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A combined dynamic economic emission dispatch and time of use demand response mathematical modelling framework

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In this paper, we integrate a Demand Response (DR) program into the multi-objective dynamic economic emission dispatch (DEED) optimization problem. The resulting optimization problem is termed DR-DEED. The DR program is a time based program known as the Time of Use DR program. The DR program has been developed using the customers' Price Elasticity Matrices, which models the customer behavior under different conditions. An interactive control strategy between utility and consumers is proposed for the combined DR-DEED model, which determines the optimal power to be generated by minimizing fuel, emissions, and DR costs and also the optimal price. The customer in light of the utility's optimal price minimizes its electricity cost and optimally schedules power consumption. Obtained results indicate that DR programs are mutually beneficial to utility and consumers alike and can bring about desired demand reduction in the power system.
[http://dx.doi.org/10.1063/1.4928875]

NOMENCLATURE

a_k, b_k, c_k	fuel cost coefficients of generator k
$B(d_i)$	total customer benefit in time i from the use of d_i kW h of electrical energy
$B(d_{0i})$	benefit at d_{0i}
B_a	energy consumption of load a in each time slot (MW h)
C_k	fuel cost of generator k
d_0	initial demand
D_i	total system demand at time i
d_{0i}	initial demand at time i
d_i^m	final system load of class m at time i
d_{0i}^m	initial load of class m at time i
DR_k	maximum ramp down rates of generator k
e_k, f_k, g_k	emission cost coefficients of generator k
E_k	emissions cost for generator k
$E(i, i)$	self-elasticity
E_a	end time slot for load a
N_g	number of generators
EL_i	total energy level of the participating customer at the last round
A	maximum number of loads a, the industrial customer wants to schedule
p_i	utility defined price/tariff for each time slot in South African Rands ZAR/kW h
p_0	initial price
p_{0j}	initial price at time j
$P_{k,i}$	power generated from generator k at time i

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$P_{k,\max}$	maximum capacity of generator k
$P_{k,\min}$	minimum capacity of generator k
S_a	start time slot for load a
T	number of dispatch interval
UR_k	maximum ramp up rates of generator k
V_{ia}	status of load a in time slot i
Z_a	total number of time slots required for load a to complete its task
Δd	change in demand
Δd_i	change in demand at time i
Δp	change in price
Δp_j	change in price at time j

I. INTRODUCTION

The dynamic economic emission dispatch (DEED) problem is a variant of the mathematical problem known as dynamic economic dispatch (DED), which is formulated to determine the optimal scheduling of the committed generating unit's output whilst supplying the load demand over a dispatch period at minimum operating cost and also satisfying ramp rate constraints among other constraints.[1–4] A review of DED is provided in Ref. 5 and presents various mathematical formulations and solution methods that have been applied to solve the problem. Electric utilities or GENCOs are requested to reduce emission of gaseous pollutants including SO_2, NO_x, CO, and CO_2 from fossil fuel fired thermal plants as they are hazardous to human health. This has given rise to the mathematical optimization problem known as emission dispatching.[6,7]

Emission dispatching can be incorporated into the DED formulation in three principal ways. The first is patterned after the DED problem with the objective as minimizing emissions in lieu of fuel costs and is widely referred to as pure dynamic economic dispatch (PDED).[8] An alternative method and by far the most popular minimizes both fuel cost and emission simultaneously under load demand constraint, ramp rate constraint, and other constraints, resulting in a multi-objective optimization problem known as DEED.[9,10] The final approach is to minimize the fuel cost alone and utilize emissions as a constraint, defining a limit for allowable emissions and is termed Emissions Constrained Dynamic Economic Dispatch (ECDED).[11]

This paper introduces a price based Demand Response (DR) program into the DEED problem. The DEED problem determines the optimal power generation schedule over a time interval whilst simultaneously minimizing fuel and emission costs. Adding the price based DR program seeks to minimize the DR cost and determine the optimal price using customers' price elasticity matrices (PEM) and load economic models. Usually, electric utilities solve the DEED with other associated tasks like unit commitment,[13,30] and due to the increasing interest in renewable energy, utilities also solve DEED with the incorporation of renewable energy sources like wind and solar power.[14,32–38] In light of the advent of the smart grid and the need to convert load customers into active interacting participants who engage with the utility to reduce system demand,[15,31] we introduce a combined DEED with DR under an interactive control strategy. This would lead to the mitigation of incidences of brownouts and blackouts caused by ever increasing customer demand, which is one of the principal benefits of DR.[31] In the formulation presented in this work, the utility minimizes the generation and DR costs and determines the optimal hourly price. Based on the utility determined price, the customers seek to minimize their electricity costs and thus determine their optimal load schedule. This interaction between the utility and the customer continues until a stabilized price and a desired level of customer participation is reached so as to maintain an optimal and sustainable market of the DR program. The major contributions of this paper are: (i) Three forms of PEM's (for different classes of customer loads) are used to represent customer load adjustments to variations in prices. These PEM's are integrated into the DEED problem via the power balance constraint and an addition of a DR cost term into the DEED objective function. (ii) The resulting multi-objective problem is transformed into a single objective function using the weighting factor approach and determines the optimal price and energy levels. (iii) A customer scheduling model is introduced which determines the optimal schedule for the three classes of customer loads in light of the utility price and energy levels. (iii) An interactive control strategy is proposed for effective coordination between the utility and the customer side and obtaining mutually acceptable prices and energy levels. (iv) The effectiveness of the final proposed mathematical model framework is shown with two test DR-DEED system setups. The first system setup involves six generators and two aggregated industrial customers while the second system setup consists of ten generators and two aggregated industrial customers. (v) Results obtained from the two system setups indicate that DR-DEED leads to reduction in customer power demand and a corresponding decrease in system emissions when compared to conventional DEED. The rest of this paper is organized as follows: Section II introduces the DEED mathematical formulations; Section III introduces demand response programs and reviews the concept of PEM and load economic models in DR programs. Section IV details the combined interactive DR DEED mathematical model. Section V focuses on numerical simulations using the developed DR-DEED model and presents obtained results. The paper is concluded in Section VI.

II. DEED PROBLEM FORMULATION

In this work, the approach is used whereby the fuel and emission costs are simultaneously minimized under load demand constraints amidst other constraints. The mathematical representation is presented below:[12]

$$\min \sum_{i=1}^{T} \sum_{k=1}^{Ng} C_k(P_{k,i}), \tag{1}$$

$$\min \sum_{i=1}^{T} \sum_{k=1}^{Ng} E_k(P_{k,i}), \tag{2}$$

with

$$C_k(P_{k,i}) = a_k + b_k P_{k,i} + c_k P_{k,i}^2, \tag{3}$$

$$E_k(P_{k,i}) = e_k + f_k P_{k,i} + g_k P_{k,i}^2, \tag{4}$$

subject to the following network constraints:

$$\sum_{k=1}^{Ng} P_{k,i} = D_i + Loss_i, \tag{5}$$

$$P_{k,\min} \le P_{k,i} \le P_{k,\max}, \tag{6}$$

$$-DR_k \le P_{k,i+1} - P_{k,i} \le UR_k, \tag{7}$$

$$Loss_i = \sum_{k=1}^{Ng} \sum_{j=1}^{Ng} P_{k,i} B_{j,k} P_{j,i}. \tag{8}$$

The following is a brief description of the constraints:

- The first constraint (5) ensures that at any time i, the total power generated equals the demand.
- The second constraint (6) ensures that the generator limits are not exceeded.
- The final constraint (7) ensures that the generator ramp rate limits are not violated.

The multi-objective optimization can be transformed into a single objective function using a weighting factor w subject to the same constraints (5)–(7).

$$\min \left[w \sum_{i=1}^{T} \sum_{k=1}^{Ng} C_k(P_{k,i}) + (1-w) \sum_{i=1}^{T} \sum_{k=1}^{Ng} E_k(P_{k,i}) \right]. \tag{9}$$

III. PRICE BASED DR PROGRAMS

In general, demand response programs are used to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized.[16] In cases where the utility is a monopoly, an advantage of demand response programs is an improvement in power system efficiency and power system reliability. There is also the advantage of a reduction in operating costs and emissions. In deregulated markets, the same advantages in monopolistic markets apply. Furthermore, there is the advantage of reduced wholesale market prices.[17,18] In incentive based DR programs, incentives are simply offered to consumers to reduce or curtail their electricity use when the power system is stressed. The incentives can be in the form of rebates or lower electricity tariffs.[19,20] In price based DR programs, there is a time variation of electricity tariffs. The price based DR program used in this work is the time of use DR (TOU-DR) program. For this kind of program, the price of elasticity is calculated for peak, off-peak, and standard times based on the energy cost in each time period. The aim is to encourage consumers to curtail their energy use to take advantage of favourable prices.[21,22]

The load economic profile for these kinds of programs is given below:[23]

The customers profit is given as

$$S_i = B(d_i) - p_i d_i. \tag{10}$$

To maximize customers' profit, $\partial S_i / \partial d_i$ should be equal to zero; therefore

$$\frac{\partial B(d_i)}{\partial d_i} = p_i. \tag{11}$$

The most common benefit function is the quadratic benefit function defined as[23]

$$B(d_i) = B(d_{0i}) + p_{0i}(d_i - d_{0i}) \left[1 + \frac{d_i - d_{0i}}{2E(i,i).d_{0i}} \right], \tag{12}$$

$$\frac{\partial B(d_i)}{\partial d_i} = p_{0i} \left[1 + \frac{d_i - d_{0i}}{E(i,i).d_{0i}} \right]. \tag{13}$$

Equating (11) and (13) we obtain that

$$d_i = d_{0i} \left[1 + E(i,i) \frac{p_i - p_{0i}}{p_{0i}} \right]. \tag{14}$$

Similarly, for the multiperiod elastic loads, it is assumed that demand rescheduling occurs. Thus, the demand at time i is a function of prices at times $i = 1, 2 \ldots T$. In this work, we assume $T = 24$ and the cross elasticity is given as $E(i,j) = \frac{\Delta d_i / d_{0i}}{\Delta p_j / p_{0j}}$.

Working with the linearity assumption that $\frac{\Delta d_i}{\Delta p_j}$ is constant for $i, j = 1, 2, 3, \ldots 24$, the following relationship is obtained between price and demand:

$$d_i = d_{0i} \left[1 + \sum_{\substack{j=1 \\ j \neq i}}^{24} E(i,j) \frac{p_j - p_{0j}}{p_{0j}} \right]. \tag{15}$$

Combining the single period (14) and multiperiod (15), we obtain

$$d_i = d_{0i}\left[1 + E(i,i)\frac{p_i - p_{0i}}{p_{0i}} + \sum_{\substack{j=1 \\ j \neq i}}^{24} E(i,j)\frac{p_j - p_{0j}}{p_{0j}}\right], \tag{16}$$

$$E_{inflexible} = \begin{bmatrix} E(1,1) & E(1,2) & 0 & 0 & 0 & 0 & 0 \\ E(2,1) & E(2,2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & E(3,3) & E(3,j) & 0 & 0 & 0 \\ 0 & 0 & E(i,3) & E(i,j) & E(i,22) & 0 & 0 \\ 0 & 0 & 0 & E(22,j) & E(22,22) & 0 & 0 \\ 0 & 0 & 0 & 0 & E(23,22) & E(23,23) & E(23,24) \\ 0 & 0 & 0 & 0 & 0 & E(24,23) & E(24,24) \end{bmatrix}, \tag{17}$$

$$E_{flexible} = \begin{bmatrix} E(1,1) & 0 & 0 & 0 & 0 & 0 & 0 \\ E(2,1) & E(2,2) & 0 & 0 & 0 & 0 & 0 \\ E(3,1) & E(3,2) & E(3,3) & 0 & 0 & 0 & 0 \\ E(4,1) & E(4,2) & E(4,3) & E(4,j) & 0 & 0 & 0 \\ E(5,1) & E(5,2) & E(5,3) & E(5,j) & 0 & 0 & 0 \\ 0 & E(6,2) & E(6,3) & E(6,j) & 0 & 0 & 0 \\ 0 & 0 & E(7,3) & E(7,j) & 0 & 0 & 0 \\ 0 & 0 & 0 & E(8,j) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E(19,22) & E(19,23) & 0 \\ 0 & 0 & 0 & 0 & E(20,22) & E(20,23) & E(20,24) \\ 0 & 0 & 0 & 0 & E(21,22) & E(21,23) & E(21,24) \\ 0 & 0 & 0 & 0 & E(22,22) & E(22,23) & E(22,24) \\ 0 & 0 & 0 & 0 & 0 & E(23,23) & E(23,24) \\ 0 & 0 & 0 & 0 & 0 & 0 & E(24,24) \end{bmatrix}, \tag{18}$$

$$E_{night-time} = \begin{bmatrix} E(1,1) & E(1,2) & \ldots & E(1,j) & \ldots & E(1,23) & E(1,24) \\ E(1,2). & E(2,2) & \ldots & E(2,j) & \ldots & E(2,23) & E(2,24) \\ 0 & 0 & E(3,3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E(i,j) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E(22,22) & 0 & 0 \\ E(1,23) & E(2,23) & \ldots & E(i,23) & \ldots & E(23,23) & E(23,24) \\ E(1,24) & E(2,24) & \ldots & E(i,24) & \ldots & E(23,24) & E(24,24) \end{bmatrix}. \tag{19}$$

IV. COMBINED INTERACTIVE DEMAND RESPONSE-DYNAMIC ECONOMIC EMISSIONS ECONOMIC DISPATCH (DR-DEED)

In this work, the DR target loads fall into three different classes.[24] The load classes are: inflexible loads, flexible loads, and night-time loads. For instance, the inflexible loads are the customer loads that must be switched on. Customers would not curtail these loads to participate in demand response programs as they impact heavily on the benefit of customers. For residential customers, examples of these kinds of loads are cookers, stoves, refrigerators, and heating systems. Depending on the industry, some loads are also inflexible for industrial customers like industrial motors for some critical processes. The other loads are loads that customers are willing to curtail. However, customers have varying load responses to price increases, hence different degrees of flexibility. Flexible loads are loads that customers are completely flexible about.

They can readily adjust these loads to price variations. Examples of such kinds of loads for residential customers are vacuum cleaners, dishwashers, and water purifiers/boilers. Industrial examples of this kind of load are industrial pumps. Finally, night-time loads are loads that the customer can schedule to hours with the lowest electricity prices, e.g., late in the night and very early hours of the morning. Examples of such kinds of loads for residential customers are washing machines, electric hot water heaters, and tumble dryers. For industrial customers, these can include furnaces. It is assumed that there is a mix of these classes of loads in the power system with the total power system load, a summation of the three different classes. It is further assumed that the utility has an estimate of each class of load and each load class has a different PEM. The PEMs are obtained through a historical analysis of customers' demand response to increases or decreases (deviations) in the price of electricity. Each load class has a different PEM, i.e., a 24×24 square matrix. The difference between the PEM of each load class is the position of the non-zero elements in the matrix. Equations (17)–(19) shows sample PEM structure for inflexible, flexible, and night-time loads, respectively.

Since there are three types of load classes $m;\ m = 1, 2, 3$, the total system load is a summation of the three load classes. We define d_{0i}^m as the initial load of class m at time i. The total initial system load $d_{0i} = \sum_{m=1}^{3} d_{0i}^m$ and d_i^m is defined as the final system load of class m at time i. The total final system load $d_i = \sum_{m=1}^{3} d_i^m$. The cost of the DR program to the utility at time i can therefore be defined as

$$cost\,DR_i = p_{0i}.d_{0i} - p_i.d_i. \tag{20}$$

Thus, the weighted single objective DR-DEED mathematical formulation from the utility perspective can be given as

$$\min\left[w\left[\sum_{i=1}^{T}\sum_{k=1}^{Ng} C_k(P_{k,i}) + \sum_{i=1}^{T} cost\,DR_i\right] + (1-w)\sum_{i=1}^{T}\sum_{k=1}^{Ng} E_k(P_{k,i})\right], \tag{21}$$

subject to the following network constraints:

$$\sum_{k=1}^{Ng} P_{k,i} = d_i + Loss_i, \tag{22}$$

$$P_{k,\min} \le P_{k,i} \le P_{k,\max}, \tag{23}$$

$$-DR_k \le P_{k,i+1} - P_{k,i} \le UR_k, \tag{24}$$

$$d_i^m = d_{0i}^m\left[1 + E_m(i,i)\frac{p_i - p_{0i}}{p_{0i}} + \sum_{\substack{j=1 \\ j \ne i}}^{24} E_m(i,j)\frac{p_j - p_{0j}}{p_{0j}}\right], \tag{25}$$

$$Loss_i = \sum_{k=1}^{Ng}\sum_{j=1}^{Ng} P_{k,i}B_{j,k}P_{j,i}. \tag{26}$$

An interactive control strategy is used in this work. The reason behind an interactive control strategy is to obtain a final optimal price and energy levels satisfactory to both the utility and customers. Thus, the utility initially determines the optimal price (p_i) and suggested energy level (d_i) using Eqs. (20)–(26). The customers respond by scheduling their appliances and loads in light of the provided utility price. The responding customers' energy levels are sent back to the utility and the utility revises the PEMs. The utility again determines the price in light of the responding customers' energy levels and revised PEMs. This process is repeated until convergence is achieved. Figure 1 shows the complete flow chart for the proposed interactive control

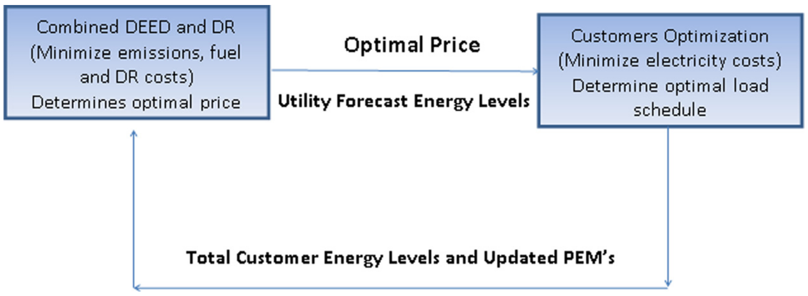

FIG. 1. Flowchart of the interactive DR-DEED program.

strategy. The intention of this interactive design of the DR program is to seek an optimal price signal and a desired level of market participation of the DR program.

A. Customer side objective function and constraints

The participating customer first needs to classify his loads into the three available load classes: flexible, inflexible, and night-time loads. The customer's optimization model has an objective function that minimizes the electricity bill/costs of all three kinds of loads. In this work, two groups of industrial customers are assumed. In the model formulation given below, i represents time slot and a represents the loads of the industrial customer. The decision variable is binary V_{ia}, which is either 1 or 0 and represents if the loads a *is* switched on or off in time slot i. The consumers are assumed to be acting rationally and seek to minimize electricity costs of all loads, devices, machines, or appliances. We assume a scheduling interval of 1 h; thus, in 1 day, there will be 24 time slots. The objective function and constraints are represented mathematically as

$$\min \sum_{i=1}^{24} \sum_{a=1}^{A} B_a V_{ia} p_i, \tag{27}$$

subject to

$$\sum_{S_a}^{E_a} V_{ia} = Z_a, \tag{28}$$

$$\sum_{a=1}^{A} B_a V_{ia} \leq EL_i. \tag{29}$$

The following is a brief explanation of the constraints:

- Constraint (28) ensures that there are sufficient time slots for a load, device, or machine to complete its tasks. This is also the constraint that handles the flexibility of the appliance/load. For instance, let us assume two time slots are required for an appliance to complete a task, i.e., $Z_a = 2$. If the customer is flexible about the appliance, i.e., the appliance must not run at specific time slots, the difference between the start time slot S_a and end time slot E_a would be greater than Z_a. If the customer is however inflexible about the appliance, the difference between start time slot S_a and end time slot E_a would be exactly Z_a. For night-time loads, both start time slot S_a and end time slot E_a would be either in the early hours of the morning or late at night.
- The final constraint (29) ensures that the customer's new energy level does not exceed the energy level of the last round. For the initial optimization, it is the maximal estimated energy level for that customer. This ensures that there is actually relief in the power system.

V. NUMERICAL SIMULATIONS, OBTAINED RESULTS, AND DISCUSSIONS

In this section, we present the parameters and results of the combined interactive DR-DEED optimization model both at the utility side and at the customer side. The proposed mathematical optimization models are tested on two example test systems. The first example test system is a six unit test system and the second is a ten unit test system. In both numerical simulations, the default weighting factor $w = 0.5$ and the following condition is required to be satisfied:

$$w + (1 - w) = 1. \tag{30}$$

This is done to give equal preference to both objective functions and not to minimize/maximize an objective function at the expense of the other.[6,12,26,29,31] Investigations are carried out to determine the effect of varying one weighting factor at the expense of the other and its impact on the power system parameters.

A. Test system 1

Test system 1 consists of six unit generators at the supply side and two aggregated industrial customers at the customer side. At the utility side, the goal is to obtain the optimal price p_i and forecast demand d_i, while at the customer side, the major aim is to obtain the optimal customer schedule in view of the utility determined optimal price and forecast demand.

The fuel cost coefficients and the emission cost coefficients modified are obtained from Ref. 12 and shown in Table X in the Appendix. The initial electricity tariff values are obtained from Eskom's (the South African utility) Tariff book[25] and shown in Table XI in the Appendix. The total initial demand is also shown in Table XI. The TOU periods are assumed to be off-peak (23:00–04:00) hours, standard (05:00–06:00, 11:00–17:00, and 21:00–22:00) hours, and peak (07:00–10:00 and 18:00–20:00) hours.[25] The assumed TOU elasticity values obtained from Ref. 24 are given in Table I. The transmission loss formula coefficients for the six unit test system are given in the below equation:

$$B = 10^{-4} \times \begin{bmatrix} 0.420 & 0.051 & 0.045 & 0.057 & 0.078 & 0.066 \\ 0.051 & 0.180 & 0.039 & 0.048 & 0.045 & 0.060 \\ 0.045 & 0.039 & 0.195 & 0.051 & 0.072 & 0.057 \\ 0.057 & 0.048 & 0.051 & 0.213 & 0.090 & 0.075 \\ 0.078 & 0.045 & 0.072 & 0.090 & 0.207 & 0.096 \\ 0.066 & 0.060 & 0.057 & 0.075 & 0.096 & 0.255 \end{bmatrix} \text{per MW}. \tag{31}$$

Most logical customers are always on the lookout for ways or measures to use energy efficiently and do so at minimal cost.[26,27] In this work, the goal of the customer is to minimize their electricity bill/costs and optimally schedule their appliance and hence their energy plan in light of the provided utility optimal price. To verify the mathematical formulations for the customer side (Eqs. (27)–(29)), two aggregated industrial consumers are assumed. Both aggregated industrial customer groups consist of 20 and 15 identical customers, respectively, and there is a regulator that can schedule these loads. The underlying principles can easily be extended to

TABLE I. TOU self and cross elasticity.

	Peak	Off-peak	Standard
Peak	−0.1	0.016	0.012
Off-peak	0.016	−0.1	0.01
Standard	0.012	0.01	−0.1

TABLE II. Load data for customer in the first group.

	B_a (MW h)	S_a (h)	E_a (h)	Z_a (h)
Flexible				
Load 1	5	1	24	12
Load 2	4	1	24	12
Inflexible				
Load 3	15	1	24	24
Load 4	10	1	24	24
Night-time				
Load 5	5	1 and 21	6 and 24	5
Load 6	1.5	1 and 21	6 and 24	4

residential or other kinds of customers. The customer just has to identify the loads that can be grouped under flexible, night-time, and inflexible. For the sake of simplicity, it is further assumed that the customer classification does not change and each customer has six loads. Tables II and III show the load data for an individual customer in the two groups.

The optimization models are built and solved using the Advanced Interactive Multidimensional Modelling System (AIMMS).[28] AIMMS is an Algebraic Modelling Language (AML) used for solving optimization and scheduling type mathematical problems. The major advantage of using AIMMS is the similarity of the software's syntax to the mathematical representation of optimization problems. The software supports the solution of a large number of optimization problem types and allows for an easy reproduction of their results without a violation of any of the constraints. The particular solver used is the CPLEX 12.6 solver, and all the constraints of the mathematical model are satisfied. After the first utility optimization (Eqs. (20)–(26)) and the corresponding customer side optimization (Eqs. (27)–(29)), the customers return their energy consumption to the utility. The utility revises the PEMs and again performs optimization. This interactive control process continues until convergence is reached. In this work, after the third round of interactive control, convergence was achieved.

In Figure 2, the load profiles at the various stages of the interactive control process are shown. It shows that there is a reduction in peak demand and also a shifting of load from peak periods (07:00 h–08:00 h and 18:00 h–20:00 h) to the standard and off peak periods. This is obvious from the difference between the initial load and the second customer load which is the load at which convergence was reached during the interactive process. Figure 3 shows the TOU price movements over a 24 h scheduling horizon during different stages of the interactive control process. It is obvious that there is a price reduction in standard and off peak periods and an increase in peak periods. The increase and decrease in prices influence the end user's energy consumption and bring about peak load reduction and load shifting. Figure 4 shows the initial system

TABLE III. Load data for customer in the second group.

	B_a (MW h)	S_a (h)	E_a (h)	Z_a (h)
Flexible				
Load 1	15.7	1	24	12
Load 2	5.3	1	24	12
Load 3	14	1	20	11
Inflexible				
Load 4	15	1	24	24
Night-time				
Load 5	5	1 and 21	6 and 24	5
Load 6	5	1 and 21	6 and 24	4

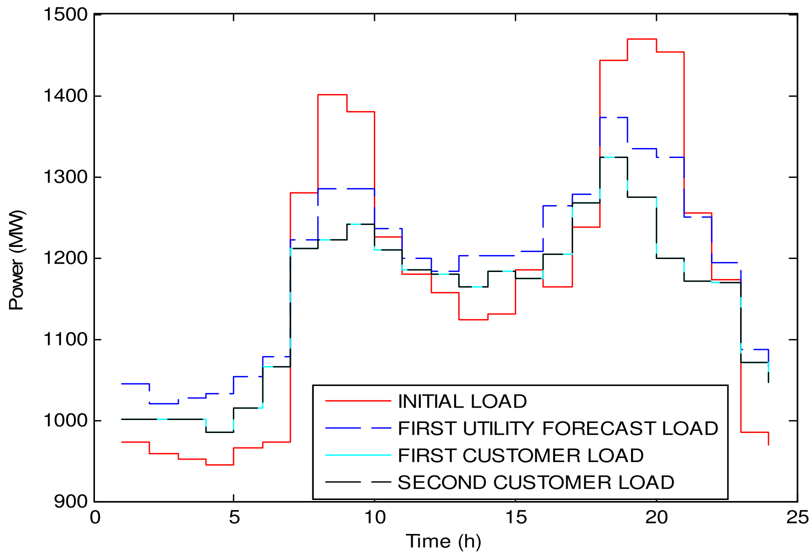

FIG. 2. Load profiles at different stages of interactive control.

load and the optimal final converged load. These two load profiles are extracted from Figure 2 and show the load profile at the beginning of the interactive control process and the optimal load profile at the end of the interactive control process. The essence is to enable a clearer view of the magnitude of load reduction and shifting. Similarly, Figure 5 is an extract from Figure 3 and shows the initial price and optimal final utility price obtained via the interactive control process. The magnitude of price movements (increase and decrease) is clearly visible.

Figures 6–11 show the optimal power generated for all six generators under initial system load (normal DEED) and optimal converged load (TOU-DEED). From these figures, we observe that due to the effect of load profile changes (see Figure 2), the power generated from the different generators reduces in peak periods and increases in standard and off peak periods. This is because the total generated power must always satisfy total demand (power balance

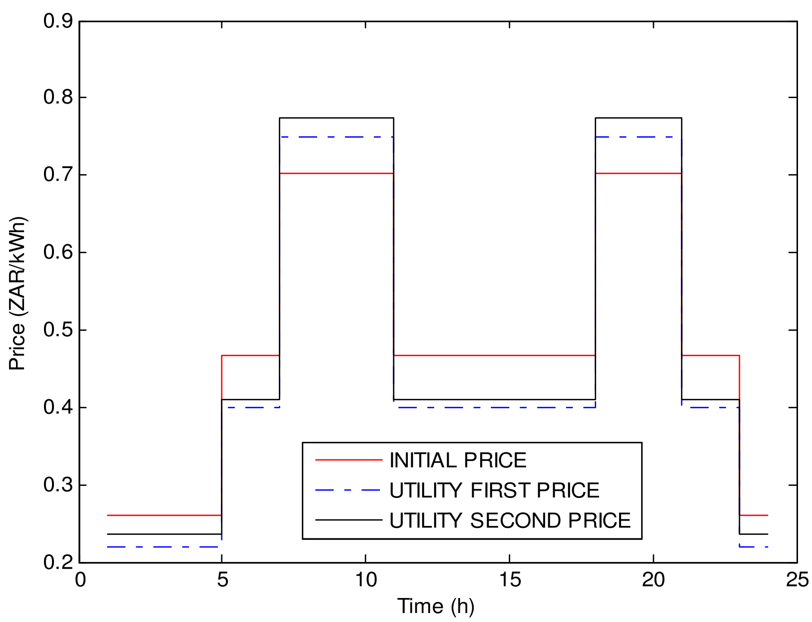

FIG. 3. Utility determined price at different stages of interactive control.

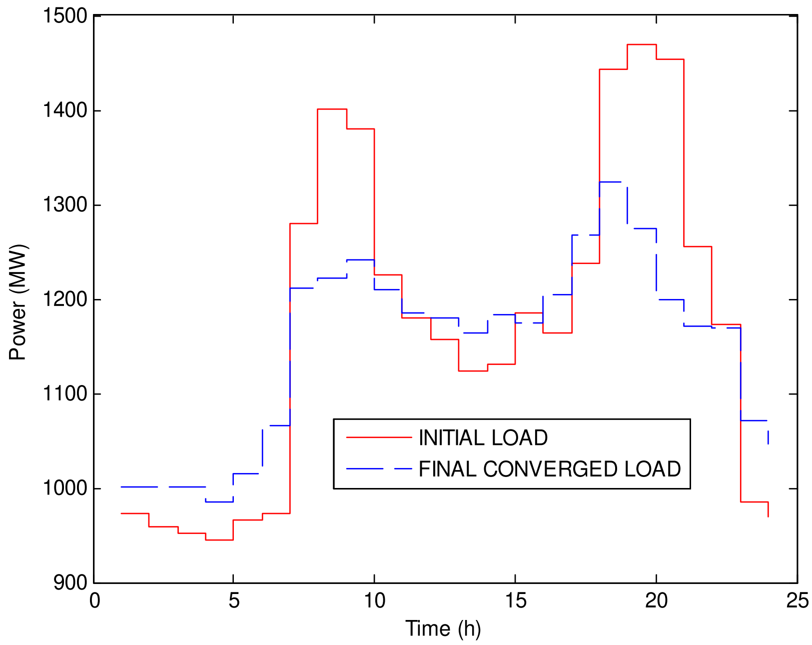

FIG. 4. Initial load and final converged load.

constraint). Table IV shows the results of varying the weighting factors on the fuel costs, emissions, and power loss under both DEED and DR-DEED for the first test system. The aim is to show the effect of a trade-off between conflicting objectives on system parameters.

The final customer optimal scheduling solution is shown in Tables V and VI for customers in the first and second groups, respectively.

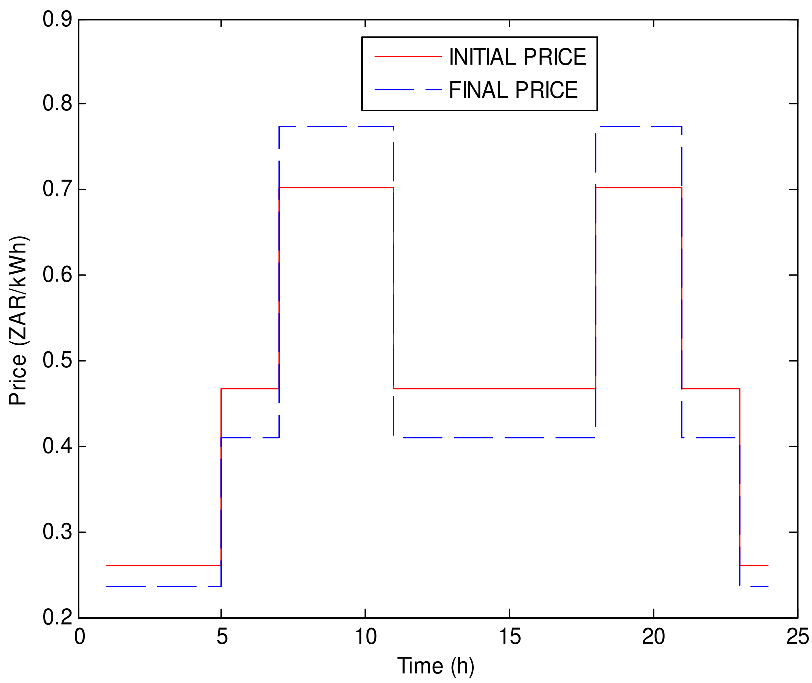

FIG. 5. Initial price and final price.

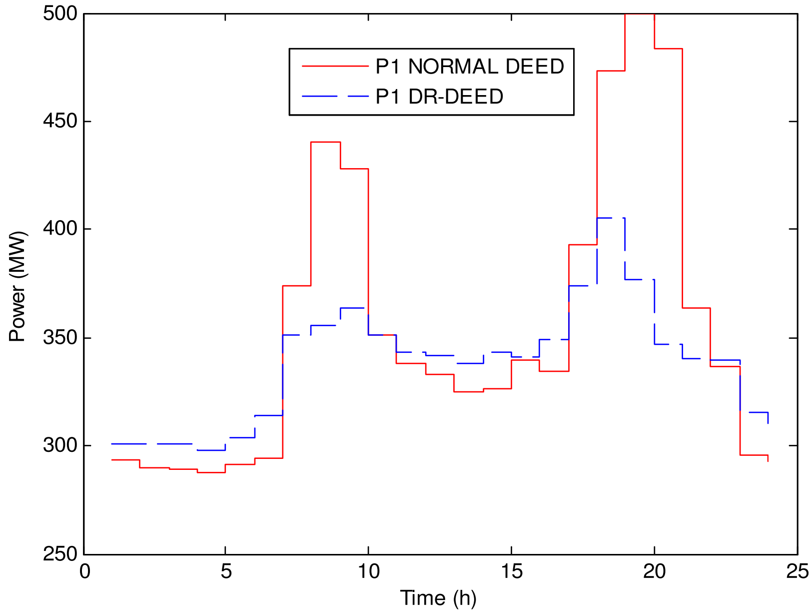

FIG. 6. Generation output of unit 1.

B. Test system 2

Test system 2 consists of ten unit generators at the supply side and two aggregated industrial customers at the customer side. Similar to the first example test system, we verify the mathematical formulations at both the utility and the customer side.

The fuel cost coefficients and the emission cost coefficients modified are obtained from Ref. 6 and shown in Table XII in the Appendix. The initial electricity tariff values are similarly obtained from Eskom's (the South African utility) Tariff book[25] and shown in Table XIII in the Appendix. The total initial demand is also shown in Table XIII. The TOU periods and elasticity values are as assumed in the first example test system given in Table I. The transmission loss formula coefficients for the ten unit test system are given as follows:

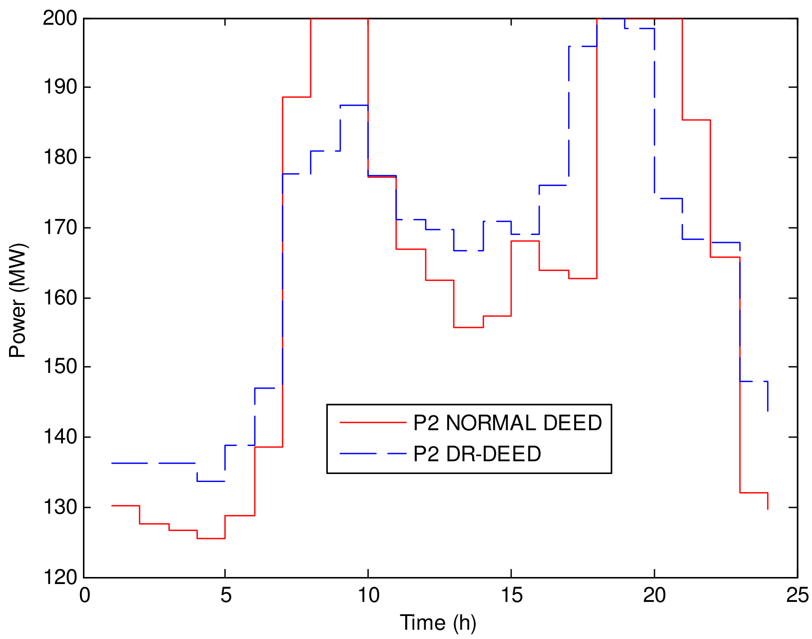

FIG. 7. Generation output of unit 2.

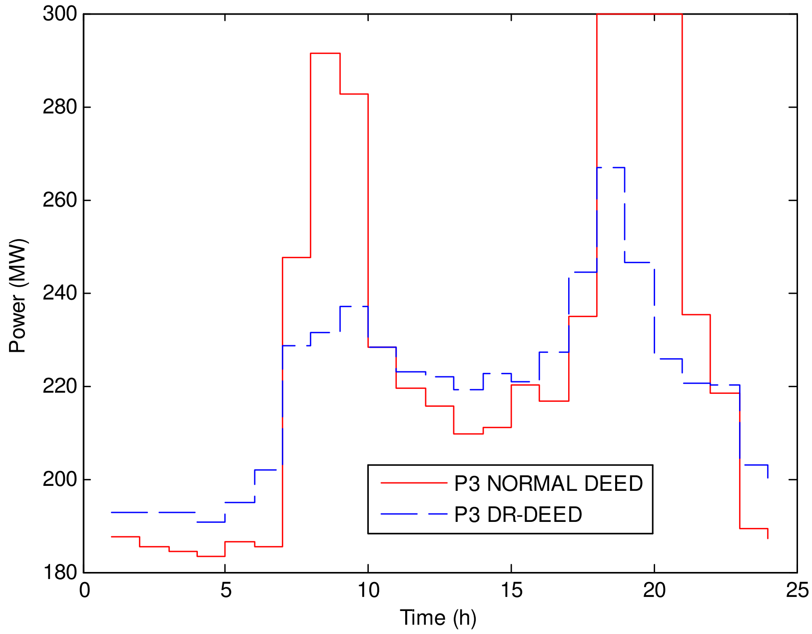

FIG. 8. Generation output of unit 3.

$$B = 10^{-5} \times \begin{bmatrix} 4.9 & 1.4 & 1.5 & 1.5 & 1.6 & 1.7 & 1.7 & 1.8 & 1.9 & 2.0 \\ 1.4 & 4.5 & 1.6 & 1.6 & 1.7 & 1.5 & 1.5 & 1.6 & 1.8 & 1.8 \\ 1.5 & 1.6 & 3.9 & 1.0 & 1.2 & 1.2 & 1.4 & 1.4 & 1.6 & 1.6 \\ 1.5 & 1.6 & 1.0 & 4.0 & 1.4 & 1.0 & 1.1 & 1.2 & 1.4 & 1.5 \\ 1.6 & 1.7 & 1.2 & 1.4 & 3.5 & 1.1 & 1.3 & 1.3 & 1.5 & 1.6 \\ 1.7 & 1.5 & 1.2 & 1.0 & 1.1 & 3.6 & 1.3 & 1.2 & 1.4 & 1.5 \\ 1.7 & 1.5 & 1.4 & 1.1 & 1.3 & 1.2 & 3.8 & 1.6 & 1.6 & 1.8 \\ 1.8 & 1.6 & 1.4 & 1.2 & 1.3 & 1.2 & 1.6 & 4.0 & 1.5 & 1.6 \\ 1.9 & 1.8 & 1.6 & 1.4 & 1.5 & 1.4 & 1.6 & 1.5 & 4.2 & 1.9 \\ 2.0 & 1.8 & 1.6 & 1.5 & 1.6 & 1.5 & 1.8 & 1.6 & 1.9 & 4.4 \end{bmatrix} \text{per MW.} \tag{32}$$

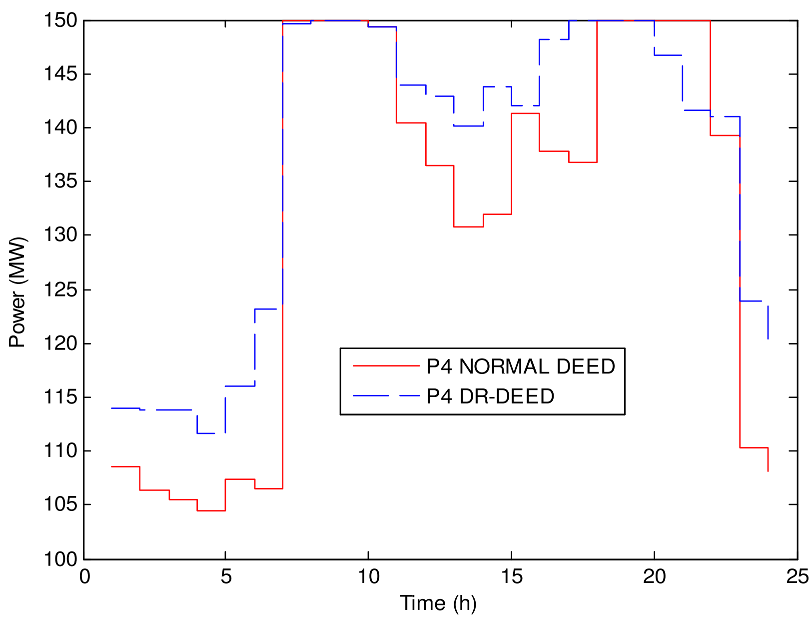

FIG. 9. Generation output of unit 4.

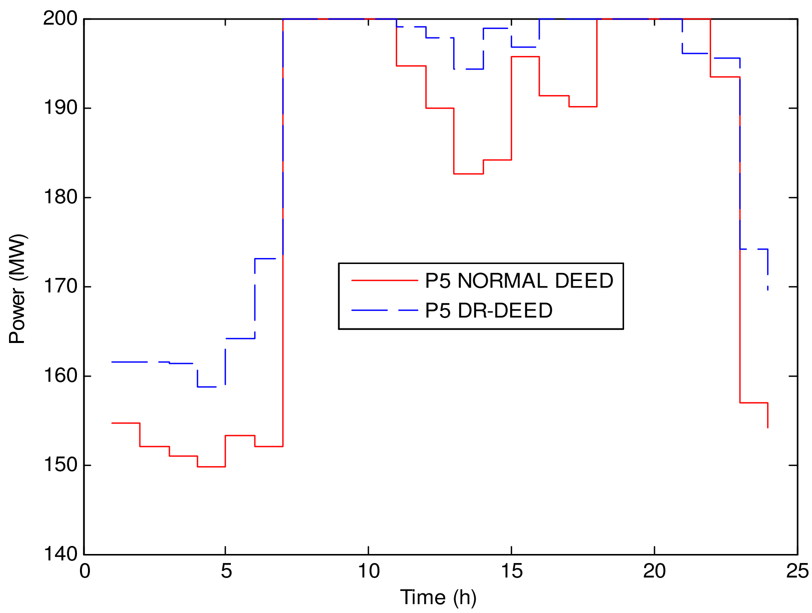

FIG. 10. Generation output of unit 5.

To verify the mathematical formulations for the customer side (Eqs. (27)–(29)), two aggregated industrial consumers are assumed. The load data for both aggregated customer groups are similar to those given in Tables IV and V, the only difference is in the number of customers within each aggregated group. It is assumed that both aggregated industrial customer groups consist of 30 and 20 identical customers, respectively, and there is a regulator that can schedule these loads.

The solution methodology employed is similar to the first example test system. AIMMS is again used to solve both optimization problems. In this work, after the third round of interactive control, convergence was achieved. Figure 12 shows the initial system load and the final

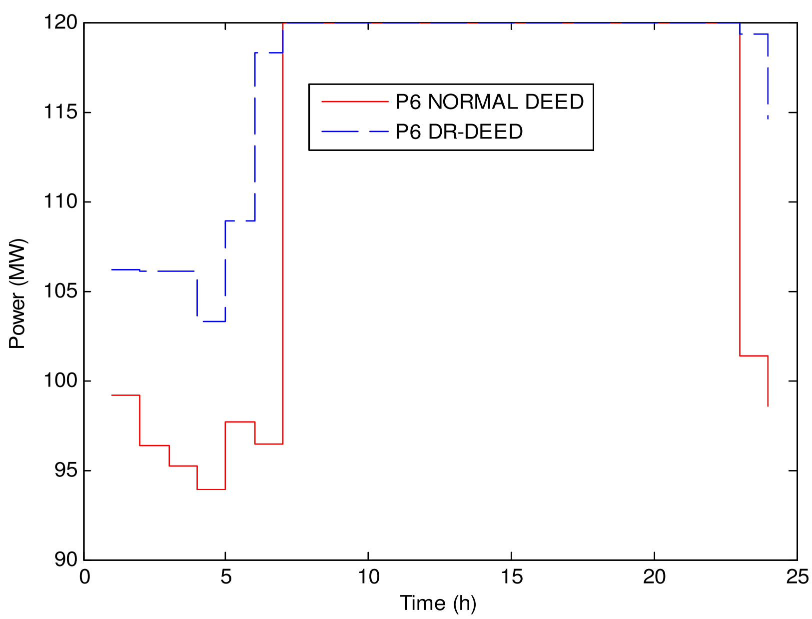

FIG. 11. Generation output of unit 6.

TABLE IV. Optimal DEED cost, emission, and loss with various weighting factor values (test system 1). Bold text indicates the base case when equal preference is given to both objective functions.

	Cost (DEED) (R)	Emissions (DEED) (lb)	Loss (DEED) (MW)	Cost (DR-DEED) (R)	Emissions (DR-DEED) (lb)	Loss (DR-DEED) (MW)
$w=0$	342 946	30 995	339	341 581.0874	29 281.74727	317.6168695
$w=0.1$	342 348	31 034	342	340 878.0754	29 331.2034	321.7372105
$w=0.2$	341 503	31 200	345	340 245.3722	29 463.33555	326.0064279
$w=0.3$	340 673	31 501	349	339 449.6479	29 766.06765	331.1171942
$w=0.4$	339 927	31 950	354	338 657.9488	30 249.67842	337.5981386
$w=0.5$	**339 105**	**32 696**	**360**	**337 873.1104**	**30 992.79329**	**345.0223506**
$w=0.6$	338 450	33 612	366	337 123.8609	32 055.83877	353.4399111
$w=0.7$	337 935	34 751	373	336 525.8092	33 411.16032	362.6221717
$w=0.8$	337 613	36 033	381	336 136.5383	34 970.12488	372.3296417
$w=0.9$	337 502	37 236	389	335 980.103	36 580.41833	382.1216955
$w=1$	337 541	38 475	397	336 021.406	38 078.86841	391.454633

TABLE V. Optimal load scheduling model solution for customer in the first group (test system 1).

Loads	Time slots (h)
1	7–11, 13–14, 16, 18–21
2	7–11, 14, 16–20, 22
3	1–24
4	1–24
5	5–6, 21–23
6	1–3, 22

TABLE VI. Optimal load scheduling model solution for customer in the second group (test system 1).

Loads	Time slots (h)
1	1–4, 11–15, 17, 23–24
2	6–10, 13, 16, 18–22
3	7–10, 12, 15–20
4	1–24
5	5–6, 21–22, 24
6	5–6, 21–22

optimal converged load. It shows that there is a reduction in peak demand and also a shifting of load from peak periods. Figure 13 shows the initial price and the final utility price. As in Figure 5, it is obvious that there is a price reduction in standard and off-peak periods and an increase in peak periods and these price movements have the intended effect on the energy consumption. Figures 14 and 15 show the optimal power generated for generators 1 and 2 under initial system load (normal DEED) and optimal converged load (DR-DEED), respectively. Due to the increase and reduction in load, there is a corresponding increase and reduction in power supplied from the generators. Table VII shows the results of varying the weighting factors on the fuel costs, emissions, and power loss under both DEED and DR-DEED for the second test system. The final customer optimal scheduling solution is shown in Tables VIII and IX for customers in the first and second groups, respectively.

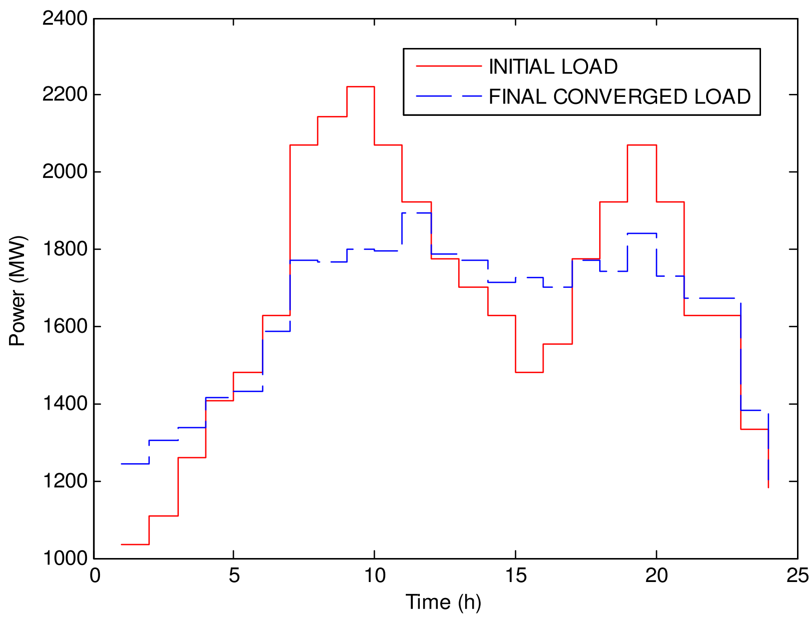

FIG. 12. Initial load and final converged load (test system 2).

C. Discussion of results

In summary, a concise and sequential description of the steps followed in this paper is described using the first test system below:

Step 1: Obtain initial load profile and initial pricing scheme (initial load in Figure 2 and initial price in Figure 3, respectively).

Step 2: The utility performs DR-DEED optimization (using Eqs. (20)–(26)) and obtains the utility forecast load and price (first utility forecast load in Figure 2 and utility first price in Figure 3, respectively).

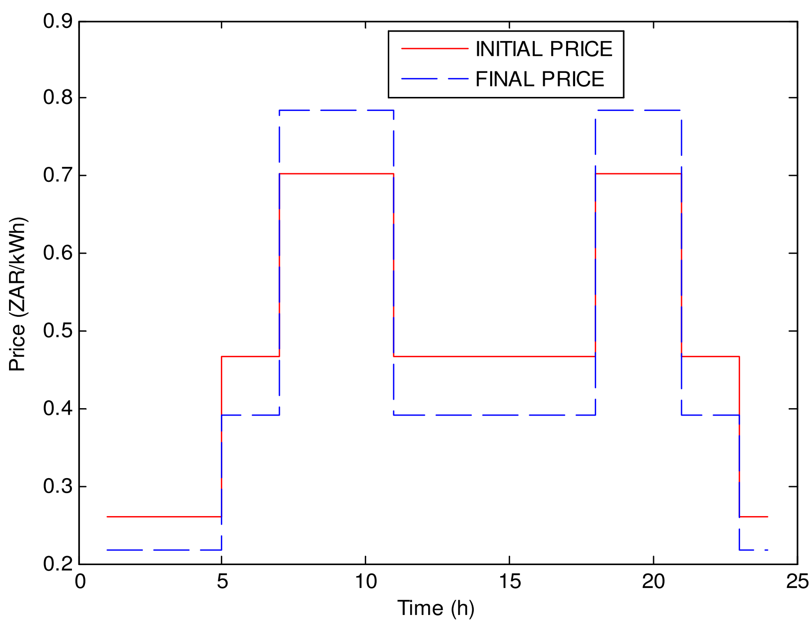

FIG. 13. Initial price and final price (test system 2).

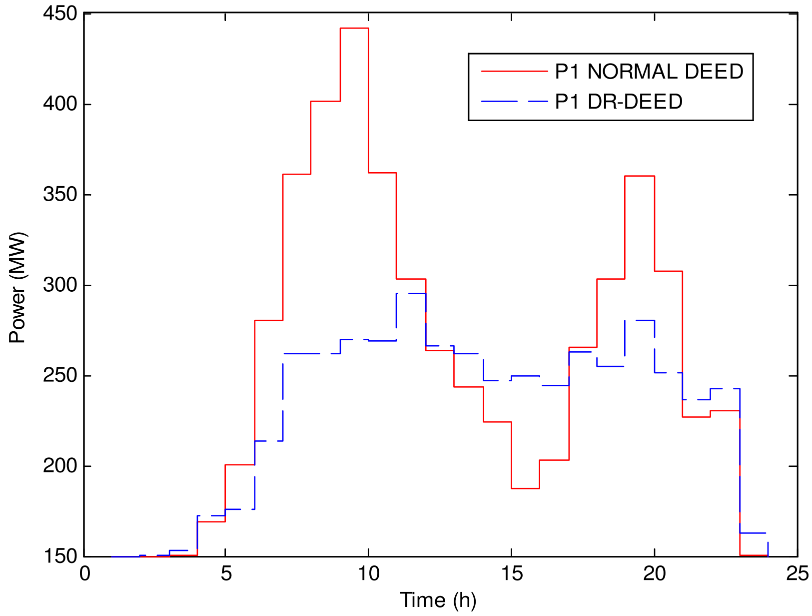

FIG. 14. Generation output of unit 1 (test system 2).

Step 3: In light of the utility's given price, the customers schedule their loads (using Eqs. (27)–(29)) and return the information back to the utility (first customer load in Figure 2).

Step 4: The utility revises the PEM and again performs DR-DEED optimization (using Eqs. (20)–(26)) and again obtains the utility price (utility second price in Figure 3).

Step 5: The customers again schedule their loads (using Eqs. (27)–(29)) in light of the new price and returns the information back to the utility (second customer load in Figure 2).

This interactive scheme continues until convergence is reached. In test system 1, this happens when the second customer load equals the first customer load (see second customer load and first customer load in Figure 2). In this context, we define convergence as when the utility's

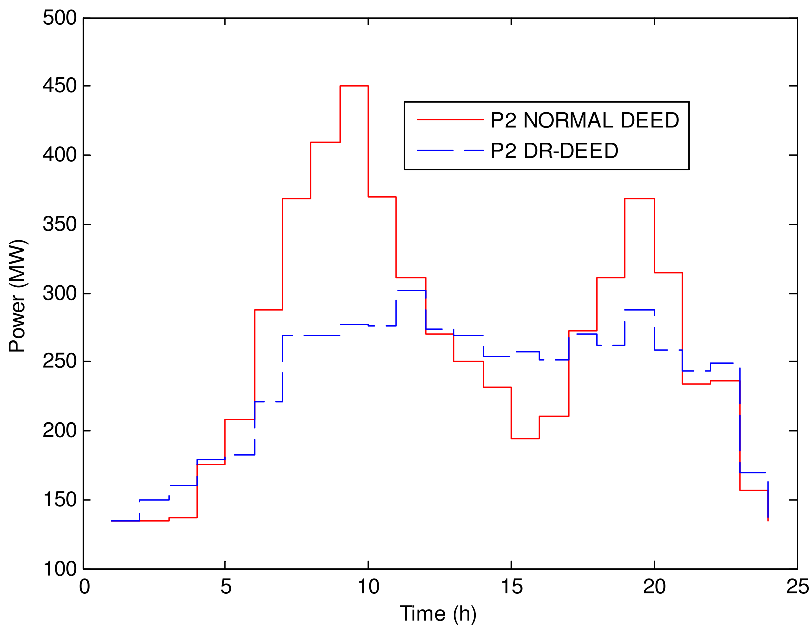

FIG. 15. Generation output of unit 2 (test system 2).

TABLE VII. Optimal DEED cost, emission, and loss with various weighting factor values (test system 2). Bold text indicates the base case when equal preference is given to both objective functions.

	Cost (DEED) (R)	Emissions (DEED) (lb)	Loss (DEED) (MW)	Cost (DR-DEED) (R)	Emissions (DR-DEED) (lb)	Loss (DR-DEED) (MW)
$w = 0$	1 054 224	248 251	1322	1 032 488	201 833.378	1208.975575
$w = 0.1$	1 053 155	248 335	1322	1 032 317.207	201 849.9257	1209.337727
$w = 0.2$	1 051 556	248 616	1322	1 031 558.711	201 990.8156	1209.8147
$w = 0.3$	1 051 049	248 799	1322	1 031 149.414	202 145.7047	1210.285814
$w = 0.4$	1 050 295	249 232	1323	1 030 376.402	202 601.3071	1210.981333
$w = 0.5$	**1 049 419**	**249 992**	**1324**	**1 028 962.793**	**203 827.1335**	**1212.25342**
$w = 0.6$	1 047 376	252 671	1326	1 025 657.354	208 270.9772	1215.233878
$w = 0.7$	1 043 923	259 564	1330	1 020 300.863	218 584.6925	1221.682126
$w = 0.8$	1 039 460	274 423	1343	1 015 861.032	233 260.7197	1233.855972
$w = 0.9$	1 034 599	308 177	1374	1 010 121.273	273 874.5636	1271.854093
$w = 1$	1 032 513	378 260	1424	1 006 570.04	385 080.8014	1353.525977

TABLE VIII. Optimal load scheduling model solution for first group of customers (test system 2).

Loads	Time slots (h)
1	7–10, 12–16, 18–20
2	7–8, 10–18, 20
3	1–24
4	1–24
5	1, 3, 5, 21, 24
6	1–2, 4, 21

TABLE IX. Optimal load scheduling model solution for second group of customers (test system 2).

Loads	Time slots (h)
1	4, 6, 9, 11–13, 16–17, 19, 21–23
2	3, 7–8, 10–15, 18, 20, 22
3	7–11, 14–15, 17–20
4	1–24
5	2, 5–6, 21–22
6	2, 5–6, 22

given price does not cause a change in the customers prior load schedule, thus obtaining an electricity price and demand mutually acceptable to the utility and customers whilst simultaneously reducing energy levels.

The DR program helps to reduce power system congestion, especially around peak times. It also shifts the load to the off peak and standard periods. Most of the generators produce more off peak/standard periods in order to reduce power production in peak periods. From the numerical simulations at the customer side, the model returns the optimal time slots to use the devices/appliances. The flexibility of the appliances is dictated by the peculiar needs of the customer and these determine the constraints of the mathematical model. The advantage of the interactive control strategy is the information flow is uni-directional, and the energy levels are obtained that provide power system relief and are acceptable to the utility and consumers. The model can be easily extended to accommodate a wider variety of consumers.

Figures 4 and 12 show the initial load and final converged load for test system 1 and test system 2, respectively. For both test systems, the benefit of DR is evident as there is load shifting and peak load reduction. This would have a commensurate effect on fuel costs reduction, emissions reduction, and power losses. Figures 5 and 13 show the initial price and final price for test system 1 and test system 2, respectively. Comparing the load graphs and the price graphs, it becomes clear that the interactive control process causes a TOU price increase in peak periods, which then has the intended effect of reducing customer loads in these periods. In standard and off-peak periods, there is a price reduction which incentivises customers to shift their load to these periods. Thus, customers shift their loads from periods when the power system is stressed, and power supply is constrained (peak periods) to periods when the power system is relatively unstressed. The loads that are shifted are the flexible loads and the night time loads. The inflexible loads are not shifted, and they account for the residual loads during peak periods. Due to the power balance constraint, which seeks to ensure that the total power supplied from the generators equals the load demand, there is an increase in power supply during standard and off-peak periods and a reduction in power supply during peak periods (see Figures 6–11, 14, and 15). It is from this power reduction from the generators that the cost and emission savings occur. Also, the reduction in power system losses stem from this reduction. Tables V and VI show the hourly optimal scheduling results for the first and second group of customers in test system 1 in light of the final utility price. Similarly, Tables VIII and IX show the results for both customer groups in test system 2.

Tables IV and VII give the optimal cost, emissions, and loss for DEED and DR-DEED for both example test systems. From both tables, the impact of DR on cost, emission, and losses can be clearly seen. DR brings a reduction in total demand, and hence, this brings about a corresponding decrease in costs, emissions, and losses. Both tables also show the variation of cost, emission, and losses when the weighting factor (w) ranges from 0 to 1. This analysis is important in multi-objective optimization problems with conflicting and competing objectives, to show how giving increased preference to one objective at the expense of the other influences the obtained results. In this case, it is observed that as w increases, the cost decreases and the emission and losses increases. This means that as the weighting factor is increased (the importance of minimizing emissions is decreased, while the importance of minimizing costs increases), emissions and losses actually increase and costs decrease. This is consistent with results obtained from the literature[29] and holds for both DEED and DR-DEED formulations.

VI. CONCLUSION

This paper presents a modification of the DEED formulation with price based DR programs. The objective in the optimization problem is to minimize the fuel, emissions, and DR costs subject to the conventional DEED constraints and some extra constraints. Investigations with different price elasticity matrices were assumed, and the TOU tariffs were used as the initial prices, giving rise to a TOU based DR-DEED problem formulation. As an interactive control strategy is used in the work, two customer mathematical models are presented where the customer classifies their loads into flexible, inflexible, and night-time loads and optimizes their demand in light of the utility suggested demand and final price. The customer schedules their load in order to minimize their electricity consumption and hence their electricity costs. Obtained simulation results indicate that DR programs reduce the total load curve and peak demand. The DR program also shifts the loads from peak periods to standard and off-peak periods. This is due to the fact that TOU prices reduce in standard and off peak periods and increase in peak periods. This reduces the likelihood of a stressed power system and minimizes instances of brown outs and blackouts. The results obtained also show that due to the reduced demand there is a corresponding reduction in fuel costs, harmful emissions, and power loss in both test systems considered. In essence, the results show that TOU-DR programs can bring about a reduction and shifting of load demand with customers who are actively interacting participants and who engage with the utility to reduce system demand. The interactive process employed also enabled the utility obtain optimal TOU prices acceptable to both utility and consumers alike. Future work will consider DR-DEED for a power system powered by combined heat and power (CHP) generators.

APPENDIX: POWER SYSTEM DATA

TABLE X. Data of the 6-unit system.

i	a_i (R/h)	b_i (R/MW h)	c_i (R/MW^2 h)	e_i (lb/h)	f_i (lb/MW h)	g_i (lb/MW^2 h)	$P_{i,\min}$ (MW)	$P_{i,\max}$ (MW)	DR_i (MW/h)	UR_i (MW/h)
1	240	7	0.007	13.8593	0.32767	0.00419	100	500	120	80
2	200	10	0.0095	13.8593	0.32767	0.00419	50	200	90	50
3	220	8.5	0.009	40.2669	−0.54551	0.00683	80	300	100	65
4	200	11	0.009	40.2669	−0.54551	0.00683	50	150	90	50
5	220	10.5	0.008	42.8955	−0.51116	0.00461	50	200	90	50
6	190	12	0.0075	42.8955	−0.51116	0.00461	50	120	90	50

TABLE XI. Initial TOU prices and total demand (test system 1).

Time (h)	TOU prices (R/kW h)	Total demand (MW)
1	0.2595	963
2	0.2595	948
3	0.2595	942
4	0.2595	935
5	0.4669	955
6	0.4669	963
7	0.7021	1263
8	0.7021	1380
9	0.7021	1360
10	0.7021	1210
11	0.4669	1165
12	0.4669	1143
13	0.4669	1110
14	0.4669	1117
15	0.4669	1170
16	0.4669	1150
17	0.4669	1221
18	0.7021	1420
19	0.7021	1445
20	0.7021	1430
21	0.4669	1238
22	0.4669	1159
23	0.2595	975
24	0.2595	960

TABLE XII. Data of the ten-unit system.

i	a_i (R/h)	b_i (R/MW h)	c_i (R/MW^2 h)	e_i (lb/h)	f_i (lb/MW h)	g_i (lb/MW^2 h)	$P_{i,\min}$ (MW)	$P_{i,\max}$ (MW)	DR_i (MW/h)	UR_i (MW/h)
1	958.2	21.6	0.00043	360.0012	−3.9864	0.04702	150	470	80	80
2	1313.6	21.05	0.00063	350.0056	−3.9524	0.04652	135	460	80	80
3	604.97	20.81	0.00039	330.0056	−3.9023	0.04652	73	340	80	80
4	471.6	23.9	0.0007	330.0056	−3.9023	0.04652	60	300	50	50
5	480.29	21.62	0.00079	13.8593	0.3277	0.0042	73	243	50	50
6	601.75	17.87	0.00056	13.8593	0.3277	0.0042	57	160	50	50
7	502.7	16.51	0.00211	40.2669	−0.5455	0.0068	20	130	30	30
8	639.4	23.23	0.0048	40.2669	−0.5455	0.0068	47	120	30	30
9	455.6	19.58	0.10908	42.8955	−0.5112	0.0046	20	80	30	30
10	692.4	22.54	0.00951	42.8955	−0.5112	0.0046	55	55	30	30

TABLE XIII. Initial TOU prices and total demand (test system 2).

Time (h)	TOU prices (R/kW h)	Total demand (MW)
1	0.2595	1036
2	0.2595	1110
3	0.2595	1258
4	0.2595	1406
5	0.4669	1480
6	0.4669	1628
7	0.7021	2072
8	0.7021	2146
9	0.7021	2220
10	0.7021	2072
11	0.4669	1924
12	0.4669	1776
13	0.4669	1702
14	0.4669	1628
15	0.4669	1480
16	0.4669	1554
17	0.4669	1776
18	0.7021	1924
19	0.7021	2072
20	0.7021	1924
21	0.4669	1628
22	0.4669	1628
23	0.2595	1332
24	0.2595	1184

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