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Optimal energy-water management in urban residential buildings through grey water recycling

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ABSTRACT

Energy and water are inseparable resources whose management in urban residential buildings is important. Continuing urbanization in developing nations is increasing the demand yet the supply is inadequate or non-existent. Decentralized urban water recycling systems are an alternative source of water that could relieve the demand from public utilities. However, there are social, economic, environmental and technological factors that affect the uptake of these systems. Although advanced water treatment technologies for decentralized systems have been developed, there are challenges in their optimal operation. This paper introduces the open loop optimal control and closed-loop model predictive control (MPC) strategies aimed at ensuring safe and reliable operation of a grey water recycling system at building level. Both controllers have shown their ability in efficiently operate the system leading to water conservation and energy cost savings. Each of these controllers has its strengths in terms of cost, ease of implementation and robustness and they should be adopted according to specific application. Their adoption can greatly improve energy and water security in urban households, reduce their demand and wastewater. Technology alone cannot solve resource insecurity, and therefore, appropriate policies, regulations, incentives and public awareness should be implemented to support such novel technologies.

1. Introduction

Energy and water are intricately entwined resources (energy-water nexus) that are vital for human survival and economic progress of any nation (Fang & Chen, 2017). Increasing global population is putting the two resources under colossal pressure. By 2050, insecurity of these resources will mostly be felt in urban areas resulting from urban population increasing by up to 70% of the total population. On one hand, urbanization increases the demand for these resources in urban areas, while on the other hand, urban centres are hotspots for innovation on their sustainable consumption (Tsolakis & Anthopoulos, 2015). In developing nations, growing cities face a momentous task of providing energy and water as utilities have limited or no capacity to adequately respond to the growing demand. These challenges are further aggravated by demographic shifts, changing lifestyles, thriving middle class and the growing impact of climate change on demand and supply chains of the two resources (Yumkella & Yillia, 2015). For instance, World Bank estimates the urban population in most African cities will double by 2030 (Jacobsen, Webster, & Vairavamoorthy, 2012). Whereas developed nations have embarked on researching on energy-water nexus in urban areas (Kontokosta & Jain, 2015; Shatat, Worall, & Riffat, 2013; Stillwell, King, Webber, Duncan, & Hardberger, 2010), developing nations have been concentrating on either energy demand management (Nwulu & Xia, 2015; Setlhaolo & Xia, 2015, 2016; Sichilalu & Xia, 2015S, 2015i, 2015c, 2015h, 2015i, 2015l, 2015a, 2015l, 2015u and Xia, 2015; Zhu, Tazvinga, & Xia, 2015), or water demand management (Cai, Yue, Xu, Yang, & Rong, 2016; Lévite, Sally, & Cour, 2003; Mutambara, Darkoh, & Atlhopheng, 2016; Savenije & Van Der Zaag, 2002), until recently when energy water nexus in buildings started gaining research interest (Wanjiru, Sichilalu, & Xia, 2016). Coordinated management of the two resources has the benefit of minimizing unaccounted indirect impact of one resource on the other (Engström et al., 2017).

Cities and urban areas are large consumers of energy and water in many countries (Ren et al., 2016). For instance, they accounted for 95% of water consumption growth in United States between 1985 and 2005 (Jeong, Gulbinas, Jain, & Taylor, 2014). Therefore, conservation of water and associated energy in buildings is a huge opportunity in realizing savings of these resources, while at the same time improving their security (Kim & Haberl, 2014), with the goal of achieving green buildings (Abdellatif & Al-Shamma'a, 2015). Conventional management of water resources concentrates on large (Lee, Tansel, & Balbin, 2011),

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Nomenclature

A_t^1, A_t^2, A_t^3	cross-sectional area of potable, grey and holding water tank (m^2) respectively
D_{grey}, D_{pot}	potable and grey water demand (m^3) respectively
h_1, h_2, h_3	height of water in potable, grey and holding tank (m) respectively
p_e	price of electricity using TOU tariff (currency/kWh)
P_1^m, P_3^m	potable and grey water pump's motor rating (kW) respectively
s_1, s_3	auxiliary variable for potable and grey water pump respectively
S_{grey}	grey water supply (m^3)
TOU	time-of-use tariff
t_s and j	sampling period (h) and jth sampling interval
u_1, u_2	Potable water pump's switch and valve respectively
u_3, u_4	grey water pump's switch and drainage valve respectively
Q_1, Q_3	flow rate of water across potable and grey water pump (m^3/h) respectively
Q_2, Q_4	flow rate of water across potable and drainage valve (m^3/h) respectively
V_1, V_2, V_3	volume of water in respective tanks (m^3)

expensive centralized water supply and sewage disposal systems that have negative environmental impact and are incompatible with modern requirements, especially in growing cities (Pahl-Wostl et al., 2008). Therefore, a paradigm shift seeking to minimize amount of pollution generated and discharged, using and reusing water very near to the point of origin as well as treating water to the required quality is required (Díaz, Stanek, Frantzeskaki, & Yeh, 2016). This shift is leading to adoption of decentralized solutions such as water recycling and rain water harvesting (Rozos & Makropoulos, 2012), further accelerated by advanced water treatment technologies (Wilcox, Nasiri, Bell, & Rahaman, 2016), change of attitude of end-users and increased awareness on the need to conserve fresh water (Stec & Kordana, 2015). In general, decentralized systems present numerous benefits, depending on the geographical area, including; cost reduction, resource efficiency, improved resource security, reduction of system failure, economic empowerment of the local community and environmental benefits. The guidelines for safe use of wastewater, excreta and greywater provided by the World Health Organization (WHO) accentuate on the prominence of grey water as an alternative water resource (World Health Organization, 2006). This is because grey water constitutes a significant volume of the waste flow from households, has nutrients that can be beneficial for irrigation, has low pathogen content and can therefore be used to reduce the demand for potable water. Water can be recycled for direct potable use as is the case of Windhoek, Namibia, where low precipitation and high evaporation necessitated augmentation of water supply with reclaimed water (Du Pisani, 2006). Indirect potable reuse can either be planned or unplanned. Planned indirect potable reuse utilizes an environmental buffer to provide further treatment and retention time such as in California and Florida. Unplanned potable reuse takes place through discharging treated waste water into the environment which is subsequently abstracted for potable use (Wilcox et al., 2016). Non-potable reuse is the most commonly applied decentralized water recycling system in urban areas. Such networks are well established in Tokyo and Fukuoka in Japan (Asano, Maeda, & Takaki, 1996), as well as Queensland in Australia (Mankad, 2012). A study conducted on university students in Pretoria

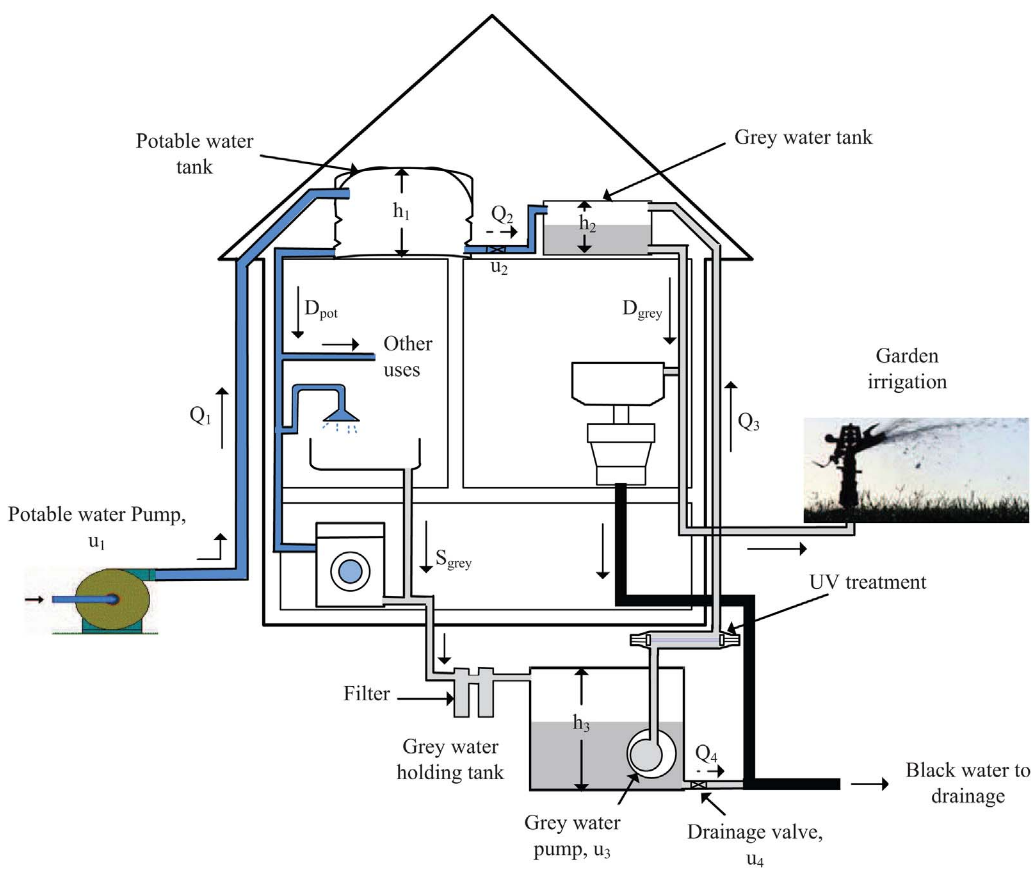

Fig. 1. Schematic of water pumping and grey water recycling system.

revealed their alacrity to adopt recycled water systems for non-potable uses, especially if it would also lead to less bills (Stoakley, 2013).

Although various studies have shown that grey water recycling in residential buildings is possible (Santasmasas, Rovira, Clarens, & Valderrama, 2013; Thirugnanasambandham, Sivakumar, & Prakash Maran, 2015), and of utmost importance (Godfrey, Labhasetwar, & Wate, 2009; Hasan, Shafiquzzaman, Nakajima, Ahmed, & Azam, 2015), there has been little research considering energy and water management involving grey water. In fact, a study conducted in a small-scale water recycling plant in England showed that the operational cost was 20 times higher than for large scale recycling plants. The high cost was attributed to operational inefficiencies whereby staff and energy accounted for 51% and 27% respectively (Wilcox et al., 2016). It is therefore important improve the operational efficiency of these plants by developing optimal technologies of operating the system that ensure energy efficiency and minimize labour cost through autonomous operation. In addition, the initial cost of implementation of grey water recycling system is high in most places (Friedler & Hadari, 2006), so government intervention is needed such as offering incentives to encourage the uptake of these solutions (Bdour, Hamdi, & Tarawneh, 2009; Imteaz & Shanableh, 2012).

This paper introduces the first attempt to design novel, cost effective and advanced optimal controllers to operate the grey water recycling system in residential areas. The open loop and closed-loop model predictive controller (MPC) are designed to ensure that water is conserved while the energy associated with the system is used efficiently. The two control strategies are designed to meet the hourly water demand for a house. Although the open loop control is more cost effective and easy to implement, it is suitable where the water demand is known to be relative stable. However, in cases where the water demand fluctuates such that it is difficult to accurately predict and the system is susceptible to external disturbances that significantly affect the grey water system, the closed-loop MPC should be espoused. It however requires installation of additional monitoring devices to the system such as level monitoring of the tanks thereby increasing the cost and complexity of the control system. The two optimal controllers, if widely adopted, would reduce the demand for potable water, energy and waste flow from utilities and municipalities. They would further lower the cost of water and sewage purification leading to lower bills incurred by the end-user associated to both resources and wastewater disposal.

2. Model development

2.1. Schematic layout

Fig. 1 shows the schematic layout of the grey water recycling system for a typical house. Two scenarios are considered in this paper. First, a building has reliable water supply from water utility meaning pumping and storing is not required. Hence, potable water pump and tank are not required in the grey water recycling system for this scenario. Water flows directly from the municipal supply pipe to potable end uses, and through potable valve, u_2, when required. Secondly, water pumping and storage is required in a building resulting from low water pressure or water rationing taking place in the area. Potable water pump, which is a fixed speed pump whose state is represented as u_1 is required to pump the water to the roof storage tank. It then flows from this tank to various end-uses by gravity. In both scenarios, grey water from some end-uses, such as, shower and washing machine, can be treated and reused for end-uses that do not require potable water such as garden irrigation and toilet flushing. Therefore, grey water is collected, filtered to remove physical impurities and stored in a holding tank, which can either be placed underground or at the back of the building. This holding tank must be emptied using the drainage solenoid valve, whose state is represented as u_4 within 24-h, to prevent formation of bacteria that produce foul smell. Grey water has to be treated, and in this model, ultra violet (UV) treatment is chosen for its adaptability, low space requirement and low power consumption. Therefore, grey water is pumped from the holding tank using the grey water pump, a fixed speed pump whose state is represented as u_3. It passes through the UV treatment chamber to the grey water tank at the roof of the building, next to the potable water tank. The treated grey water also flows by gravity to the suitable end uses. There are instances, however, when there won't be sufficient grey water to meet the grey water demand. The potable water valve u_2 allows potable water from the potable water tank to flow to the grey water tank to assist in meeting the demand. Black water, that is, water that cannot easily be recycled (e.g. water from the toilet) is allowed to flow to the drainage directly. The aim, therefore, is to control the potable and grey water pumps, drainage and potable water valves to ensure that water supply in the house is reliable while enhancing energy and water efficiency.

2.2. Dynamics of water flow

When potable water supply is reliable, only grey and holding tanks would be required. However, if the supply is unreliable, water is stored in three tanks to meet the overall demand in the house. It is assumed that all the tanks used in this paper have uniform cross-sectional area. The dynamics of water flow in each tank is mathematically modelled below.

2.2.1. Potable water tank

When potable water supply is unreliable, it has to be pumped by potable water pump into the potable water tank, to meet the overall potable water demand and supplement grey water whenever needed to. The dynamics of volume of water in this tank, $\dot{V}_1$ (m^3/h), is,

$$\dot{V}_1 = A_1^t \dot{h}_1 = Q_1 u_1 - Q_2 u_2 - \dot{D}_{\text{pot}} \tag{1}$$

where A_1^t is the cross-sectional area of the tank (m^2) while $\dot{h}_1$ is the rate of change of the height of water in the tank (m/h). $\dot{D}_{\text{pot}}$ is the potable water demand (m^3/h) in the house while Q_1 and Q_2 are the flow rates (m^3/h) of the potable water pump and potable solenoid valve respectively. The dynamic equation (1) can be expressed in discrete-time domain by a first order difference equation as follows;

$$h_1(j+1) = h_1(j) + \frac{1}{A_1^t}[t_s Q_1 u_1(j) - t_s Q_2 u_2(j) - D_{\text{pot}}(j)] \tag{2}$$

where j the sampling interval and t_s is the sampling period during a full operating cycle of 24-h. The equation is modelled in terms of the height of water in the tank in a sampling interval, $h_1(j)$, because level sensors are the most economical and easy to use in measuring the volume of water in a tank. Since the tanks have been assumed to have a uniform cross-section, then the height reading will be converted to the volume by the controller. Through recurrence manipulation, Eq. (2) can further be modelled as,

$$h_1(j) = h_1(0) + \frac{t_s}{A_1^t}\sum_{i=1}^{j}[Q_1 u_1(i) - Q_2 u_2(i)] - \frac{1}{A_1^t}\sum_{i=1}^{j} D_{\text{pot}}(i) \quad 1 \le j \le N. \tag{3}$$

where N is the total number of cycles during the full operating cycle given as $N = \frac{24}{t_s}$.

2.2.2. Grey water tank

This tank receives the treated grey water from the grey water pump and stores it for use by non-potable water end-uses. In case there is no grey water available, potable water is allowed to flow from the potable water tank to supplement the grey water. Therefore, the volumetric rate of change, $\dot{V}_2$ (m^3/h), of the water in the tank is,

$$\dot{V}_2 = A_2^t \dot{h}_2 = Q_2 u_2 + Q_3 u_3 - \dot{D}_{\text{grey}} \tag{4}$$

where A_2^t is the cross-sectional area (m^2) of the tank, $\dot{h}_2$ is the rate of change of the height of water (m/h) in the tank, $\dot{D}_{\text{grey}}$ is the grey water demand (m^3/h) while Q_3 is the water flow rate (m^3/h) through the grey water pump. The equation can be expressed in the discrete-time domain as

$$h_2(j+1) = h_2(j) + \frac{1}{A_2^t}[t_s Q_2 u_2(j) + t_s Q_3 u_3(j) - D_{\text{grey}}(j)], \tag{5}$$

which can further be expressed as

$$h_2(j) = h_2(0) + \frac{t_s}{A_2^t}\sum_{i=1}^{j}[Q_2 u_2(i) + Q_3 u_3(i)] - \frac{1}{A_2^t}\sum_{i=1}^{j} D_{\text{grey}}(i) \quad 1 \le j \le N. \tag{6}$$

2.2.3. Holding tank

The holding tank temporarily stores filtered grey water that is collected from the recyclable potable water end-uses. The tank acts as a temporary reservoir for pumping grey water after treatment to the grey water tank. To avoid foul smell from developing, the tank must be emptied at least every 24 h. Therefore, the dynamics of the volume of water in this tank, $\dot{V}_3$ (m^3/h), is,

$$\dot{V}_3 = A_3^t \dot{h}_3 = \dot{S}_{\text{grey}} - Q_3 u_3 - Q_4 u_4, \tag{7}$$

where A_3^t is the cross-sectional area (m^2) of the tank, $\dot{h}_3$ is the rate of change of the height (m/h) of water in the tank, $\dot{S}_{\text{grey}}$ is the amount (m^3/h) of water supplied from the recyclable potable water end-uses in an hour and Q_4 is the flow rate (m^3/h) of the grey water through the drainage valve. Eq. (7) can be written in discrete-time domain as,

$$h_3(j+1) = h_3(j) + \frac{1}{A_3^t}[S_{\text{grey}}(j) - t_s Q_3 u_3(j) - t_s Q_4 u_4(j)], \tag{8}$$

which transforms to,

$$h_3(j) = h_3(0) + \frac{1}{A_3^t}\sum_{i=1}^{j} S_{\text{grey}}(i) - \frac{t_s}{A_3^t}\sum_{i=1}^{j}[Q_3 u_3(i) + Q_4 u_4(i)] \quad 1 \le j \le N. \tag{9}$$

The dynamic equations are used in designing two model based controllers using advanced optimal control concept to ensure that water demand in the house is reliably met through efficient and optimal operation of the grey water system. The performance indicators of the control systems are;

- Minimize the cost of pumping energy of both potable and grey water.
- Minimize the maintenance cost of the pumps.
- Minimize consumption of potable water in the house.

The design of the open loop controller and closed-loop MPC are designed and their performance compared in the following sections.

2.3. Open loop controller model

The open loop controller uses a feed forward principle whereby, measurements of the water demand are used by the controller to predict the future behaviour of the system in meeting the demand throughout the operating cycle. Therefore the previously mentioned performance indicators are modelled in the following objective function,

$$J = \sum_{j=1}^{N}[\alpha_1 t_s p_e(j) P_1^m u_1(j) + \alpha_2 t_s Q_2 p_w(j) u_2(j) + \alpha_3 t_s p_e(j) P_3^m u_3(j)] + \alpha_4 \sum_{j=1}^{N}[s_1(j) + s_3(j)] \tag{10}$$

where P_1^m (kW) and P_3^m (kW) are the potable and grey water power pump's rating respectively, while p_w, p_e and t_s are the cost of water, electricity during the jth sampling interval and the sampling time respectively. $s_1(j)$ and $s_3(j)$ are auxiliary variables used to minimize the maintenance cost for potable and grey water pumps respectively. Each auxiliary variable is represented by a value 1 whenever a pump's state changes from off to on Mathaba, Xia, and Zhang (2014), Wanjiru and Xia (2015). Weights α_1 to α_4 are used to tune the controller according to user's preference. The first and third terms in Eq. (10) minimize the cost of energy consumed by the pumps, the second term minimizes the cost of potable water consumed by grey water end-uses and the fourth term is responsible to minimize the maintenance cost of the pumps.

The objective function is subject to physical and operational constraints. The constraints are represented mathematically as;

$$h_1^{\min} \le h_1(j) \le h_1^{\max}, \tag{11}$$

$$h_2^{\min} \le h_2(j) \le h_2^{\max}, \tag{12}$$

$$h_3^{\min} \le h_3(j) \le h_3^{\max}, \tag{13}$$

$$h_3(N) = h_3^f, \tag{14}$$

$$u_1(1) - s_1(1) \le 0,$$

$$u_1(j) - u_1(j-1) - s_1(j) \le 0, \tag{16}$$

$$u_3(1) - s_3(1) \le 0, \tag{17}$$

$$u_3(j) - u_3(j-1) - s_3(j) \le 0, \tag{18}$$

$$u_m(j) \in \{0, 1\} \text{ where } m = 1, 2, 3, 4, \tag{19}$$

$$s_1(j), s_3(j) \in \{0, 1\}. \tag{20}$$

Inequalities (11), (12) and (13) limit the state variables, that is, height of water in respective tanks between minimum and maximum allowable levels. Potable and grey water tanks are set never to run completely empty during the full operating cycle. However, the holding tank must be emptied within the 24-h operating cycle, in order to avoid formation of bacteria producing foul smell. This is given by Eq. (14) where h_3^f is the final water level in the tank. Inequalities (15) and (17) initialize the auxiliary variables as the initial state of the respective u while inequalities (16) and (18) favour the control that involve less switching frequency of the respective pumps. Finally Eqs. (19) and (20) are bounds for the control variables, that is, the status of the pumps and switches and the auxiliary variables respectively. The algorithm for open loop optimal controller is shown in A.

2.4. Closed-loop control model

The closed-loop model predictive control (MPC) strategy is formulated in this paper due to its predictive nature, ability to cope with constraints in the design process and the ability to deal with disturbances that are always there in any system, whether external or errors within the system (Wanjiru, Zhang, & Xia, 2016). The closed-loop MPC uses both the feed forward and feed back measurements from the system to compute the control law on-line (Mayne, Rawlings, Rao, & Scokaert, 2000).

The control and state variables in this strategy are the same as those for the open loop control model. Denoting the control variables as u_m, with $m = 1, 2, 3, 4$, s_1 and s_3, the objective function encompassing the previously listed performance index for the closed-loop model, J_{mpc}, is derived from the open loop objective (10) as,

$$J_{\text{mpc}} = \sum_{j=k}^{k+N_c-1}[\alpha_1 t_s p_e(j) P_1^m u_1(j|k) + \alpha_2 t_s Q_2 u_2(j|k) + \alpha_3 t_s p_e(j) P_3^m u_3(j|k)] + \alpha_4 \sum_{j=k}^{k+N_c-1}[s_1(j) + s_3(j)], \tag{21}$$

where N_c is the control horizon, $u_1(j|k)$, $u_2(j|k)$, $u_3(j|k)$, $s_1(j|k)$ and $s_3(j|k)$ are the predicted values at the jth sampling interval based in information available at time k. Normally, MPC problems include both predicting, N_p, and control, N_c, horizons. However, since none of the state variables (height of water in the tank) is included in the objective function, this MPC problem does not include the predicting horizon, N_p. The control horizon is therefore given as

$$N_c = N - k + 1. \tag{22}$$

The state equations are modified from Eqs. (3), (6) and (9) to,

$$h_1(j|k) = h_1(k) + \frac{t_s}{A_1^t}\sum_{i=k}^{j}[Q_1u_1(i|k) - Q_2u_2(i|k)] - \frac{1}{A_1^t}\sum_{i=k}^{j}D_{\text{pot}}(i), \tag{23}$$

$$h_2(j|k) = h_2(k) + \frac{t_s}{A_2^t}\sum_{i=k}^{j}[Q_2u_2(i|k) + Q_3u_3(i|k)] - \frac{1}{A_2^t}\sum_{i=k}^{j}D_{\text{grey}}(i), \tag{24}$$

$$h_3(j|k) = h_3(k) + \frac{1}{A_3^t}\sum_{i=k}^{j}S_{\text{grey}}(i) - \frac{t_s}{A_3^t}\sum_{i=k}^{j}[Q_3u_3(i|k) + Q_4u_4(i|k)],$$
$$k \le j \le k + N_c - 1. \tag{25}$$

Additionally, physical and operational constraints are similar to those discussed and mathematically modelled in constraints and Eqs. (11)–(20), with the following modifications,

$$h_1^{\min} \le h_1(j|k) \le h_1^{\max}, \tag{26}$$

$$h_2^{\min} \le h_2(j|k) \le h_2^{\max}, \tag{27}$$

$$h_3^{\min} \le h_3(j|k) \le h_3^{\max},$$

$$h_3(N) = h_3^f, \tag{29}$$

$$u_1(1|k) - s_1(1|k) \le 0, \tag{30}$$

$$u_1(j|k) - u_1(j-1|k) - s_1(j|k) \le 0, \tag{31}$$

$$u_3(1|k) - s_3(1|k) \le 0, \tag{32}$$

$$u_3(j|k) - u_3(j-1|k) - s_3(j|k) \le 0, \tag{33}$$

$$u_{m,c}(j|k) \in \{0,1\} \text{ where } m = 1, 2, 3, 4, \tag{34}$$

$$s_1(j|k), s_3(j|k) \in \{0,1\}. \tag{35}$$

At a particular time, k, the controller solves an open loop optimization problem for N_c horizon. Only the first element of each of the control variables $u_m(j|k)$, $s_1(k|k)$ and $s_3(j|k)$ obtained is implemented to the plant. The states (heights of water in respective tanks, $h_m(j|k)$ is measured and fed back to the controller to be used as the initial heights during the next time step, $k + 1$. Other input variables are also updated and the optimization continues up to a predetermined operating cycle.

The work flow of the MPC controller, whose algorithm is detailed in Appendix B, is as follows (Zhu et al., 2015);

(1) For time, k, find the control horizon ($N_c(k)$) using Eq. (22).
(2) *Optimization:* Find the optimal solution within the control horizon; minimize objective function (21),
(3) subject to constraints (26)–(35).
(4) From the optimal solution, implement $[u_1(1|k), u_2(1|k), u_3(1|k), u_4(1|k)]^T$ to the plant.
(5) *Feed back:* Measure the states $h_1(j|k)$, $h_2(j|k)$ and $h_3(j|k)$.
(6) Set $k = k + 1$ and update system states and inputs and outputs.
(7) Repeat steps 1–5 until k reaches a predefined value.

2.5. Effect of monthly water block tariff

The control models are run over a 24-h operating cycle as the demand pattern is assumed to be repeated over this cycle, though different for week days and days of the weekend. However, since water is priced monthly using the block tariff in Table 2, it is important to investigate the effect increasing price of water has on the optimal operation of the system. Both open loop and MPC controllers are run for a month, by considering each week to have 5 weekdays and 2 days of the weekend. In this study, the weekday water demand profile, $D_{pot}(weekday)$, is assumed to be the same for all the 5 week days, while the weekend demand profile, $D_{pot}(weekend)$, is also same for the 2 days of the weekend. Further, the first day of the month is taken as a Monday, and the month has exactly four weeks (28 days). Therefore, the cumulative volume of water consumed up to a certain weekday, $D_{pot,wkdy}$, or a weekend, $D_{pot,wknd}$, is obtained as;

$$\begin{aligned} D_{\text{pot,wkdy}} &= (5q)D_{\text{pot}}(\text{weekday}) + (2q-2)D_{\text{pot}}(\text{weekend}) \\ D_{\text{pot,wknd}} &= (5q)D_{\text{pot}}(\text{weekday}) + (2q-1)D_{\text{pot}}(\text{weekend}) \end{aligned} \tag{36}$$

where q is the number of the week in the month ($q = 1, 2, 3, 4$). This volume is then used to compute the cost of water.

3. General data

3.1. Case study

A house in Pretoria, which is forced to pump and store the water due to the unreliability of municipal water supply was studied. The water consumption and energy associated with pumping water in this house is used as the baseline, as the house uses only potable water for all its end-uses. The pump, rated as 0.8 kW and 0.75 m^3/h, is controlled by level switches that just detect empty and full levels. Whenever the tank is empty, the pump switches on until the tank is full, regardless of TOU period. Water then flows from the potable water tank to the end-uses through gravity. Various end-uses were categorized to enable identify those that could be used for water recycling and those that could use treated grey water. The hourly water demand pattern of these uses was measured using digital flow meters connected with data loggers. Therefore, the hourly water demand for a typical week day and a weekend is shown in Fig. 2. The weekday water demand has a high peak early in the morning and in the evening caused by the house occupants preparing to leave the house and coming back in the evening, respectively. However, during the weekend, the peak demand occurs later than during weekdays, and remains relatively high during daytime as the occupants carry on with the weekend chores throughout the daytime. It can be seen from the curves that the grey water supply, S_{grey}, is always less than the potable water demand, D_{pot}, as some of this potable water qualifies to be recycled. On the contrary, the grey water demand, D_{grey}, does not necessarily follow the others, as this demand entirely depends on the human behaviour.

The cylindrical potable water tank has the dimensions given in Table 1. In order to incorporate grey water recycling, two tanks; grey and holding water tanks would be required and their dimensions and capacity constraints are given in Table 1. In this paper, the sensors in all tanks would be put to limit the water level between minimum and maximum water levels shown in Table 1. This is meant to avoid either running the tank completely empty or spilling the water in the tank hence damaging the roof of the house. The only tank that is allowed to

Table 1
Dimensions and capacity of the tanks.

Tank	Radius (m)	Height (m)	Min	Max
Potable	0.55	1.2	0.1	1.0
Grey	0.36	1.0	0.1	0.8
Holding	0.30	0.6	0	0.5

Table 2
City of Tshwane water tariff for 2014/2015.

Volume (m^3/ month)	0–6	7–12	13–18	19–24	25–30	31–42	43–72	> 72
Rates (R/m^3)	6.81	9.72	12.77	14.77	16.89	18.25	19.53	20.91

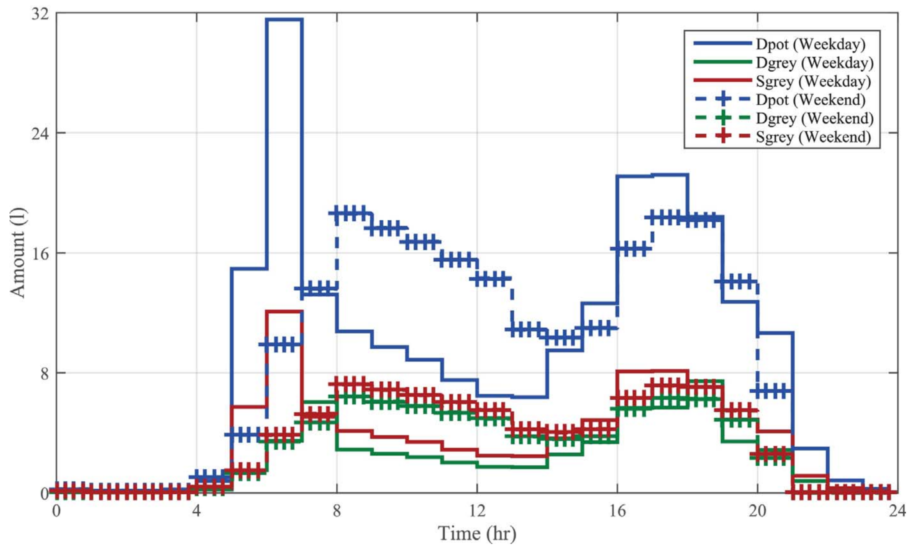

Fig. 2. Hourly water profile for a typical week day and weekend.

run completely empty is the grey water holding tank, which is meant to avoid formation foul smell caused by bacteria. The grey water pump to be incorporated would be rated at 650 W with flow rate of 0.35 m^3/h.

3.2. Time-of-use electricity tariff

The time-of-use (TOU) tariff is commonly used globally to encourage shifting of peak load (Zhang, Xia, & Zhang, 2014) and it can vary by time of day, day of week and season (Zhuan & Xia, 2013). Eskom's TOU Homeflex structure for residential consumers given below is used (Setlhaolo & Xia, 2015).

$$p_e(t) = \begin{cases} p_{\text{off}} = 0.5510\,\text{R/Kwh} & \text{if } t \in [0, 6] \cup [10, 18] \cup [20, 24] \\ p_{\text{peak}} = 1.748\,\text{R/Kwh} & \text{if } t \in [7, 10] \cup [18, 20] \end{cases} \quad (37)$$

where p_{off} is the off peak price, p_{peak} is the peak time price, R is the South African currency, Rand, and t is the time of day in hours.

3.3. Water tariffs

Table 2 shows the water tariffs for domestic consumers in the city of Tshwane (Wanjiru & Xia, 2015).

3.4. Uncertainty analysis

Uncertainty or error analysis is required to determine the confidence level of measurements carried out. In this case, the analysis of measured water demand data is carried out using the approach taken by Sichilalu and Xia, 2015S, 2015i, 2015c, 2015h, 2015i, 2015l, 2015a, 2015l, 2015u and Xia (2015). Random and instrument's error are assumed to affect the measurements. Random errors are generated in MATLAB software with a distribution mean and standard deviation of 0 and 1 respectively while the instrument's absolute uncertainty of ± 0.01 is provided by the manufacturer. The measured value, S_{meas}, is therefore given as,

$$S_{\text{meas}} = S_{\text{actual}} + (\text{Err}_{\text{random}} \times \text{Err}_{\text{inst}}) \quad (38)$$

where Err_{random} and Err_{inst} are the random and instrument errors respectively, while S_{actual} is the true value. The relative error, $Err_{relative}$, is then obtained as,

$$\text{Err}_{\text{relative}} = \frac{\text{Err}_{\text{eff}}}{S_{\text{meas}}}\% \quad (39)$$

From rule of the weakest link, the measurement with the largest relative error is used to determine the final absolute error of the performance index (Wamalwa, Sichilalu, & Xia, 2017), which is the cost of energy and water in this case.

4. Simulation results and discussion

The simulation results for the two control strategies are discussed below. Simulations for both open loop and closed-loop MPC models are done over a 24-h operating cycle, with the sampling period, t_s = 15 min. For both controllers, simulations are done for a weekday, a weekend and finally for one month to investigate the effect of increasing water tariff.

4.1. Open loop optimal control model

The open loop optimal operation of the grey water recycling system in a weekday, which mainly involves switching of pumps and solenoid valves is shown in Fig. 3. The legend showing peak and off peak periods of the TOU tariff is used throughout the paper. Moreover, only pumps, whose status are represented by u_1 and u_3, are considered to consume power hence subjected to the TOU tariff as solenoid valves u_2 and u_4 consume negligible amount of energy. It can be seen that the optimal controller seeks to operate both pumps during the off-peak, effectively shifting the load to the desired period, while meeting the household potable and grey water demand. However, the grey water pump is operated during the morning TOU peak for 15 minutes, due to increased grey water demand in the same period and there is sufficient grey water collected in the holding tank. The open loop controller switches both pumps twice during the 24-h operating cycle. This is in line with the objective that seeks to also minimize the maintenance cost of the pumps represented as the number of pumps' switching taking place. Frequent switching destroys a pump's motor as it tries to overcome the dead load (water) while changing from off to on status. The valves are however allowed to switch any number of times. The open loop controller allows the use of potable water for grey uses early in the morning by opening the potable water valve, u_2, as it awaits more grey water to be collected so that it can be pumped into the grey water tank.

The optimal operation for a weekend is shown in Fig. 4. Unlike in the weekday optimal operation, the controller manages to operate the pumps during the cheaper off-peak TOU period. Since less grey water has been collected by early morning, the controller is forced to use more potable water, through valve u_2 to ensure that grey water demand is met. Thereafter, the required grey water is pumped into the grey water tank to meet the demand. Both pumps are only switched twice

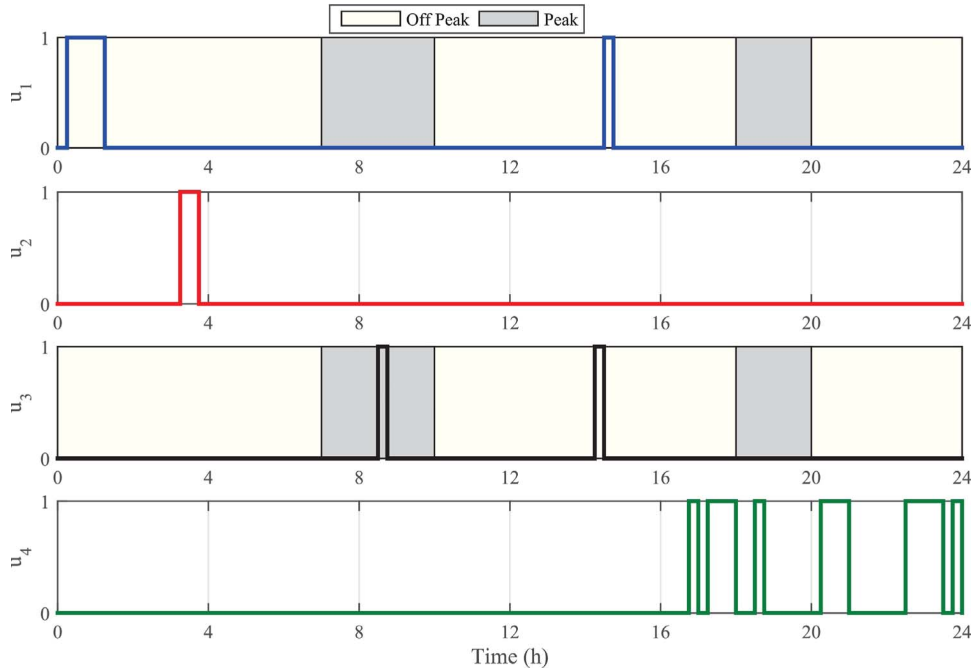

Fig. 3. Open loop optimal switching for a weekday.

throughout the operating cycle as desired.

Optimal operation of the pumps and valves in a weekday and weekend leads to water variation in the respective tanks as shown in Fig. 5. In all tanks, none of the constraints is violated throughout the 24-h operating cycle. During the weekday, the controller predicts that the water available in the potable water tank is not sufficient to meet the potable water demand therefore switching on the pump for 1-h in the beginning of the operating cycle. This leads to a rise in water level, h_1, in the potable water. Thereafter the level of this water drops to meet the potable water demand in the house until 14:30 h when the pump switches on again for 15 min. This amount of water is sufficient to meet the potable water demand for the remaining period of the 24-h operating cycle. In the same day, the height of water in the grey water tank, h_2, is mainly reducing due the grey water demand in the house. The water level rises at 03:15 when the potable water valve, u_2, is opened for 30 min. Thereafter, the level declines as grey water demand rises until 08:30 when the controller realises that the holding tank has enough grey water collected while the grey water tank is at the risk of running dry. Therefore, grey water pump is switched on for 15 min leading to simultaneous rise in water levels h_2 and drop in h_3. This is only experienced again at 14:15 when grey water tank needs more grey water, which is sufficient to meet the remaining duration of the operating cycle. Once grey water tank has sufficient water, the controller then opens drainage valve, u_4 to release grey water being collected, so as to ensure the tank is emptied by the end of the 24-h operating cycle.

In a weekend, the optimal controller ensures that both pumps are operated during the off-peak TOU period. Water level in potable tank,

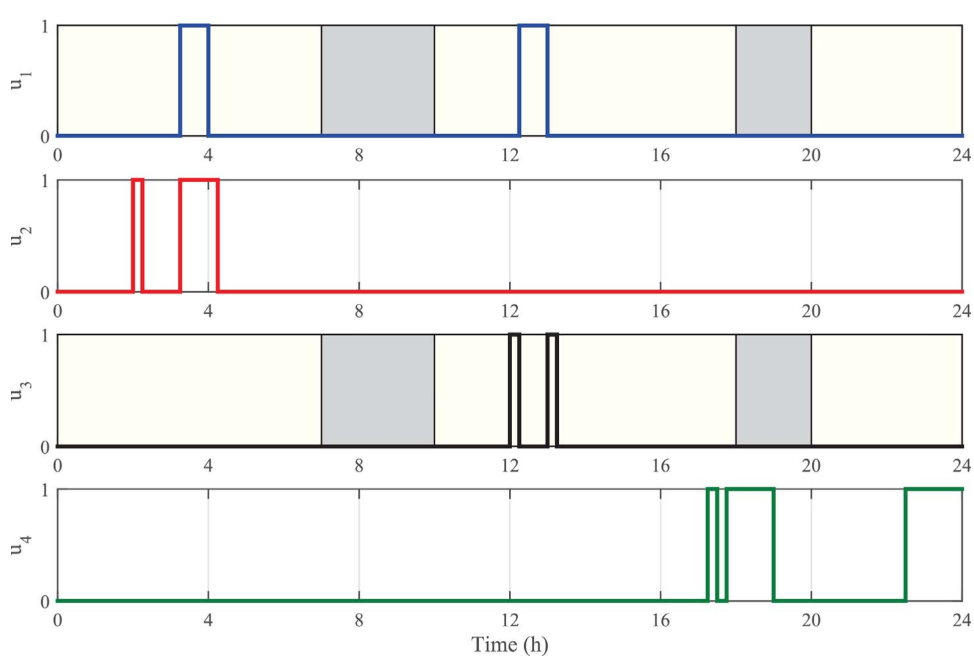

Fig. 4. Open loop optimal switching for a weekend.

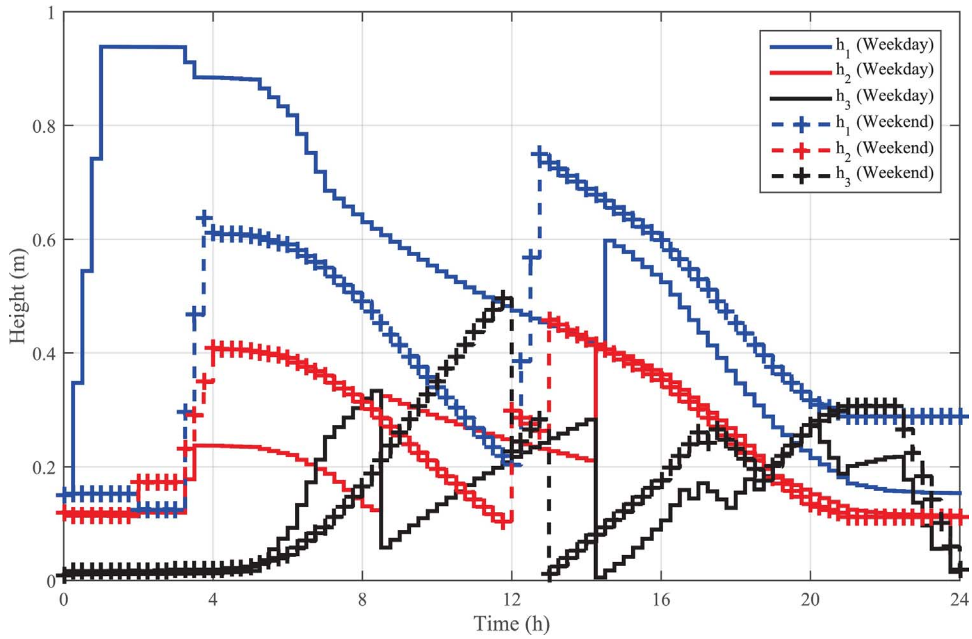

Fig. 5. Water height variation in respective tanks with open loop controller.

h_1 rises at 03:15 when the potable water pump is switched on for 45 min. Subsequently, this water level starts to decline as the water is used to meet potable water demand until 12:15 when the pump is again switched on for another 45 min. This water is sufficient to meet the remaining potable water demand. During the weekend, however, the grey water demand, D_{grey}, increases almost at the same rate as the amount of grey water being collected, S_{grey}. The controller is therefore forced to use potable water through valve, u_2, between 02:00-04:15 to meet the grey water demand. This causes a rise in the water level h_2 in the grey water tank which is used to meet grey water demand until 12:00. The controller then operates grey water pump twice for 15 minutes each at 12:00 and 13:00 causing a simultaneous rise in height, h_2, and fall in h_3 as the water is transferred from the holding tank to the grey water tank to meet the demand for the remaining duration of the operating cycle. At 17:15, the controller opens drainage valve u_4 severally to ensure the grey water being collected is allowed to flow to the drainage as it is no longer required.

4.2. Closed-loop MPC model

The optimal operation of the grey water recycling system obtained while using the closed-loop MPC for a typical weekday is shown in Fig. 6. Similar to the weekday open loop operation, the pumps operate twice each throughout the 24-h operating cycle. Potable water pump operates in the off-peak TOU periods in meeting the potable water demand while grey water pump operates in the morning TOU peak for 15 min due to rising grey water demand and sufficient amount of grey water has been collected in the holding tank. Before operating the grey water pump in the morning peak, the controller uses potable water for use in grey water uses while waiting for sufficient grey water to be

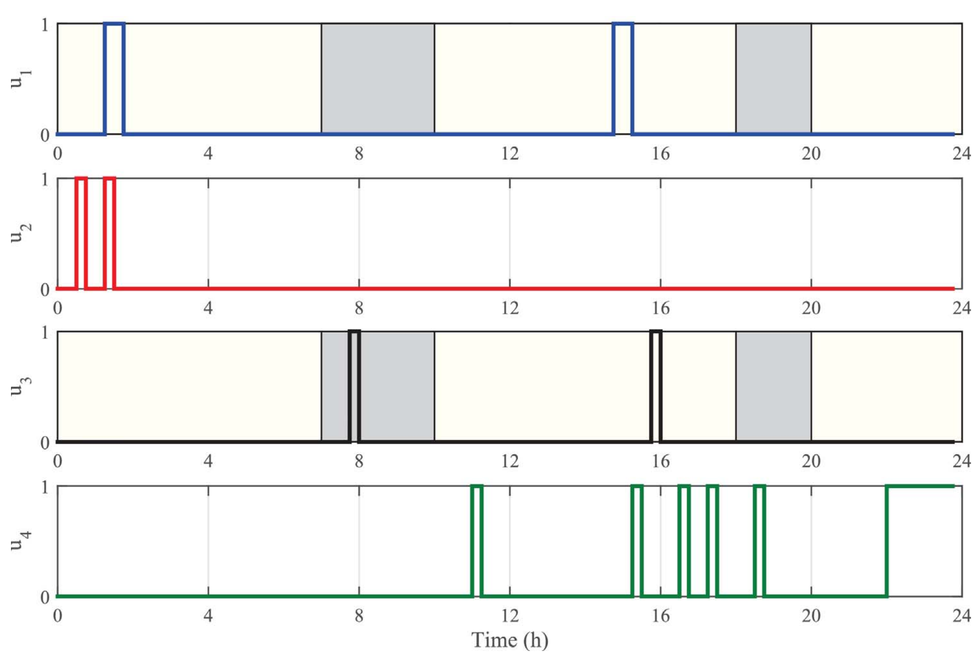

Fig. 6. Week day closed-loop optimal switching.

collected in the holding tank. Further, the controller operates the drainage valve severally in the afternoon in order to ensure that the holding tank is emptied by the end of the operating cycle.

The optimal operation of the system for a typical weekend is shown in Fig. 7. The controller operates both pumps in the off-peak TOU periods as desired. Potable water pump is switched on three times while grey water pump operates twice. Just like the open loop controller during the weekend, the MPC uses potable water for grey water purposes in the morning as it awaits sufficient water to be collected in the holding tank. Further the controller ensures that the holding tank is empty by the end of the control horizon by opening drainage valve, u_4, whenever necessary.

The optimal operation using MPC for both weekday and weekend leads to variation of water levels in various tanks as shown in Fig. 8. The water levels are maintained within the prescribed maximum and minimum levels in all tanks in both days. In the weekday, water level in the potable water tank, h_1, rises at 01:15 when the controller switches on the potable water pump for 30 min. The level, subsequently, decreases while meeting the potable water demand until 14:45 when the pump is again switched on for another 30 min. This water in the tank is sufficient to meet potable water demand for the remaining duration. Since the holding tank is empty, the controller open potable water valve, u_2, at 00:30 and 01:15 for 15 min each to meet the expected grey water demand. Consequently, water level, h_2, rises and this water is used to meet grey water demand until 07:45 when the holding tank has sufficient water. This makes the controller to switch on the grey water pump for 15 min, resulting in simultaneous increase and drop of water levels h_2 and h_3 respectively. This event only happens again at 15:45 when the grey water pump is operated again for another 15 min, and thereafter, the controller predicts that the grey water in the grey water tank is sufficient to meet the remaining grey water demand. Since more grey water is collected, the controller keeps opening the drainage water valve, especially towards the end of the operating cycle in order to ensure the holding tank.

During the weekend, water level in potable water tank, h_1, rises when the MPC controller switches on the potable water pump at 02:30 for 45 min. This water then declines as it is used up in meeting potable water demand until 10:30 when the pump is again switched on for 15 min. This causes an increase in the water level, which is used up until 16:00 when the pump is switched on for 30 min making the water level to rise again. The controller predicts that this water is sufficient for the remaining period of the operating cycle. Moreover, the MPC controller has to use potable water through potable water valve, u_2, in the early morning so as to ensure grey water tank has sufficient water to meet grey water demand, while waiting for grey water to be collected in the holding tank. This leads to the increase in water levels in grey water tank between 01:15–07:30. This water is sufficient to meet grey water demand until 11:30 when grey water pump is operated for 15 min leading to a simultaneous rise and drop in h_2 and h_3 respectively. This occurs again at 16:30 for another 15 min, and thereafter the controller predicts that the water in the grey water tank is sufficient to meet the grey water demand for the remaining period. The MPC controller keeps opening the drainage water valve for short intervals, which causes the water level, h_3, to fall but it eventually opens the valve at 22.15 till the end to ensure the tank is left empty as required.

4.3. Effect of monthly water block tariff

The open loop and MPC optimal schedules of potable water flowing through valve u_2, for a month are shown in Fig. 9. Both open loop and MPC controllers open u_2 for the first two weeks, and the weekday of the third week. In the first and second week, both controllers open u_2 2 and 5 times during the weekday and weekend respectively. However, in the third week, they open u_2 twice during the weekday but none on the weekend and following weeks. u_2 remains off due to a very high weight being given to the cost of water minimizing term in the objective function which was increasing as the cost of water increased. Table 3 shows the consumption of potable water in the house during the month, as well as use of potable water for grey water uses. The baseline and potable water columns show the cumulative amount of water used and the unit price during the month. The weekday cumulative potable water is the amount of potable water used in the house at the end of 5 week days, while the weekend's is the amount consumed at the end of the 2 days of the weekend. Further, $\Sigma Q_2 t_s u_2 (m^3)$ is the amount of potable water used to supplement the grey water uses through valve u_2 in each period. Since more potable water is used in the baseline than while using grey water with control strategies, its unit cost increases faster as weeks go by. By the end of the month, the consumer pays 18.25 R/m^3 as opposed to 16.89 R/m^3 charged while recycling grey water. This means that consumers will have the added benefit of lower cost of potable water in addition to conserving it. Grey water recycling system operated by either open loop or MPC control strategy use 0.05 m^3 and 0.13 m^3 of potable water for grey uses in a week day and weekend, respectively, during the first two weeks. Thereafter, 0.05 m^3 is used

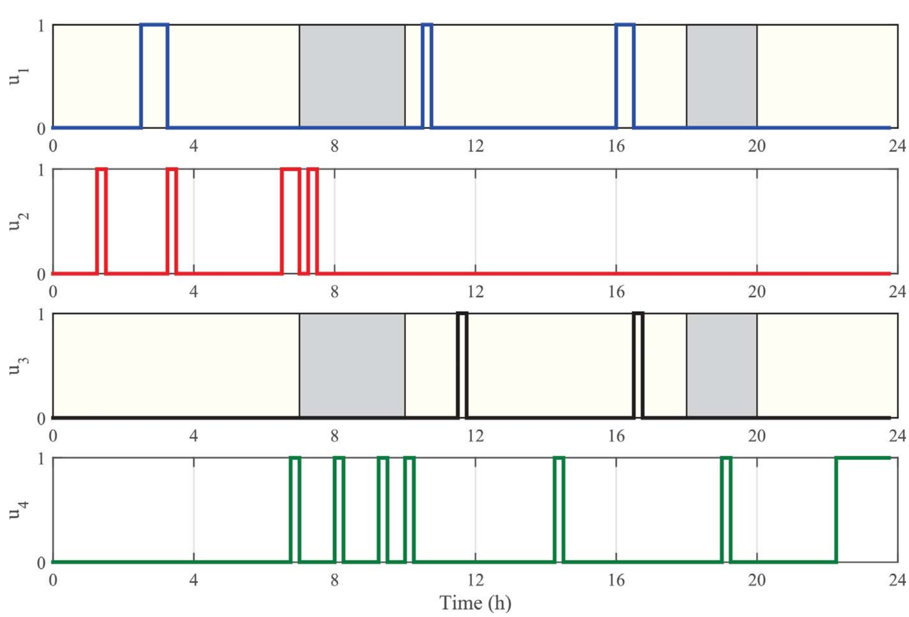

Fig. 7. Closed-loop optimal switching for a weekend.

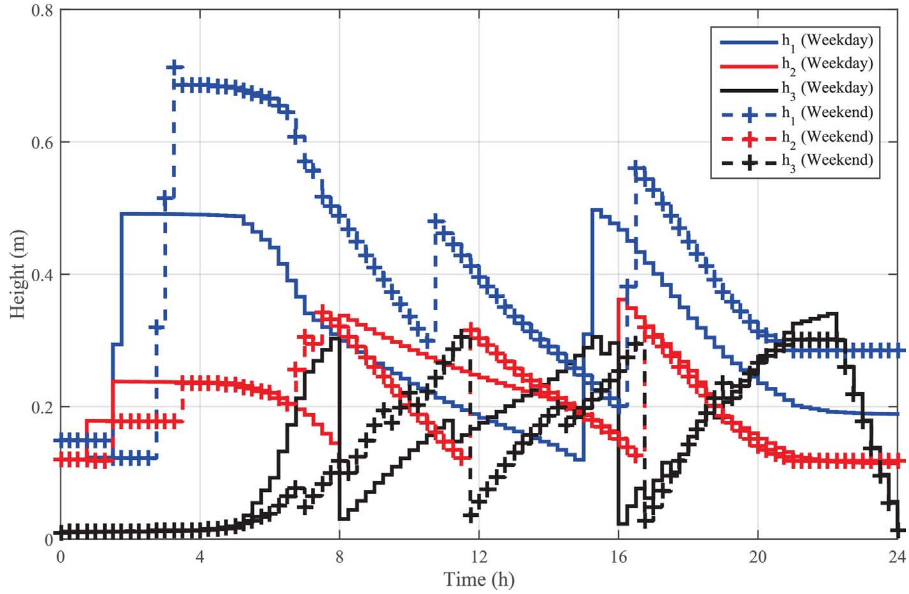

Fig. 8. Water height variation using MPC controller.

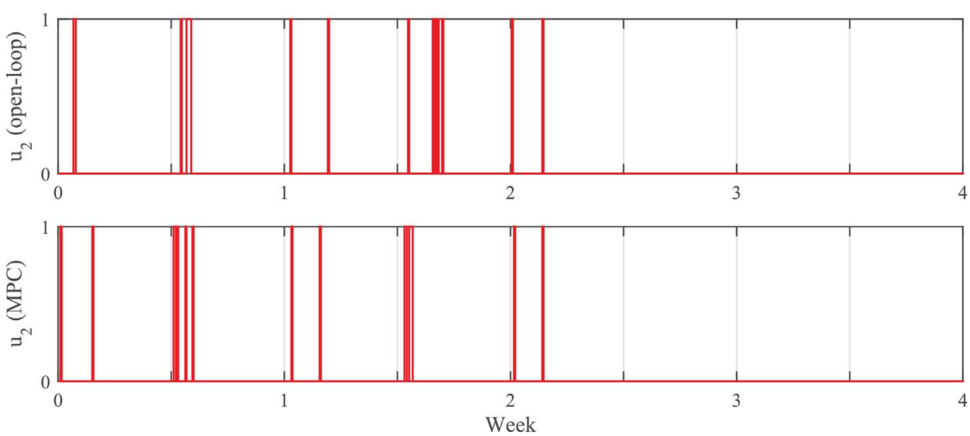

Fig. 9. Optimal use of potable water in grey uses in a month.

during the week day of the third week. Up to this point, the cost of water has risen to 12.77 R/m^3. However, when the price reaches 14.77 R/m^3, during the weekend of the third week, both controllers do not use potable water for grey end-uses. This is caused by weight of the term responsible for minimizing the cost of water in objective functions (10) and (21) increasing significantly, such that both controllers give this term more preference as compared to the other terms. Finally, the increasing weighting factor leads to an increase in the use of grey water from 0.18 m^3 to 0.26 m^3 for both controllers.

4.4. Discussion

Table 4 shows monthly water and associated energy consumption together with their associated costs. Given that the baseline has unreliable potable water which is used for all the household end-uses, about 31.61 m^3/month is required to meet the overall demand, costing 409.48 R/month (inclusive of pumping energy cost). This amount meets both the potable and grey water demands, meaning that recyclable water is effectively wasted. With grey water recycling system in place, which is operated by either open loop or MPC controllers, the

Table 3
Comparison of weekly water consumption.

Wk	Day	Baseline		Potable water		$\Sigma Q_2 t_s u_2 (m^3)$		Grey water (m^3)	
		Amount (m^3)	Price (R/m^3)	Amount (m^3)	Price (R/m^3)	Open loop	MPC	Open loop	MPC
1	Weekday	5.80	6.81	4.47	6.81	0.05	0.05	0.18	0.18
	Weekend	8.14	9.72	6.34	9.72	0.13	0.13	0.18	0.18
2	Weekday	13.94	12.77	10.81	9.72	0.05	0.05	0.18	0.18
	Weekend	16.29	12.77	12.68	12.77	0.13	0.13	0.18	0.18
3	Weekday	22.09	14.77	17.15	12.77	0.05	0.05	0.18	0.18
	Weekend	24.43	16.89	18.89	14.77	0	0	0.26	0.26
4	Weekday	30.23	18.25	23.31	14.77	0	0	0.26	0.26
	Weekend	31.61	18.25	24.18	16.89	0	0	0.26	0.26

Table 4
Water and energy consumption using open loop and MPC controllers.

	Baseline	Open loop	MPC
Potable water			
Amount (m^3/month)	31.61	24.18	24.18
Cost (R/month)	395.15	267.46	267.46
Potable pump			
Energy (kWh/month)	13.04	8.00	7.80
Cost (R/month)	14.33	5.84	6.54
Grey pump			
Energy (kWh/month)	0[a]	3.25	3.25
Cost (R/month)	0[a]	3.37	3.37
Total cost (*R/month*)[b]	409.48	276.67	277.37

[a] The household was only using potable water.
[b] Cost of water and pumping energy.

amount of potable water used in a month reduces by about 23.5%. At the same time, open loop and MPC controllers can save the cost of pumping energy by up to 59.2% and 54.3% respectively, through load shifting. The energy consumed by the pumps while using both controllers reduced by 38.7%, which is attributed to the use of a lower power rated grey water pump that ends up using less power than the potable water pump used in the baseline case. Therefore, open loop and MPC controllers lead to overall cost saving of 32.5% and 32.3% respectively. The MPC controller incurs slightly more cost of energy than open loop controller as MPC controller does not give the solution at the global minimum as open loop optimal controller does.

With reliable potable water supply, the baseline would not require pumping and storing hence the total cost would be 395.15 R/month. Therefore, the grey water system with both controllers in such a house would incur a total cost (potable water and grey water pumping) of 270.83 R/month, which is a cost saving of 31.5%. This means that optimal operation of grey water recycling has both conservation and economic benefits whether there is reliable potable water supply or not. If widely adopted, these savings would be of immense benefit to both energy utilities and municipal companies over a long time. In addition, the recycled water leads to less water going down the drain, which leads to less costs incurred by domestic users and municipalities in transporting and purifying the waste water.

The uncertainty analysis for measurements done is conducted for a typical weekday. The maximum relative error, $Err_{relative} = 13.6\%$ is obtained, whose effect on the performance index is shown in Table 5. The performance index when actual values are used in both open loop and MPC controllers leads to 28.61% cost savings compared to the baseline. Therefore, final relative error of the performance index of both controllers is (7.44–6.04)/7.44 = 18.82%.

During implementation, all systems have disturbances. It has been shown by Wanjiru, Zhang, et al. (2016) that closed-loop MPC is more robust and superior than open-loop controller in dealing with disturbances. This however comes at a higher cost and more complexity as it would need extra components to enable the feedback of height of water in the tanks to take place. It is therefore recommended that each controller is adopted depending on the nature of each application. If the demand pattern does not change significantly, then the open loop controller is suitable. However, if the disturbances to the system cause the demand pattern to change, the closed-loop MPC is suitable.

The two control strategies for operating a domestic grey water recycling system aim at reducing water and energy demand. In order to determine the period taken by an investor to recover the investment, the simple payback method is used as it is commonly used for estimating the economic potential of a project (Valdiserri & Biserni, 2016). This method, however, does not take into account the time value of money and long term inflows and therefore provides the hypothetical payback period (Wong, Tay, Wong, Ong, & Sia, 2003). The simple payback period (*SPP*) is given as,

$$SPP = \frac{I_0}{S} \tag{40}$$

where I_0 is the initial investment while S is the annual savings achieved from using the grey water system using either controllers. By using the market cost of grey water recycling systems in South Africa, the grey water recycling system operated by open loop optimal or MPC controller has a payback period of 15 and 16 years respectively. Despite the technological and conservation benefits of the proposed interventions, the two strategies take a long time to recover the capital cost. The long payback period cannot motivate home owners to invest in such systems unless policies encouraging the same with monetary benefits are implemented in the country. In comparison, a study done in two universities in South Africa (Ilemobade, Olanrewaju, & Griffioen, 2013), as well as in other parts of the world such as Ireland (Li, Boyle, & Reynolds, 2010), Greece (Fountoulakis, Markakis, Petousi, & Manios, 2016) and Austria (Jabornig, 2014) have found that such systems do not necessarily pay back within their lifetime. Low water tariffs significantly influence end users' willingness to embrace water recycling. Therefore, government subsidies are therefore necessary in order to create the market for these technologies that will help in preventing water insecurity around the country and the region (Adewumi, Ilemobade, & Van Zyl, 2010). In cities with intermittent or no supply infrastructure, the cost of buying potable water as well as waste disposal could be much higher, and the system would make much more economic sense besides ensuring security of water supply.

4.5. Adoption of water recycling

Integrated urban water management (IUWM) seeks to achieve a more sustainable solution for water and sewage systems, with a trade-off among water, energy and land use. The optimal solution is a balance between energy intensive technologies and land intensive forms of water supply and treatment (Makropoulos & Butler, 2010). In this regard, water recycling provides an opportunity to increase the available water for consumption at a lower cost and sustainable environmental and social outcomes. In any country, or city, there exists localised and complex relationship between political, social, economic, environmental and technological factors that affect decision and policy making for urban water recycling. Although this study provides a technological solution to reliably operate decentralized water recycling systems, a lot of effort is required to deal with social, political and economic factors as they could hinder the uptake of the systems.

Cities with existing and functional centralized water and waste water systems, adoption of decentralized water recycling systems requires financial incentives for development and implementation. Public perception is one of the main hindrances to the implementation of water recycling systems (Rice, Wutich, White, & Westerhoff, 2016). Therefore, public participation, education and adoption of publicly visible standards on water recycling can lead to public support on these systems. Such policies have worked well in areas with well established policies and regulations such as in the USA (Verrecht et al., 2011). Government policies can dictate market behaviour as seen in Japan, where policies with wide public acceptance led to development and adoption of water recycling systems. Without effective policies, there is no motivation for home owners and developers to invest in and adopt

Table 5
Uncertainty of the performance index.

	Cost (R/day)		
	Baseline	Open loop	MPC
Measured	8.46	7.44	7.44
Actual	8.46	6.04	6.04

these alternatives or augmenting systems, considering the centralized systems are still functional. In South Africa, where cities are rapidly expanding and the infrastructure is ageing, abstraction of fresh water will soon go beyond their hydrological limits. Therefore, there needs a paradigm shift in policy making to incorporate water recycling in buildings. Such policies with incentives have successfully been implemented in the energy sector in the country, and with water becoming more scarce, water recycling policies should be in place before water insecurity becomes irreversible.

In cities with intermittent or no centralized water supply and waste water systems such as Nairobi in Kenya, Jakarta in Indonesia and Lima in Peru, home owners have to rely on water vendors to augment water supply and septic tanks for sanitation. In fact, it is estimated that 25% of the population in cities in developing nations buy water from vendors at exhorbitant prices of up to 20 times higher than the utility supply (Alaerts & Dickinson, 2008). Worse still, continued urbanization is increasing the demand for housing requiring water and sanitation infrastructure. It is therefore prudent for government in such countries to develop proper and acceptable policies that would increase water and sanitation security. Water recycling, such as the one developed in this paper, would be a relief to such residents, as it would greatly increase the efficiency of water usage at reduced cost of water, energy and operation. In addition, less water would go down the drain meaning that it would take a longer period before the septic tank requires emptying.

5. Conclusion

The incessant strain on energy and water resources in developing nations, such as South Africa, resulting from urbanization and increasing population, is causing further energy and water insecurity. Water conservation through recycling and efficient use of energy in urban areas are important in improving the security of these resources.

This paper presents two optimal controllers that enable efficient operation of water recycling in a house. The controllers are designed using the TOU tariff in South Africa, where a case was considered. Grey water recycling can conserve about 23.5% of water while leading to 32.3% cost savings on water in a month. Both the open loop and the closed-loop MPC controllers can potentially lead to 59.2% and 54.3% cost of energy savings respectively. Although the open loop model is easier and more cost effective to implement, the closed-loop MPC is more robust and reliable in controlling the grey water recycling system in domestic houses. Importantly, both controllers adapt well when subjected to the monthly block water tariff that increases as more potable water is consumed.

Such a grey water recycling system has a huge potential to conserve water while ensuring efficient use of energy when either control strategies is employed. If widely adopted, the environmental impact would be great as the demand for energy and water from the utilities and municipalities would reduce. In addition, the stress put in the sewerage system would greatly reduce as less water would be running down the drain. The system would also reduce the costs incurred by end-users for energy, water and sewage, making significant savings. These benefits would go a long way in improving the security of both resources in countries that are facing major challenges of sufficiently providing the resources to the growing urban population such as South Africa. The system however has a high payback period that could discourage home owners from adopting them besides their benefit. Government intervention is therefore necessary to provide conducive policies and incentives that will create the market for the grey water systems. In cities where water supply is intermittent or non-existent, the optimal grey water recycling system is necessary so as to reduce cost incurred while buying water from vendors while ensuring resource efficiency and security. The systems are expected to make more economic sense in such cities, where they could be adopted as a vital and cheap alternative water supply for non-potable uses.

Public perception is normally a huge barrier against implementation of such initiatives. Lack of public awareness as well as lack of evidence of such working systems can lead to general objection of such systems by the public. Governments should formulate and implement policies that are clear to the public, which can easily change the perception by providing clear safety standards. Establishment of such regulations could encourage adoption grey water recycling systems especially for non-potable uses.

Acknowledgements

The authors would like to thank the National Hub for Energy Efficiency and Demand Side Management (EEDSM) for supporting this research.

Appendix A. Open loop algorithm

In the open loop control model, the objective function and constraints are solved using the following canonical form (Numbi & Xia, 2015; Numbi, Zhang, & Xia, 2014),

$$\min f^T X \tag{A.1}$$

subject to

$$\begin{cases} AX \leq b \text{ (linear inequality constraint)}, \\ A_{eq}X = b_{eq} \text{ (linear equality constraint)}, \\ L_B \leq X \leq U_B \text{ (lower and upper bounds)}. \end{cases} \tag{A.2}$$

Vector X consists all the control variables in the optimization problem. That is,

$$X = [u_1(1), \ldots, u_1(N),\ u_2(1) \ldots, u_2(N),\ u_3(1), \ldots, u_3(N),\ u_4(1), \ldots, u_4(N),\ s_1(1), \ldots, s_1(N),\ s_3(1), \ldots, s_3(N)]^T_{6N\times 1}. \tag{A.3}$$

This means that vector f^T in the canonical form (A.1) can be obtained from the objective function (10) as,

$$f^T = [\alpha_1 t_s P_1^m p_e(1), \ldots, \alpha_1 t_s P_1^m p_e(N),\ \alpha_2 t_s Q_2 \ldots, \alpha_2 t_s Q_2,\ \alpha_3 t_s P_3^m p_e(1), \ldots, \alpha_3 t_s P_3^m p_e(N),\ 0, \ldots, 0,\ \alpha_4, \ldots, \alpha_4,\ \alpha_4, \ldots, \alpha_4]_{1\times 6N}. \tag{A.4}$$

The linear inequality constraint (11) is tranformed to

$$\begin{aligned} A_1 X &\leq b_1 \\ -A_1 X &\leq b_2 \end{aligned} \tag{A.5}$$

such that

$$A_1 = \begin{bmatrix} -Q_1t_s & 0 & \dots & 0 & Q_2t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ -Q_1t_s & -Q_1t_s & \dots & 0 & Q_2t_s & Q_2t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ -Q_1t_s & -Q_1t_s & \dots & -Q_1t_s & Q_2t_s & Q_2t_s & \dots & Q_2t_s & 0 & \dots & \dots & 0 \end{bmatrix}_{N\times 6N}, \tag{A.6}$$

$$b_1 = \begin{bmatrix} -D_{\text{pot}}(1) - A_1^t\{h_1^{\min} - h_1(0)\} \\ -\{D_{\text{pot}}(1) + D_{\text{pot}}(2)\} - A_1^t\{h_1^{\min} - h_1(0)\} \\ \vdots \\ -\{D_{\text{pot}}(1)+\dots+D_{\text{pot}}(N)\} - A_1^t\{h_1^{\min} - h_1(0)\} \end{bmatrix}_{N\times 1}, \tag{A.7}$$

and

$$b_2 = \begin{bmatrix} D_{\text{pot}}(1) + A_1^t\{h_1^{\max} - h_1(0)\} \\ \{D_{\text{pot}}(1) + D_{\text{pot}}(2)\} + A_1^t\{h_1^{\max} - h_1(0)\} \\ \vdots \\ \{D_{\text{pot}}(1)+\dots+D_{\text{pot}}(N)\} + A_1^t\{h_1^{\max} - h_1(0)\} \end{bmatrix}_{N\times 1}. \tag{A.8}$$

Similarly, inequality constraint (12) is transformed to

$$\begin{aligned} &A_2X \le b_3 \\ &-A_2X \le b_4 \end{aligned} \tag{A.9}$$

such that

$$A_2 = \begin{bmatrix} 0 & \dots & 0 & -Q_2t_s & 0 & \dots & 0 & -Q_3t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & -Q_2t_s & -Q_2t_s & \dots & 0 & -Q_3t_s & -Q_3t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -Q_2t_s & -Q_2t_s & \dots & -Q_2t_s & -Q_3t_s & -Q_3t_s & \dots & -Q_3t_s & 0 & \dots & \dots & 0 \end{bmatrix}, \tag{A.10}$$

$$b_3 = \begin{bmatrix} -D_{\text{grey}}(1) - A_2^t\{h_2^{\min} - h_2(0)\} \\ -\{D_{\text{grey}}(1) + D_{\text{grey}}(2)\} - A_2^t\{h_2^{\min} - h_2(0)\} \\ \vdots \\ -\{D_{\text{grey}}(1)+\dots+D_{\text{grey}}(N)\} - A_2^t\{h_2^{\min} - h_2(0)\} \end{bmatrix}, \tag{A.11}$$

and

$$b_4 = \begin{bmatrix} D_{\text{grey}}(1) + A_2^t\{h_2^{\max} - h_2(0)\} \\ \{D_{\text{grey}}(1) + D_{\text{grey}}(2)\} + A_2^t\{h_2^{\max} - h_2(0)\} \\ \vdots \\ \{D_{\text{grey}}(1)+\dots+D_{\text{grey}}(N)\} + A_2^t\{h_2^{\max} - h_2(0)\} \end{bmatrix}, \tag{A.12}$$

while inequality (13) is remodelled to,

$$\begin{aligned} &A_3X \le b_5 \\ &-A_3X \le b_6 \end{aligned} \tag{A.13}$$

such that,

$$A_3 = \begin{bmatrix} 0 & \dots & \dots & 0 & Q_3t_s & 0 & \dots & 0 & Q_4t_s & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & Q_3t_s & Q_3t_s & \dots & 0 & Q_4t_s & Q_4t_s & \dots & 0 & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & 0 & Q_3t_s & Q_3t_s & \dots & Q_3t_s & Q_4t_s & Q_4t_s & \dots & Q_4t_s & 0 & \dots & \dots & 0 \end{bmatrix}, \tag{A.14}$$

$$b_5 = \begin{bmatrix} S_{\text{grey}}(1) - A_3^t\{h_3^{\min} - h_3(0)\} \\ \{S_{\text{grey}}(1) + S_{\text{grey}}(2)\} - A_3^t\{h_3^{\min} - h_3(0)\} \\ \vdots \\ \{S_{\text{grey}}(1)+\dots+S_{\text{grey}}(N)\} - A_3^t\{h_3^{\min} - h_3(0)\} \end{bmatrix}, \tag{A.15}$$

and

$$b_6 = \begin{bmatrix} -S_{\text{grey}}(1) + A_3^t\{h_3^{\max} - h_3(0)\} \\ -\{S_{\text{grey}}(1) + S_{\text{grey}}(2)\} + A_3^t\{h_3^{\max} - h_3(0)\} \\ \vdots \\ -\{S_{\text{grey}}(1)+\dots+S_{\text{grey}}(N)\} + A_3^t\{h_3^{\max} - h_3(0)\} \end{bmatrix}. \tag{A.16}$$

Finally, the auxiliary variables in inequalities (15)–(18) are remodelled as

$$A_4X \le b_7 \tag{A.17}$$

where,

$$A_4 =$$
$$\left[\begin{array}{ccccc|c|ccccc|c|cccc|cccc} 1 & 0 & \dots & 0 & 0 & 0\dots0 & 1 & 0 & \dots & 0 & 0 & 0\dots0 & -1 & 0 & \dots & 0 & -1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 & 0 & 0\dots0 & -1 & 1 & \dots & 0 & 0 & 0\dots0 & 0 & -1 & \dots & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\ddots\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\ddots\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1 & 0\dots0 & 0 & 0 & \dots & -1 & 1 & 0\dots0 & 0 & 0 & \dots & -1 & 0 & 0 & \dots & -1 \end{array}\right], \tag{A.18}$$

and

$$b_7 = [0 \ \dots \ 0]^T. \tag{A.19}$$

Matrices A_1 to A_4 have ($N \times 6N$) dimension while vectors b_1 to b_7 have a dimension of ($N \times 1$). Therefore, linear inequality in the canonical form (A.2) becomes,

$$\begin{bmatrix} A_1 \\ -A_1 \\ A_2 \\ -A_2 \\ A_3 \\ -A_3 \\ A_4 \end{bmatrix}_{7N\times 6N} X = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix}_{7N\times 1} \tag{A.20}$$

In the same manner, linear equality constraint (14) becomes,

$$A_{\text{eq}} X = b_{\text{eq}}, \tag{A.21}$$

where

$$A_{eq} = \left[\begin{array}{ccc|cccccc|ccc} 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & Q_3 t_s & \dots & Q_3 t_s & Q_4 t_s & \dots & Q_4 t_s & 0 & \dots & 0 \end{array}\right]_{N\times 6N}, \tag{A.22}$$

and

$$b_{\text{eq}} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ A_3^t\{h_3(0) - h_3^f\} + \{S_{\text{grey}}(1) + \ldots + S_g\text{rey}(j)\} \end{bmatrix}_{N\times 1}, \tag{A.23}$$

while the bounds given in Eqs. (19) and (20) become,

$$L_B = [0 \ \dots \ 0]^T_{6N\times 1} \quad \text{and} \quad U_B = [1 \ \dots \ 1]^T_{6N\times 1}. \tag{A.24}$$

This binary integer optimization problem is solved using the SCIP solver in OPTI toolbox, a free Matlab optimization toolbox. This solver is used as it is reported as the fastest non-commercial optimization solver (Setlhaolo & Xia, 2015, 2016).

Appendix B. Closed-loop algorithm

Closed-loop MPC obtains the current control action by solving, in each sampling time, a finite horizon open loop optimal control problem using the current state of the plant as the initial state. The optimization yields an optimal control sequence and the first control in this sequence is applied to the plant. This process is repeated throughout the entire control period (Mayne et al., 2000). Using the principle of the receding horizon control in closed-loop MPC, only the first element in the control vector X^{mpc} is implemented after each iteration, ignoring the rest of the elements (Wang, 2009). The state of the plant (water level in the tanks) is measured. During the next iteration, k + 1, the objective function and the constraints are updated while taking the previous state of the tanks (water level at sampling time k) as the initial state. The process of optimization is carried out in real time over the new control horizon ($N_c = N - k + 1$) to give the receding horizon control law. Similar to the open loop control algorithm, the control vector, X^{mpc}, contains the control variables such that,

$$\begin{aligned} X^{\text{mpc}} = \ & [u_1(k|k),\, u_1(k+1|k), \ldots, u_1(k+N_c-1|k)),\, u_2(k|k),\, u_2(k+1|k), \ldots, u_2(k+N_c-1|k), \\ & u_3(k|k),\, u_3(k+1|k), \ldots, u_3(k+N_c-1|k)),\, u_4(k|k),\, u_4(k+1|k), \ldots, u_4(k+N_c-1|k), \\ & s_1(k|k),\, s_1(k+1|k), \ldots, s_1(k+N_c-1|k),\, s_3(k|k),\, s_3(k+1|k), \ldots, s_3(k+N_c-1|k)]^T_{6N\times 1}. \end{aligned} \tag{B.1}$$

This is also a binary integer optimization problem solved using the SCIP solver in OPTI toolbox, a free Matlab optimization toolbox.

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