

## Research Article

# Hybrid DE-SQP Method for Solving Combined Heat and Power Dynamic Economic Dispatch Problem

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Combined heat and power dynamic economic dispatch (CHPDED) plays a key role in economic operation of power systems. CHPDED determines the optimal heat and power schedule of committed generating units by minimizing the fuel cost under ramp rate constraints and other constraints. Due to complex characteristics, heuristic and evolutionary based optimization approaches have become effective tools to solve the CHPDED problem. This paper proposes hybrid differential evolution (DE) and sequential quadratic programming (SQP) to solve the CHPDED problem with nonsmooth and nonconvex cost function due to valve point effects. DE is used as a global optimizer and SQP is used as a fine tuning to determine the optimal solution at the final. The proposed hybrid DE-SQP method has been tested and compared to demonstrate its effectiveness.

## 1. Introduction

In the past decades, increasing demand for power and heat resulted in the existence of combined heat and power (CHP) units, known as cogeneration or distributed generation. It produces electricity and useful heat simultaneously. While the efficiency of the normal power generation is between 50% and 60%, the power and heat cogeneration increases the efficiency around 90% [1]. Utilization of CHP units besides conventional thermal power generating units and heat-only units to satisfy heat and power load demands in an economical manner emphasizes the need to combined heat and power economic dispatch (CHPED). The objective of the CHPED problem is to determine both power generation and heat production from units by minimizing the fuel cost such that both heat and power demands are met while the combined heat and power units are operated in a bounded heat versus power plane. For most CHP units, the heat production capacities depend on the power generation. This mutual dependency of the CHP units introduce a complication to the problem [2]. In addition, considering valve point effects in the CHPED problem makes the problem nonsmooth with

multiple local optimal point which makes finding the global optimal challenging.

Over the past few years, a number of approaches have been developed for solving the CHPED problem with complex objective functions or constraints such as Lagrangian Relaxation (LR) [3, 4], Semidefinite Programming (SDP) [5], augmented Lagrange combined with Hopfield neural network [6], Harmony Search (HS) algorithm [1, 7], Genetic Algorithm (GA) [8], Ant Colony Search Algorithm (ACSA) [9], Mesh Adaptive Direct Search (MADS) algorithm [10], Self Adaptive Real-Coded Genetic Algorithm (SARGA) [2], Particle Swarm Optimization (PSO) [11, 12], Artificial Immune System (AIS) [13], and Evolutionary Programming (EP) [14]. In [11, 13], the valve point effects and the transmission line losses are incorporated into the CHPED problem.

The main drawbacks of the CHPED is that it may fail to deal with the large variations of the heat and power load demands due to the ramp rate limits of the units; moreover, it does not have the look-ahead capability. To overcome these drawbacks, combined heat and power dynamic economic dispatch (CHPDED) problem is formulated with the objective to determine the optimal heat and power schedule of

the committed units so as to meet the predicted heat and power load demands over a time horizon at minimum operating cost under ramp rate constraints and other constraints [15]. CHPDED has a look-ahead capability which is necessary to schedule the load beforehand so that the system can anticipate sudden changes in power and heat demands in the near future. The ramp rate constraint is a dynamic constraint which is important to maintain the life of the generators [16]. Since the ramp rate constraint couples the time intervals, the CHPDED problem is a difficult optimization problem. If the ramp rate constraint is not included in the optimization problem, the CHPDED problem is reduced to a set of uncoupled CHPED problems that can easily be solved. The traditional dynamic economic dispatch (DED) problem which considers only conventional thermal units that provide only electric power has been studied by several authors (see e.g., [17, 18] and the review paper [16]). However, the CHPDED problem has only been considered in [15, 19].

Differential Evolution (DE) algorithm, which was proposed by Storn and Price [20] is a population-based stochastic parallel search technique. DE uses a rather greedy and less stochastic approach to problem solving compared to other evolutionary algorithms. DE has the ability to handle optimization problems with nonsmooth/nonconvex objective functions [20]. Moreover, it has a simple structure and a good convergence property, and it requires a few robust control parameters [20]. DE has been applied to the CHPED problem with nonsmooth and nonconvex cost functions in [21].

The DE shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA) techniques. The system is initialized with a population of random solutions and searches for optima by updating generations. DE has evolution operators such as crossover and mutation. Although DE seems to be a good method to solve the CHPDED problem with nonsmooth and nonconvex cost functions, solutions obtained are just near global optimum with long computation time. Therefore, hybrid methods such as DE-SQP can be effective in solving the CHPDED problem with valve-point effects. Hybrid DE-SQP method has been used for solving the DED problem in [22, 23].

The aim of this paper is to propose a hybrid DE-SQP method for solving the CHPDED problem with nonsmooth and nonconvex objective function. DE is used as a base level search for global exploration and SQP is used as a local search to fine-tune the solution obtained from DE. The effectiveness of the proposed method is shown for test system.

## 2. Problem Formulation

In this section, we formulate the CHPDED problem. The system under consideration has three types of generating units, conventional thermal units (TU), CHP units, and heat-only units ( $H$ ). The power is generated by conventional thermal units and CHP units, while the heat is generated by CHP units and heat-only units. The objective of the CHPDED problem is to minimize the system's production cost so as to meet the predicted heat and power load demands

over a time horizon under ramp rate and other constraints. The following objectives and constraints are taken into account in the formulation of the CHPDED problem.

*2.1. Objective Functions.* In this section, we introduce the cost function of three types of generating units, conventional thermal units, CHP units, and heat-only units.

*Conventional Thermal Units.* The cost function curve of a conventional thermal unit can be approximated by a quadratic function [24, 25]. Power plants commonly have multiple valves which are used to control the power output of the unit. When steam admission valves in conventional thermal units are first open, a sudden increase in losses is registered which results in ripples in the cost function [16, 26]. This phenomenon is called as valve-point effects. The generator with valve-point effects has very different input-output curve compared with smooth cost function. Taking the valve-point effects into consideration, the fuel cost is expressed as the sum of a quadratic and sinusoidal functions [17, 19, 27]. Therefore, the fuel cost function of the conventional thermal units is given by

$$C_i^{\text{TU}}(P_{i,t}^{\text{TU}}) = a_i + b_i P_{i,t}^{\text{TU}} + c_i (P_{i,t}^{\text{TU}})^2 + |e_i \sin(f_i (P_{i,\min}^{\text{TU}} - P_{i,t}^{\text{TU}}))|, \quad (1)$$

where  $a_i$ ,  $b_i$ , and  $c_i$  are positive constants,  $e_i$  and  $f_i$  are the coefficients of conventional thermal unit  $i$  reflecting valve-point effects,  $P_{i,t}^{\text{TU}}$  is the power generation of conventional thermal unit  $i$  during the  $t$ th time interval  $[t - 1, t)$ ,  $P_{i,\min}^{\text{TU}}$  is the minimum capacity of conventional thermal unit  $i$ , and  $C_i^{\text{TU}}(P_{i,t}^{\text{TU}})$  is the fuel cost of conventional thermal unit  $i$  to produce  $P_{i,t}^{\text{TU}}$ .

*CHP Units.* A CHP unit has a convex cost function in both power and heat. The form of the fuel cost function of CHP units can be given by [5, 19]

$$C_j^{\text{CHP}}(P_{j,t}^{\text{CHP}}, H_{j,t}^{\text{CHP}}) = \bar{a}_j + \bar{b}_j P_{j,t}^{\text{CHP}} + \bar{c}_j (P_{j,t}^{\text{CHP}})^2 + \bar{d}_j H_{j,t}^{\text{CHP}} + \bar{e}_j (H_{j,t}^{\text{CHP}})^2 + \bar{f}_j P_{j,t}^{\text{CHP}} H_{j,t}^{\text{CHP}}, \quad (2)$$

where  $C_j^{\text{CHP}}(P_{j,t}^{\text{CHP}}, H_{j,t}^{\text{CHP}})$  is the generation fuel cost of CHP unit  $j$  to produce power  $P_{j,t}^{\text{CHP}}$  and heat  $H_{j,t}^{\text{CHP}}$ . Constants  $\bar{a}_j$ ,  $\bar{b}_j$ ,  $\bar{c}_j$ ,  $\bar{d}_j$ ,  $\bar{e}_j$ , and  $\bar{f}_j$  are the fuel cost coefficients of CHP unit  $j$ .

*Heat-Only Units. Cost:* The cost function of heat-only units can take the following form [5, 19]:

$$C_k^{\text{H}}(H_{k,t}^{\text{H}}) = \tilde{a}_k + \tilde{b}_k H_{k,t}^{\text{H}} + \tilde{c}_k (H_{k,t}^{\text{H}})^2, \quad (3)$$

where  $\tilde{a}_k$ ,  $\tilde{b}_k$ , and  $\tilde{c}_k$ , are the fuel cost coefficients of heat-only unit  $k$  and they are constants.

Let  $N$  be the number of dispatch intervals and let  $N_p + N_c + N_h$  be the number of committed units, where  $N_p$  is the number of conventional thermal units,  $N_c$  is the number of the CHP units, and  $N_h$  is the number of the heat-only units. Then the total fuel cost over the dispatch period  $[0, N]$  is given by

$$\begin{aligned} C(\mathbf{PH}) &= \sum_{t=1}^N \left( \sum_{i=1}^{N_p} C_i^{\text{TU}} (P_{i,t}^{\text{TU}}) + \sum_{j=1}^{N_c} C_j^{\text{CHP}} (P_{j,t}^{\text{CHP}}, H_{j,t}^{\text{CHP}}) \right. \\ &\quad \left. + \sum_{k=1}^{N_h} C_k^{\text{H}} (H_{k,t}^{\text{H}}) \right), \end{aligned} \quad (4)$$

where  $\mathbf{PH} = (\mathbf{PH}_1, \mathbf{PH}_2, \dots, \mathbf{PH}_t, \dots, \mathbf{PH}_N)'$ ,  $\mathbf{PH}_t = (\mathbf{P}_t^{\text{TU}}, \mathbf{P}_t^{\text{CHP}}, \mathbf{H}_t^{\text{CHP}}, \mathbf{H}_t^{\text{H}})'$ ,  $\mathbf{P}_t^{\text{TU}} = (P_{1,t}^{\text{TU}}, P_{2,t}^{\text{TU}}, \dots, P_{N_p,t}^{\text{TU}})'$ ,  $\mathbf{P}_t^{\text{CHP}} = (P_{1,t}^{\text{CHP}}, P_{2,t}^{\text{CHP}}, \dots, P_{N_c,t}^{\text{CHP}})'$ ,  $\mathbf{H}_t^{\text{CHP}} = (H_{1,t}^{\text{CHP}}, H_{2,t}^{\text{CHP}}, \dots, H_{N_c,t}^{\text{CHP}})'$ , and  $\mathbf{H}_t^{\text{H}} = (H_{1,t}^{\text{H}}, H_{2,t}^{\text{H}}, \dots, H_{N_h,t}^{\text{H}})'$ .

The CHPDED problem can be mathematically formulated as a nonlinear constrained optimization problem as

$$\min_{\mathbf{PH}} C(\mathbf{PH}) \quad (5)$$

subject to the constraints.

Power production and demand balance:

$$\sum_{i=1}^{N_p} P_{i,t}^{\text{TU}} + \sum_{j=1}^{N_c} P_{j,t}^{\text{CHP}} = P_{D,t} + \text{Loss}_t, \quad t = 1, \dots, N, \quad (6)$$

where  $P_{D,t}$  and  $\text{Loss}_t$  are the system power demand and transmission line losses at time  $t$  (i.e., the  $t$ th time interval), respectively. The B-coefficient method is one of the most commonly used methods by power utility industry to calculate the network losses. In this method, the network losses are expressed as a quadratic function of the unit's power outputs that can be approximated in the following:

$$\text{Loss}_t = \sum_{i=1}^{N_p+N_c} \sum_{j=1}^{N_p+N_c} \mathbf{PL}_{i,t} B_{ij} \mathbf{PL}_{j,t} \quad t = 1, \dots, N, \quad (7)$$

where

$$\mathbf{PL}_{i,t} = \begin{cases} P_{i,t}^{\text{TU}}, & i = 1, \dots, N_p, \\ P_{i-N_p,t}^{\text{CHP}}, & i = N_p + 1, \dots, N_p + N_c, \end{cases} \quad (8)$$

and  $B_{ij}$  is the  $ij$ th element of the loss coefficient square matrix of size  $N_p + N_c$ .

Heat production and demand balance:

$$\sum_{j=1}^{N_c} H_{j,t}^{\text{CHP}} + \sum_{k=1}^{N_h} H_{k,t}^{\text{H}} = H_{D,t}, \quad t = 1, \dots, N, \quad (9)$$

where  $H_{D,t}$  is the system heat demand at time  $t$ .

Capacity limits of conventional thermal units:

$$P_{i,\min}^{\text{TU}} \leq P_{i,t}^{\text{TU}} \leq P_{i,\max}^{\text{TU}}, \quad i = 1, \dots, N_p, \quad t = 1, \dots, N, \quad (10)$$

where  $P_{i,\min}^{\text{TU}}$  and  $P_{i,\max}^{\text{TU}}$  are the minimum and maximum power capacities of conventional thermal unit  $i$ , respectively.

Capacity limits of CHP units:

$$\begin{aligned} P_{j,\min}^{\text{CHP}} (H_{j,t}^{\text{CHP}}) &\leq P_{j,t}^{\text{CHP}} \leq P_{j,\max}^{\text{CHP}} (H_{j,t}^{\text{CHP}}), \\ j &= 1, \dots, N_c, \quad t = 1, \dots, N, \\ H_{j,\min}^{\text{CHP}} (P_{j,t}^{\text{CHP}}) &\leq H_{j,t}^{\text{CHP}} \leq H_{j,\max}^{\text{CHP}} (P_{j,t}^{\text{CHP}}), \\ j &= 1, \dots, N_c, \quad t = 1, \dots, N, \end{aligned} \quad (11)$$

where  $P_{j,\min}^{\text{CHP}} (H_{j,t}^{\text{CHP}})$  and  $P_{j,\max}^{\text{CHP}} (H_{j,t}^{\text{CHP}})$  are the minimum and maximum power limits of CHP unit  $j$ , respectively, and they are functions of generated heat ( $H_{j,t}^{\text{CHP}}$ ).  $H_{j,\min}^{\text{CHP}} (P_{j,t}^{\text{CHP}})$  and  $H_{j,\max}^{\text{CHP}} (P_{j,t}^{\text{CHP}})$  are the heat generation limits of CHP unit  $j$  which are functions of generated power ( $P_{j,t}^{\text{CHP}}$ ).

Capacity limits of heat-only units:

$$H_{k,\min}^{\text{H}} \leq H_{k,t}^{\text{H}} \leq H_{k,\max}^{\text{H}}, \quad k = 1, \dots, N_h, \quad t = 1, \dots, N, \quad (12)$$

where  $H_{k,\min}^{\text{H}}$  and  $H_{k,\max}^{\text{H}}$  are the minimum and maximum heat capacities of heat-only unit  $k$ , respectively.

Upper/down ramp rate limits of conventional thermal units:

$$\begin{aligned} -DR_i^{\text{TU}} &\leq P_{i,t+1}^{\text{TU}} - P_{i,t}^{\text{TU}} \leq UR_i^{\text{TU}}, \\ i &= 1, \dots, N_p, \quad t = 1, \dots, N-1, \end{aligned} \quad (13)$$

where  $UR_i^{\text{TU}}$  and  $DR_i^{\text{TU}}$  are the maximum ramp up/down rates for conventional thermal unit  $i$  [16].

Upper/down ramp rate limits of CHP units:

$$\begin{aligned} -DR_j^{\text{CHP}} &\leq P_{j,t+1}^{\text{CHP}} - P_{j,t}^{\text{CHP}} \leq UR_j^{\text{CHP}}, \\ j &= 1, \dots, N_c, \quad t = 1, \dots, N-1, \end{aligned} \quad (14)$$

where  $UR_j^{\text{CHP}}$  and  $DR_j^{\text{CHP}}$  are the maximum ramp up/down rates for CHP unit  $j$  [19].

### 3. Differential Evolution Method

DE is a simple yet powerful heuristic method for solving nonlinear, nonconvex, and nonsmooth optimization problems. DE algorithm is a population-based algorithm using three operators: mutation, crossover, and selection to evolve from randomly generated initial population to final individual solution [20]. In the initialization, a population of  $NP$  target vectors (parents)  $X_i = \{x_{1i}, x_{2i}, \dots, x_{Di}\}$ ,  $i = 1, 2, \dots, NP$  is randomly generated within user-defined bounds, where  $D$  is the dimension of the optimization problem. Let  $X_i^G = \{x_{1i}^G, x_{2i}^G, \dots, x_{Di}^G\}$  be the individual  $i$  at the current generation

TABLE 1: Data of the CHP units and heat-only unit of the eleven-unit system.

CHP units	$\bar{a}_j$	$\bar{b}_j$	$\bar{c}_j$	$\bar{d}_j$	$\bar{e}_j$	$\bar{f}_j$	$DR_j^{\text{CHP}} = UR_j^{\text{CHP}}$
$j = 1$	2650	14.5	0.0345	4.2	0.030	0.031	70
$j = 2$	1250	36	0.0435	0.6	0.027	0.011	50
Heat-only units		$H_{k,\max}^H$	$H_{k,\min}^H$	$\bar{a}_k$	$\bar{b}_k$	$\bar{c}_k$	
$k = 1$		2695.2	0	950	2.0109	0.038	

TABLE 2: Heat load demand of the eleven-unit system.

Time (h)	Demand (MWh)
1	390
2	400
3	410
4	420
5	440
6	450
7	450
8	455
9	460
10	460
11	470
12	480
13	470
14	460
15	450
16	450
17	420
18	435
19	445
20	450
21	445
22	435
23	400
24	400

G. A mutant vector  $V_i^{G+1} = (v_{1i}^{G+1}, v_{2i}^{G+1}, \dots, v_{Di}^{G+1})$  is generated according to

$$V_i^{G+1} = X_{r_1}^G + \mathcal{F} \times (X_{r_2}^G - X_{r_3}^G), \quad (15)$$

$$r_1 \neq r_2 \neq r_3 \neq i, \quad i = 1, 2, \dots, NP$$

with randomly chosen integer indexes  $r_1, r_2, r_3 \in \{1, 2, \dots, NP\}$ . Here  $\mathcal{F}$  is the mutation factor.

According to the target vector  $X_i^G$  and the mutant vector  $V_i^{G+1}$ , a new trial vector (offspring)  $U_i^{G+1} = \{u_{1i}^{G+1}, u_{2i}^{G+1}, \dots, u_{Di}^{G+1}\}$  is created with

$$u_{ji}^{G+1} = \begin{cases} v_{ji}^{G+1} & \text{if } (\text{rand}(j) \leq CR) \text{ or } j = \text{rnb}(i) \\ x_{ji}^G & \text{otherwise,} \end{cases} \quad (16)$$

where  $j = 1, 2, \dots, D$ ,  $i = 1, 2, \dots, NP$ , and  $\text{rand}(j)$  is the  $j$ th evaluation of a uniform random number between  $[0, 1]$ .

$CR \in [0, 1]$  is the crossover constant which has to be determined by the user.  $\text{rnb}(i)$  is a randomly chosen index from  $1, 2, \dots, D$  which ensures that  $U_i^{G+1}$  gets at least one parameter from  $V_i^{G+1}$  [20].

The selection process determines which of the vectors will be chosen for the next generation by implementing one-to-one competition between the offsprings and their corresponding parents. If  $f$  denotes the function to be minimized, then

$$X_i^{G+1} = \begin{cases} U_i^{G+1} & \text{if } f(U_i^{G+1}) \leq f(X_i^G) \\ X_i^G & \text{otherwise,} \end{cases} \quad (17)$$

where  $i = 1, 2, \dots, NP$ . The value of  $f$  of each trial vector  $U_i^{G+1}$  is compared with that of its parent target vector  $X_i^G$ . The above iteration process of reproduction and selection will continue until a user-specified stopping criteria is met.

In this paper, we define the evaluation function for evaluating the fitness of each individual in the population in DE algorithm as follows:

$$f = C + \lambda_1 \sum_{t=1}^N \left( \sum_{i=1}^{N_p} P_{i,t}^{\text{TU}} + \sum_{j=1}^{N_c} P_{j,t}^{\text{CHP}} - (P_{D,t} + \text{Loss}_t) \right)^2 + \lambda_2 \sum_{t=1}^N \left( \sum_{j=1}^{N_c} H_{j,t}^{\text{CHP}} + \sum_{k=1}^{N_h} H_{k,t}^H - H_{D,t} \right)^2, \quad (18)$$

where  $\lambda_1$  and  $\lambda_2$  are penalty values. Then the objective is to find  $f_{\min}$ , the minimum evaluation value of all the individuals in all iterations. The penalty term reflects the violation of the equality constraints. Once the minimum of  $f$  is reached, the equality constraints are satisfied.

#### 4. Sequential Quadratic Programming Method

SQP method can be considered as one of the best nonlinear programming method for constrained optimization problems [28]. It outperforms every other nonlinear programming method in terms of efficiency, accuracy, and percentage of successful solutions over a large number of test problems. The method closely resembles Newton's method for constrained optimization, just as is done for unconstrained optimization. At each iteration, an approximation is made of the Hessian of the Lagrangian function using Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton updating method. The result of the approximation is then used to generate a Quadratic Programming (QP) subproblem whose

TABLE 3: Hourly heat and power schedule obtained from CHPDED using DE-SQP for eleven-unit system.

H	$P_1^{TU}$	$P_2^{TU}$	$P_3^{TU}$	$P_4^{TU}$	$P_5^{TU}$	$P_6^{TU}$	$P_7^{TU}$	$P_8^{TU}$	$P_1^{CHP}$	$P_2^{CHP}$	Loss	$H_1^{CHP}$	$H_2^{CHP}$	$H_1^H$
1	150.0000	135.0000	74.5372	72.0784	124.5129	124.4302	20.0000	10.0000	236.8041	110.1974	21.5630	57.3450	135.5994	197.0556
2	150.0000	135.0000	98.1135	122.0784	122.2113	101.6179	48.2025	10.0000	236.8011	110.1974	24.2248	57.3614	135.5994	207.0392
3	150.0000	135.0000	178.1135	172.0784	120.7640	98.7468	78.2025	10.0000	235.3275	110.1974	30.4319	65.6496	135.5994	208.7509
4	150.0000	135.0000	188.0106	218.5077	160.0000	126.3142	80.0000	40.0000	235.2182	110.1974	37.2496	66.2643	135.5994	218.1363
5	150.0000	135.0000	268.0106	244.7145	128.0292	129.9179	80.0000	42.2707	233.2313	110.1974	41.3736	77.4390	135.5994	226.9616
6	150.0000	135.0000	334.4706	294.7145	160.0000	130.0000	80.0000	48.0931	235.6609	110.1974	50.1383	63.7746	135.5994	250.6260
7	150.0000	199.1593	340.0000	300.0000	160.0000	130.0000	80.0000	49.7990	238.0991	110.1974	55.2549	50.0614	135.5994	264.3392
8	189.7336	229.5497	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	242.2569	110.1974	60.7377	26.6766	135.5994	292.7240
9	265.3596	309.5497	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	247.0000	110.1974	73.1068	0.0	135.5994	324.4006
10	303.6024	378.5162	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	246.9410	110.1974	82.2580	0.3317	135.5994	324.0689
11	368.8317	405.6648	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	247.0000	110.1974	90.6945	0.0	135.5994	334.4006
12	367.7179	455.4472	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	247.0000	110.1974	95.3624	0.0	135.5994	344.4006
13	352.0071	385.0034	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	247.0000	110.1974	87.2079	0.0	135.5994	334.4006
14	272.0071	305.0034	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	244.9090	110.1974	73.1169	11.7604	135.5994	312.6402
15	193.6233	225.0034	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	242.9121	110.1974	60.7362	22.9917	135.5994	291.4089
16	150.0000	145.0034	296.8330	250.8703	160.0000	129.9573	80.0000	43.4626	233.2660	110.1974	45.5900	77.2439	135.5994	237.1567
17	150.0000	135.0000	260.0109	250.0000	160.0000	100.0000	80.0000	40.9143	235.3888	110.1974	41.5121	65.3046	135.5994	219.0959
18	150.0000	151.0646	319.4485	300.0000	160.0000	130.0000	80.0000	40.0577	237.4722	110.1974	50.2419	53.5869	135.5994	245.8137
19	229.4141	231.0646	313.3779	300.0000	160.0000	130.0000	80.0000	46.0360	237.0065	110.1974	61.0988	56.2062	135.5994	253.1943
20	309.4141	311.0646	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	247.0000	116.9757	77.4552	0.0	90.7694	359.2306
21	272.4577	300.8037	340.0000	300.0000	160.0000	130.0000	80.0000	55.0000	247.0000	111.8344	73.0959	0.0	124.7723	320.2277
22	192.4577	220.8037	260.6669	250.0000	160.0000	124.1397	80.0000	45.9763	234.6724	110.1974	50.9154	69.3338	135.5994	230.0668
23	150.0000	140.8037	180.6669	200.0000	127.6584	130.0000	50.0000	40.0000	236.4213	110.1974	33.7482	59.4980	135.5994	204.9026
24	150.0000	135.0000	100.6669	177.0362	123.2649	128.6636	42.3316	10.0000	234.6572	109.5624	27.1834	69.4196	135.0513	195.5291

Cost (\$) =  $2.5257 \times 10^6$ ; total loss (MW) =  $1.3443 \times 10^3$ .

solution is used to form a search direction for a line search procedure. Since the objective function of the CHPDED problem is nonconvex and nonsmooth, SQP ensures a local minimum for an initial solution. In this paper, DE is used as a global search and finally the best solution obtained from DE is given as initial condition for SQP method as a local search to fine-tune the solution. SQP simulations can be computed by the `fmincon` code of the MATLAB Optimization Toolbox.

### 5. Simulation Results

In this section, we present an eleven-unit test system. The hybrid DE-SQP method is applied to the CHPDED problem, where three types of generating units, conventional thermal units, CHP units, and heat-only units, are considered. In DE-SQP method, the control parameters are chosen as  $NP = 80$ ,  $\mathcal{F} = 0.423$ , and  $CR = 0.885$ . The maximum number of iterations is selected as 20,000. The results represent the average of 30 runs of the proposed method. All computations are carried out by MATLAB program.

*Eleven-Unit System.* This system consists of eight conventional thermal units, two CHP units, and one heat-only unit. The CHPDED problem is solved by hybrid DE-SQP method. The technical data of conventional thermal units, the matrix  $B$ , and the power demand are taken from the ten-unit system presented in [27]. The 5th and 8th conventional units in [27] were replaced by two CHP units. The technical data of the two

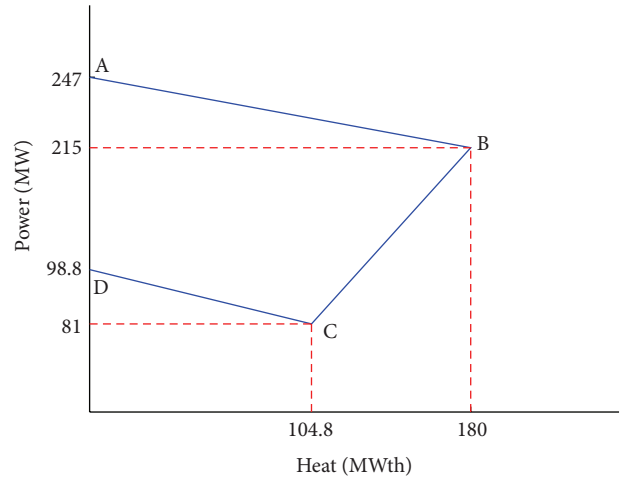


FIGURE 1: Heat-power feasible operating region for CHP unit 1 of the eleven-unit system.

CHP units and the heat-only unit are taken from [19] and are given in Table 1. The heat demand for 24 hours is given in Table 2. The feasible operating regions of the two CHP units are taken from [3] and are given in Figures 1 and 2.

The best solution of the CHPDED problem obtained by DE-SQP algorithm is given in Table 3. The best cost and transmission line losses are also given in Table 3.



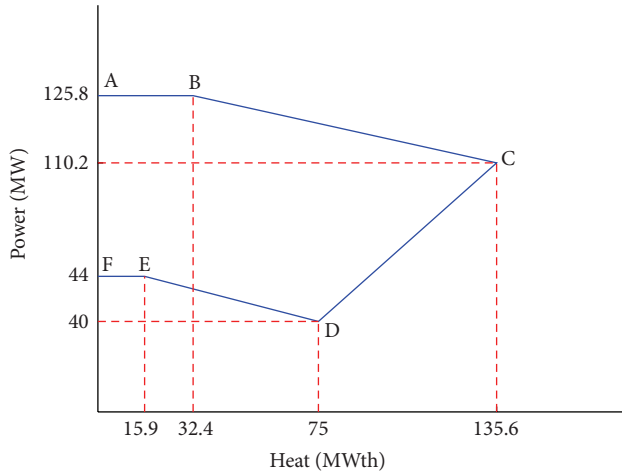


FIGURE 2: Heat-power feasible operating region for CHP unit 2 of the eleven-unit system.

## 6. Conclusion

This paper presents a hybrid method combining DE and SQP for solving the CHPDED problem with valve-point effects. In this paper, DE is first applied to find the best solution. This best solution is given to SQP as an initial condition to fine-tune the optimal solution at the final. The feasibility and efficiency of the DE-SQP were illustrated by conducting case study with eleven-unit test system.

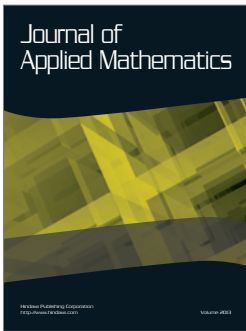
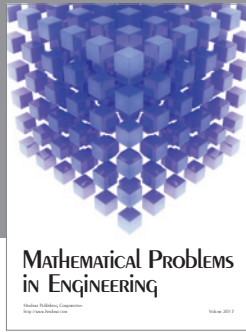
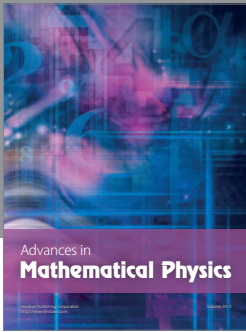
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