convergence with stability theory applications. *Automatica* 2006; **42**(4):573–587.
20. Ahmed-Ali T, Karafyllis I, Krstic M, Lamnabhi-Lagarrigue F. Robust stabilization of nonlinear globally Lipschitz delay systems. In *Recent Results on Nonlinear Time Delayed Systems*, Malisoff M, Mazenc F, Pege P, Karafyllis I (eds), in the series advances in delays and dynamics (ADD@S). Springer: Heidelberg, 2015.
21. Karafyllis I. A system-theoretic framework for a wide class of systems I: Applications to numerical analysis. *Journal of Mathematical Analysis* 2007; **328**(2):876–899.
22. Karafyllis I. A system-theoretic framework for a wide class of systems II: Input-to-output stability. *Journal of Mathematical Analysis* 2007; **328**(1):466–486.
23. Karafyllis I, Jiang ZP. A small-gain theorem for a wide class of feedback systems with control applications. *SIAM Journal on Control and Optimization* 2007; **46**(4):1483–1517.
24. Ahmed-Ali T, Van Assche V, Massieu J, Dorleans P. Continuous-discrete observer for state affine systems with sampled and delayed measurements. *IEEE Transactions on Automatic Control* 2013; **58**(4):1085–1091.
25. Tsinias J. A theorem on global stabilization of nonlinear systems by linear feedback. *System & Control Letters* 1991; **17**:357–362.
26. Lei H, Lin W. Universal adaptive control of nonlinear systems with unknown growth rate by output feedback. *Automatica* 2006; **42**:1783–1789.
27. Gauthier J, Hammouri H, Othman S. A simple observer for nonlinear systems applications to bioreactors. *IEEE Transactions on Automatic Control* 1992; **37**:875–880.
28. Liu Y, Wang Z, Liu X. On global exponential stability of generalized stochastic neural networks with mixed time-delays. *Neurocomputing* 2006; **70**(1):314–326.
29. Coddington EA, Levinson N. *Theory of Ordinary Differential Equations*. McGraw-Hill: New York, 1955.
30. Isidori A. *Nonlinear Control Systems*. Springer: New York, 1995.
31. Marino R, Tomei P. *Nonlinear Control Design: Geometric, Adaptive and Robust*. Prentice Hall International: UK, 1995.