Available online at www.sciencedirect.com
ScienceDirect
automatica
Automatica 44 (2008) 242–247
www.elsevier.com/locate/automatica

Brief paper

Speed regulation with measured output feedback in the control of heavy haul trains ☆

X. Zhuan*, X. Xia

Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa

Received 14 February 2006; received in revised form 6 November 2006; accepted 20 May 2007
Available online 16 August 2007

Abstract

An approach of output regulation with measurement feedback is proposed for the control of heavy haul trains. The objective is to regulate all cars' speeds to a prescribed speed profile. The output regulation problem of nonlinear systems with measurement feedback is formulated and solved for the first time in this paper. Its application to train control is detailed. Simulation result shows the feasibility of the approach, in terms of its simplicity, cost effectiveness and its implementation convenience.

Keywords: Output regulation; Measured output feedback; Quadratic programming; Heavy haul trains; ECP braking system

1. Introduction

Trains play an important role in the transportation of mineral resources such as in South Africa. In terms of the schedule and the number of people involved, the cost is less for a larger load per car or per train. This has resulted in long heavy haul trains with multi-locomotives. The application of electronically controlled pneumatic (ECP) braking systems in the 1990s makes it possible to have very long trains stretching over 10 km. An introduction of ECP is seen in Kull (2001). This braking system was firstly rolled out on a large scale by Spoornet, a train operator in South Africa, on its COALink line.

Energy consumption, running time (speed tracking) and in-train force are of much concern to transportation corporations (Zhuan & Xia, 2007). Various studies have tried to achieve different objectives. Some studies have been done in Cheng and Howlett (1992, 1993), Howlett, Milroy, and Pudney (1994), Howlett (1996), Khmelnitsky (2000), Liu and Golovitcher (2003) to schedule a train to travel from one station to the next one in a given time with minimal energy consumption. "To schedule" means to determine a driving sequence in terms of locomotives' power notches and wagons' braking pressure along a specific railway track. In those papers, a train is modelled as a mass point, and the dynamics within a train are ignored. Those approaches are essentially open loop control.

The other models a train as a cascade of mass points, for example, in Yang and Sun (2001), Gruber and Bayoumi (1982), where a speed profile is assumed first, and the aims are to drive a train according to a speed profile with some objectives considered. For the study of high speed (passenger) trains in Yang and Sun (2001), Astolfi and Menini (2002), speed tracking is emphasized without considering in-train forces because in-train forces are not so important for such short trains. An early study for in-train forces can be seen in Gruber and Bayoumi (1982), where a linear quadratic regulator (LQR) approach is employed to optimize in-train forces and/or speed deviation from a reference speed with a largely simplified model. Recently, an LQR approach is employed in Chou and Xia (2007) to optimize in-train force, energy consumption and velocity tracking based on a validated model in Chou, Xia, and Kayser (2007) with the operation data from Spoornet. The methods in those papers are all within a linear system theory. The off-line scheduling

☆ A preliminary version of this paper was presented at the 4th IFAC-Symposium on Mechatronic Systems, Heidelberg, Germany, 11–16 September, 2006. This paper was recommended for publication in revised form by Associate Editor Pedro Albertos under the direction of Editor Mituhiko Araki.

* Corresponding author. Tel.: +27 12 420 4341; fax: +27 12 362 5000.
E-mail addresses: zhuan.xiangtao@up.ac.za (X. Zhuan), xxia@postino.up.ac.za (X. Xia).

0005-1098/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.05.002

is based on some heuristic assumptions. In Rao (2005), based on an infinite dimensional linear model, flatness based methods are used to design open loop control for a train. In Zhuan and Xia (2006), an approach is applied to get an equilibrium optimizing in-train force. The paper (Zhuan & Xia, 2007) integrates the ideas of optimal scheduling in Zhuan and Xia (2006) and LQR closed-loop control in Chou and Xia (2007). However, the closed-loop controller is designed based on full state feedback, which is not practical since not all the states can be measured.

An observer could be designed to supplement the LQR controller if partial states are measured. This is, however, not the approach taken in this paper. Instead, output regulation is adopted on the assumption that only speed measurement of locomotives is available: while optimality is retained in open loop control, closed-loop control is done by employing an output regulation theory. This approach is practically feasible and is easily integrable with human drivers who generally drive a train according to the train's speed. Instead of a linear system theory, a nonlinear system theory is adopted such that without a linear approximation philosophy, the control is closer to reality. Another advantage is the assumption that only locomotives' speeds are measurable.

Output regulation problem with state/error feedback in the local version has been well studied. A seminal contribution was made in Isidori and Byrnes (1990). Recent studies include Huang and Chen (2004), Chen and Huang (2005) in which general frameworks for global version have been proposed. However, all those studies, by their nature, consider state/error feedback. In practice, the states are usually not measurable, and sometimes measured outputs differ from regulation error.

The output regulation problem of nonlinear systems with measurement feedback is first formulated and solved in this paper. Its application to train control is also detailed. To make applicable the nonlinear regulator theory, the reference speed trajectories are carefully redesigned according to the prescribed speed profile. Simulation result shows the feasibility of such a controller in terms of its simplicity, cost-effectiveness and implementation convenience.

The structure of this paper is: the output regulation problem of nonlinear systems with measurement feedback is formulated and solved in Section 2. A train model is briefly described in Section 3 while Section 4 discusses the application issues of output regulation to train control. Simulation result is shown in Section 5.

2. Measurement feedback output regulation

$$\begin{aligned}\dot{x} &= f(x) + g(x)u + p(x)w,\\ e &= h(x) + q(w),\\ y_m &= h_m(x, w).\end{aligned} \tag{1}$$

Consider a nonlinear system (1), where the first equation defines a plant with state $x \in X \subset R^n$ and input $u \in U \subset R^m$, $e \in R^p$ is the output to be regulated, and $y_m \in R^{p_m}$ is the measured output. The variable w is the state of the exosystem defined by

$$\dot{w} = s(w). \tag{2}$$

The mappings $f(x), g(x), p(x), h(x), q(w), h_m(x, w), s(w)$ are assumed to be smooth satisfying $f(0) = 0$, $h(0) + q(0) = 0$, $h_m(0, 0) = 0$, $s(0) = 0$ and the composite system of (1) and (2) has an equilibrium at the origin $col(x, w) = [x^{\mathrm{T}}, w^{\mathrm{T}}]^{\mathrm{T}} = col(0, 0)$ with the input $u = 0$.

The class of controllers considered here is in the form of measured output feedback

$$\begin{aligned}\dot{z} &= \eta(z, y_m), \quad \eta(0, 0) = 0,\\ u &= k(z, y_m), \quad k(0, 0) = 0.\end{aligned} \tag{3}$$

As a result, the closed-loop system can be written as (4), of which the first two are denoted as $\dot{x}_z = f_c(x_z, w)$.

$$\begin{aligned}\dot{x} &= f(x) + g(x)k(z, y_m) + p(x)w,\\ \dot{z} &= \eta(z, y_m),\\ \dot{w} &= s(w),\\ e &= h(x) + q(w), \quad y_m = h_m(x, w).\end{aligned} \tag{4}$$

The output regulation problem is to find a controller of the form (3) such that the closed-loop system (4) satisfies

P1. $(\partial f_c)/(\partial x_z)$ is Hurwitz at the origin $x_z = 0$.
P2. For all sufficiently small x_{z0}, w_0,

$$\lim_{t\to\infty} e(x(t), w(t)) = 0.$$

The exosystem is assumed to be neutrally stable, i.e., $S = \partial s(0)/\partial w$ has all its eigenvalues on the imaginary axis. One also has the following notations.

$$A = \frac{\partial f(0)}{\partial x}, \quad Q_m = \frac{\partial h_m(0,0)}{\partial w}, \quad C_m = \frac{\partial h_m(0,0)}{\partial x},$$
$$G = \frac{\partial \eta(0,0)}{\partial y_m}, \quad F = \frac{\partial \eta(0,0)}{\partial z}, \quad H_1 = \frac{\partial k(0,0)}{\partial y_m},$$
$$B = g(0), \quad P = p(0), \quad H = \frac{\partial k(0,0)}{\partial z}.$$

Theorem 1. *For the nonlinear system* (1), *assuming the exosystem is neutrally stable*, (A, B) *is controllable and the pair of*

$$\left(\begin{bmatrix} A & P\\ 0 & S\end{bmatrix}, [C_m, Q_m]\right)$$

is detectable, the local dynamic measurement feedback output regulation problem is solvable if and only if there exist smooth mappings $x = \pi(w)$, $u = c(w)$ *with* $\pi(0) = 0$, $c(0) = 0$ *satisfying*

$$\begin{aligned}\frac{\partial \pi(w)}{\partial w}s(w) &= f(\pi(w)) + g(\pi(w))c(w) + p(\pi(w))w,\\ 0 &= h(\pi(w)) + q(w).\end{aligned} \tag{5}$$

Proof. The necessity is as follows. The closed-loop system of (4) has the form

$$\begin{aligned}\dot{x} &= (A + BH_1C_m)x + BHz + (P + BH_1Q_m)w\\ &\quad + \phi(x, z, w),\\ \dot{z} &= GC_m x + Fz + GQ_m w + \chi(x, z, w),\\ \dot{w} &= Sw + \psi(w),\end{aligned} \tag{6}$$

where $\phi(x, z, w)$, $\chi(x, z, w)$, $\psi(w)$ vanish at the origin with their first-order derivatives. By P1, the eigenvalues of the matrix

$$\begin{bmatrix} A + BH_1C_m & BH \\ GC_m & F \end{bmatrix}$$

are in C^-, and those of S on the imaginary axis. Thus, the system has a center manifold at (0, 0, 0), the graph of a mapping $x = \pi(w)$, $z = \sigma(w)$, with $\pi(0) = 0$, $\sigma(0) = 0$ satisfying,

$$\frac{\partial\pi}{\partial w}s(w) = f(\pi(w)) + g(\pi(w))c(w) + p(\pi(w))w,$$
$$\frac{\partial\sigma}{\partial w}s(w) = \eta(\sigma(w), h_m(\pi(w), w)).$$

The first equation of (5) is satisfied.

Suppose the second equation of (5) is not true at some $(\pi(w^o), w^o)$ sufficiently close to (0, 0). Then, $M = \|h(\pi(w^o)) + q(w^o)\| > 0$ and there exists a neighborhood V of $(\pi(w^o), w^o)$ such that $\|h(\pi(w)) + q(w)\| > M/2$ at each $(\pi(w), w) \in V$. If condition P2 holds for a trajectory starting at $(\pi(w^o), w^o)$, there exists T such that $\|h(\pi(w)) + q(w)\| < M/2$ for all $t > T$. However, if $(\pi(w^o), w^o)$ is Poisson stable, then for some $t' > T$,$(\pi(w(t')), w(t')) \in V$ and this contradicts the previous inequality. So the second equation of (5) must be true, that is, P2 can hold only if this center manifold is annihilated by the error map $e = h(x) + q(w)$.

The proof of sufficiency is constructive. In Theorem 1, assuming the equations (5) are satisfied with some $\pi(w)$ and $c(w)$, consider the following control law,

$$u = \theta(z),$$
$$\dot{z} = \eta(z, y_m), \tag{7}$$

with $z = col(z_1, z_2)$ and $\theta(z)$ and $\eta(z)$ defined by,

$$\theta(z) = c(z_2) + H(z_1 - \pi(z_2)),$$
$$\eta(z, y_m) = col(\eta_1(z, y_m), \eta_2(z, y_m)),$$
$$\eta_1(z, y_m) = f(z_1) + p(z_1)z_2 + g(z_1)c(z_2) + H(z_1 - \pi(z_2)) - G_1(h_m(z_1, z_2) - y_m),$$
$$\eta_2(z, y_m) = s(z_2) - G_2(h_m(z_1, z_2) - y_m),$$

where H, G_1 and G_2, with the assumptions of Theorem 1, can be chosen such that $A + BH$ and

$$\begin{bmatrix} A - G_1C_m & P - G_1Q_m \\ -G_2C_m & S - G_2Q_m \end{bmatrix}$$

have all their eigenvalues with negative real parts, which guarantees the Jacobian matrix of (4) is Hurwitz. Thus P1 is satisfied.

Because of the continuity of $e = h(x) + q(w)$ and the limit of w, it is easy to reach

$$\lim_{t\to\infty} e(x(t), w(t)) = \lim_{x\to\pi(w)} e(x, w) = 0.$$

The property P2 is satisfied. □

Remark 1. The above theorem is very similar to the one in Isidori and Byrnes (1990) for error feedback as well as its proof. However, the controller in Theorem 1 is in the form of measurement feedback. The measurement generally differs from the output to regulate. For example in this study of train handling, the outputs to be regulated are all the cars' speeds while only part of the speeds can be practically measured. On the other hand, measurement feedback covers the form of error or state feedback.

Remark 2. In particular, when w is known, for example, $y_m = col(y_m^1, w)$, the problem can be solved by

$$\dot{z} = f(z) + g(z)u + p(z)w + G_1(y_m^1 - h_m(z, w)),$$
$$u = c(w) + H(z - \pi(w)), \tag{8}$$

where G_1, H are chosen such that $A + BH$ and $A - G_1C_m^1$ are Hurwitz ($C_m^1 = \partial y_m^1(0, 0)/\partial x$.)

Remark 3. The parameters K, G can be determined with different kinds of methods, such as with pole placement. Although K, G are chosen with a linear system theory, the output regulation problem is solved with them.

3. Train model

A heavy haul train is composed of k locomotives and $n - k$ wagons (both referred to as cars). The longitudinal dynamics of a train can be modeled as

$$m_i\dot{v}_i = u_i + f_{in_{i-1}} - f_{in_i} - f_{a_i}, \quad i = 1, 2, \ldots, n,$$
$$\dot{x}_j = v_j - v_{j+1}, \quad j = 1, 2, \ldots, n - 1, \tag{9}$$

where m_i is the ith car's mass, v_i, u_i are the speed and the effort of the ith car. The variable $f_{a_i} = f_{aero_i} + f_{p_i}$ is the force undertaken by the ith car from the environment. $f_{aero_i} = m_ic_{0_i} + m_ic_{1_i}v_i + m_ic_{2_i}v_i^2$, where c_{0_i}, c_{1_i} and c_{2_i} are constants determined by experiments. The variable f_{p_i} is a sum of f_{g_i}, the gravity force in longitudinal direction, and f_{c_i}, the curvature resistance. The variable f_{in_j} is the in-train force between the jth and $(j + 1)$th cars, which is a function of x_j, the relative displacement between the two neighboring cars.

A more detailed description of the model is referred to Chou and Xia (2007) and Zhuan and Xia (2006).

4. Output regulation of heavy haul trains

In this paper, the 2–2 control strategy (Zhuan & Xia, 2006), an ECP/iDP-only strategy is assumed, i.e., all the inputs may be different. For (9), some changes are required to be done for the application of output regulation. On the one hand, the origin is not an equilibrium. On the other hand, there are a lot of trajectories to regulate cars' speeds to the reference speed. However, for train handling, the choice of the trajectories involves the balance between energy consumption and in-train forces. In this paper, optimal scheduling in Zhuan and Xia (2007) is employed. With this optimal scheduling, the equilibrium can be denoted as $f_{in_j}^0(x_j^0)$, $v_i^0(v_r)$, u_i^0, which are in-train forces (coupler displacements), speeds (reference speed) and efforts of cars. The train model is rewritten as:

$$\delta\dot{v}_i = (\delta u_i + \delta f_{in_{i-1}} - \delta f_{in_i} - \delta f a_i)/m_i,$$
$$\delta\dot{x}_j = \delta v_j - \delta v_{j+1}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n-1, \quad (10)$$

where $\delta v_i = v_i - v_i^0$, $\delta u_i = u_i - u_i^0$, $\delta f_{in_i} = f_{in_i} - f_{in_i}^0$, $\delta x_j = x_j - x_j^0$. With $X = [\delta v_1, \ldots, \delta v_n, \delta x_1, \ldots, \delta x_{n-1}]^{\mathrm{T}}$ and $U = [\delta u_1, \ldots, \delta u_n]^{\mathrm{T}}$, the system (10) is also denoted in the form

$$\dot{X} = f(X) + g(X)U. \quad (11)$$

The outputs to be regulated are the cars' speeds, i.e., assuming the reference speed is w_1, $e_i = v_i - w_1 = X_i + v_r - w_1$. The measurement is part of the cars' speeds, i.e., $y_m = C_m(X + v_r)$, where $C_m = (C_{ij})_{p_m \times (2n-1)}$ and all the entries of the row vectors of C_m are zeros only except one of the first n ones which is one. For example,

$$C_m = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \end{bmatrix}$$

if only the first two cars' speeds are measured. Notice that the measurement is different from the error output.

The linearized system of (11) has system matrixes

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} \mathrm{diag}\left(\dfrac{1}{m_1}, \ldots, \dfrac{1}{m_n}\right) \\ 0_{(n-1)\times n} \end{bmatrix},$$

$$A_{11} = -\mathrm{diag}(c_{1_1} + c_{2_1} v_r, \ldots, c_{1_n} + c_{2_n} v_r),$$

$$A_{12} = \begin{bmatrix} -\dfrac{k_1}{m_1} & 0 & \cdots & 0 & 0 \\ \dfrac{k_1}{m_2} & -\dfrac{k_2}{m_2} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \dfrac{k_{n-2}}{m_{n-1}} & -\dfrac{k_{n-1}}{m_{n-1}} \\ 0 & \cdots & 0 & 0 & \dfrac{k_{n-1}}{m_n} \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}, \quad A_{22} = 0_{(n-1)\times(n-1)}.$$

It can be verified that (A, B) is controllable and (A, C_m) is observable with the *PBH* criterion in Kailath (1980).

$$[\lambda I - A \mid B] = \begin{bmatrix} \lambda I_{n\times n} - A_{11} & -A_{12} & \mathrm{diag}\left(\dfrac{1}{m_i}\right) \\ -A_{21} & \lambda I_{(n-1)\times(n-1)} & 0_{(n-1)\times n} \end{bmatrix}$$
$$\sim \begin{bmatrix} 0_{n\times n} & & 0_{n\times(n-1)} & I_{n\times n} \\ I_{(n-1)\times(n-1)} & 0 & \lambda I_{(n-1)\times(n-1)} & 0_{(n-1)\times n} \end{bmatrix},$$

from which one can get $\mathrm{rank}([\lambda I - A \mid B]) = 2n - 1$. If the first or last car's speed is measured, the pair (A, C_m) is observable. With the first car's speed available, one has

$$\Gamma = \begin{bmatrix} A - \lambda I \\ C_m \end{bmatrix} = \begin{bmatrix} A_{11} - \lambda I_{n\times n} & A_{12} \\ A_{21} & -\lambda I_{(n-1)\times(n-1)} \\ 1 \quad 0 \quad \cdots \quad 0 & 0_{1\times(n-1)} \end{bmatrix}$$
$$\sim \begin{bmatrix} I_{(2n-1)\times(2n-1)} \\ 0_{1\times(2n-1)} \end{bmatrix},$$

from which one knows that $\mathrm{rank}(\Gamma) = 2n - 1$, and the pair (A, C_m) is observable according to the *PBH* criterion. Actually the first car of a train is usually a locomotive, whose speed is available. So the above assumption does not lose generality.

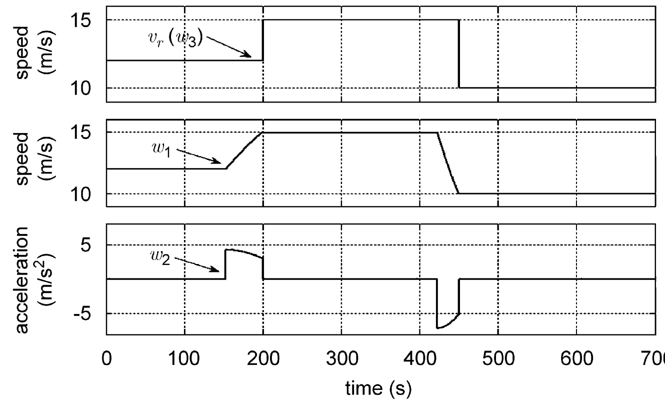

Fig. 1. Modified speed profile.

The speed maintenance phase, speed acceleration and speed deceleration phases are discussed in this paper. The cars' speeds are the subject of regulation. According to the reference speed profile, the trajectory of the reference speed can be redesigned as w_1 of

$$\dot{w}_1 = a w_2, \quad \dot{w}_2 = -a(w_1 - w_3), \quad \dot{w}_3 = 0, \quad (12)$$

whose solution is $w_1 = w_3(0) + A\sin(at + \phi_0)$, $w_2 = A\cos(at + \phi_0)$, $w_3 = w_3(0)$, where A and ϕ_0 are determined by the initial conditions $(w_1(0), w_2(0), w_3(0))$. Within the cruise phase, the initial conditions are chosen as $(w_1(0), w_2(0), w_3(0)) = (v_r, 0, v_r)$, where v_r is the cruise speed. Assuming the reference speed before acceleration is v_{r_1} and the reference speed after acceleration is v_{r_2}, then the initial conditions are chosen such that $w_3(0) = v_{r_1}$, $\phi_0 = 0$, $A = \sqrt{2}(v_{r_2} - v_{r_1})$. The variable a in (12) is chosen with considering the acceleration limit a_r or deceleration limit a_c of the train, which is determined by the effort capacity of the train. In simulation, $a = a_r/A$ within an acceleration phase and $a = a_c/A$ within a deceleration phase. For example, one chooses $a_r = 0.07\,\mathrm{m/s^2}$, $a_c = -0.2\,\mathrm{m/s^2}$, $\phi_0 = 0$, and the time interval $T_1 = \pi/4a$ as acceleration/deceleration phase.The modified speed file according to the speed profile is shown in Fig. 1.

From the above designed trajectories, the conditions in Theorem 1 are satisfied if the regulator equations (5) are solved. Actually, one can verify that $X = \pi(w) = (w_1 - w_3)col(1_{1\times n}, 0_{1\times(n-1)})$, $U = c(w) = w_2 B_1^{-1} \cdot 1_{n\times 1} - B_1^{-1} f^1(\pi(w))$, with f^1 is the first n entries of f and B_1 is the first n rows of B, is a pair of the solution of (5).

According to Remark 2, the output regulating controller with measurement feedback is

$$\dot{z} = f(z) + g(z)U + G_1(y_m - C_m z),$$
$$U = c(w) + K(z - \pi(w)), \quad (13)$$

where G_1, K are chosen such that $A + BK$ and $A - G_1 C_m$ are Hurwitz.

Based on the optimal scheduling and the output regulating controller, the complete closed-loop controller is $u = U + u^0$.

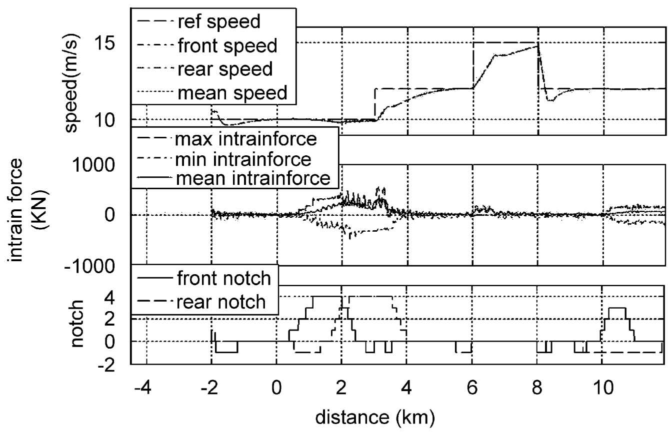

Fig. 2. Speed regulation with $K_f = 1, K_v = 1, K_e = 1$.

In simulation, K, G is chosen with a linear quadratic algorithm (Chou & Xia, 2007). These choices of K and G are consistent with Remark 3.

Since the throttle of the locomotives takes discrete values and the braking capacities of the wagons are constrained, the input u in the complete controller may violate these constraints. When this happens, an anti-windup technique is employed in simulation. For wagons, the application of the anti-windup technique is very simple. For locomotives, inputs are discrete with some operation constraints. Similar methods as described in Slotine and Li (1991) are used to smooth continuous control inputs. Assuming the required force of jth locomotives is F_j and the output of admitted kth notch at current velocity v_j is $g(k, v_j)$, the output force F_j^r of the jth locomotives can be determined by

$$F_j^r = g(k, v_j) \quad \text{if} \quad G(k-1, v_j) \leqslant F_j < G(k, v_j),$$
$$G(k, v) = g(k, v_j) + \alpha(g(k+1, v_j) - g(k, v_j)),$$

where $G(k, v_j)$ and $G(k-1, v_j)$ are upper and lower boundaries of the admitted notch k, respectively. The variable α is the ratio of the separation for the boundary.

5. Simulation

The simulation setting and parameters are the same as those in Zhuan and Xia (2007) except that the deceleration limit is $-0.2\,\mathrm{m/s^2}$. α is chosen as 0.5. The observer is designed with the assumption that the front and rear locomotive group speeds are available.

Simulation result is shown in Fig. 2. The first subplot shows the front locomotive group speed, rear locomotive group speed and the mean speed of all the cars. The second subplot shows maximum and minimum in-train forces and the mean value of the absolute values of all the in-train forces at a specific time. The third shows the front and rear locomotive groups' notches. The track profile is the same as that in Zhuan and Xia (2007) and omitted here. From Fig. 2, it can be seen the train tracks the reference speed well except within acceleration/deceleration phases. This is because of the application of observer in latter, which needs some time to track the states of the train. The in-train forces in Fig. 2 are smaller than those of the LQR controller in Zhuan and Xia (2007) at steady state. This is because the slower response (result of application of an observer) leads to more gentle output. Table 1 shows the simulation results of LQR controllers with state feedback $S_i, i = 2, 3, 4$ advanced in Zhuan and Xia (2007) and measurement feedback controllers $M_i, i = 2, 3, 4$ proposed in this paper with different parameters.The indices 2,3,4 denote the different sets of parameters $(K_e, K_f, K_v) = (1, 1, 10)$, $(K_e, K_f, K_v) = (1, 10, 1)$, $(K_e, K_f, K_v) = (100, 1, 1)$, respectively. $|\delta\bar{v}|$ is the absolute value of the difference between the reference velocity and the mean value of all the cars' velocities at a specific point. $\overline{|f_{in}|}$ is the mean value of the absolute values of all the couplers' in-train forces at a specific point. The items max, mean and std are the maximum value, mean value and standard deviation of the statistical variable, respectively. From Table 1, it can be seen there are more energy consumed in LQR controllers with state feedback than in output regulation controllers with measurement feedback, no matter what the optimal parameters are. This is because for the state feedback controllers are sensitive to the state deviation from the equilibrium and the energy optimization is local, the locomotives' traction efforts and the cars' braking change more frequently, which leads to consume more energy.

For the speed tracking, the optimal controller with state feedback is a little better than the output regulating controller with measurement feedback, and for in-train forces, the former is worse than the latter.

6. Conclusion

An approach of output regulation with measurement feedback is proposed for the control of heavy haul trains. The output regulation problem of nonlinear systems with measurement feedback is formulated and solved for the first time, extending

Table 1
Performance comparison

	$\lvert\delta\bar{v}\rvert$(m/s)			$\overline{\lvert f_{in}\rvert}$(kN)			E(MJ)
	max	mean	std	max	mean	std	
S2	3.02	0.24	0.50	408.70	74.07	76.34	16,524
S3	3.01	0.37	0.47	405.70	70.77	78.04	15,007
S4	3.25	0.49	0.47	297.27	78.90	63.27	13,422
M2	2.98	0.30	0.53	329.39	54.28	65.28	12,713
M3	2.97	0.33	0.52	329.00	56.74	64.14	12,570
M4	3.61	0.90	0.62	405.34	98.41	73.50	10,493

the existing results. Based on a cascade-mass-point model, the application of output regulation to train control is detailed. The conditions of the application are verified. Optimal scheduling is integrated in the controller of output regulation. Simulation shows the feasibility of the output regulating controller with only measurement of the locomotive speeds, in terms of its simplicity, cost-effectiveness and its implementation convenience.

References

Astolfi, A., & Menini, L. (2002). Input/output decoupling problem for high speed trains. In *Proceedings of the American Control Conference*, (pp. 549–554). Anchorage, May 2002.

Chen, Z., & Huang, J. (2005). A general formulation and solvability of the global robust output regulation problem. *IEEE Transactions on Automatic Control*, *50*, 448–462.

Cheng, J., & Howlett, P. (1992). Application of critical velocities to minimization of fuel consumption in the control of trains. *Automatica*, *28*, 165–169.

Cheng, J., & Howlett, P. (1993). A note on the calculation of optimal strategies for the minimization of fuel consumption in the control of trains. *IEEE Transactions on Automatic Control*, *38*, 1730–1734.

Chou, M., Xia, X., & Kayser, C. (2007). Modelling and model validation of heavy-haul trains equipped with electronically controlled pneumatic brake systems. *Control Engineering Practice*, *15*(5), 501–509.

Chou, M., & Xia, X. (2007). Optimal cruise control of heavy-haul trains equipped with electronically controlled pneumatic brake systems. *Control Engineering Practice*, *15*(5), 511–519.

Gruber, P., & Bayoumi, M. (1982). Suboptimal control strategies for multilocomotive powered trains. *IEEE Transactions on Automatic Control*, *27*, 536–546.

Howlett, P. G., Milroy, I. P., & Pudney, P. J. (1994). Energy-efficient train control. *Control Engineering Practice*, *2*, 193–200.

Howlett, P. (1996). Optimal strategies for the control of a train. *Automatica*, *32*, 519–532.

Huang, J., & Chen, Z. (2004). A general framework for tackling the output regulation problem. *IEEE Transactions on Automatic Control*, *49*, 2203–2218.

Isidori, A., & Byrnes, C. I. (1990). Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, *35*, 131–140.

Kailath, T. (1980). *Linear systems*. Englewood Cliffs, NJ: Prentice-Hall.

Khmelnitsky, E. (2000). On an optimal control problem of train operation. *IEEE Transactions on Automatic Control*, *45*, 1257–1266.

Kull, R. C. (2001). WABTEC ECP system update. *IEEE vehiclur technology society land transportation division and ASME rail transportation division joint railroad conference*, (pp. 129–134). Toronto, April 2001.

Liu, R., & Golovitcher, I. M. (2003). Energy-efficient operation of rail vehicles. *Transportation Research Part A: Policy and Practice*, *37*, 917–932.

Rao, G. (2005). Modellbildung und Regelung für lange güterzüge unter verwendung unendlichdimensionaler modelle. Master's Thesis, Technische Unversität Dresden, Germany.

Slotine, J. J. E., & Li, W. (1991). *Applied nonlinear control*. New Jersey: Prentice-Hall.

Yang, C., & Sun, Y. (2001). Mixed H_2/H_∞ cruise controller design for high speed train. *International Journal of Control*, *74*, 905–920.

Zhuan, X., & Xia, X. (2006). Cruise control scheduling of heavy haul trains. *IEEE Transactions on Control Systems Technology*, *14*(4), 757–766.

Zhuan, X., & Xia, X. (2007). Optimal scheduling and control of heavy haul trains equipped with electronically controlled pneumatic braking systems. *IEEE Transactions on Control Systems Technology*, to appear.

Xiangtao Zhuan obtained his B.Eng. from Wuhan University of Hydraulic and Electrical Engineering, China in 1999. He received a master's degree from Wuhan University in 2002. He obtained his PhD degree at the University of Pretoria. He has been employed as a lecturer in the Department of Automation at Wuhan University since 2002. Now he is a post-doctor at the University of Pretoria. His research interests include power plant automation, power station automation, supervising system of dam/river systems, control of heavy haul trains, dam/river system modelling and automation.

Xiaohua Xia obtained his PhD degree at Beijing University of Aeronautics and Astronautics, Beijing, China, in 1989. He stayed at the University of Stuttgart, Germany, as an Alexander von Humboldt fellow in May 1994 and for two years, followed by two short visits to Ecole Centrale de Nantes, France and National University of Singapore during 1996 and 1997, respectively, both as a post-doctoral fellow. He joined the University of Pretoria, South Africa, since 1998, and became a full professor in 2000. He also held a number of visiting positions, as an invited professor at IRCCYN, Nantes, France, in 2001, 2004 and 2005, as a guest professor at Huazhong University of Science and Technology, China, and as a Cheung Kong chair professor at Wuhan University, China. He is a Senior IEEE member, served as the South African IEEE Section/Control Chapter Chair. He also serves for IFAC as the vice-chair of the Technical Committee of Non-linear Systems. He has been an Associate Editor of Automatica, IEEE Transactions on Circuits and Systems II, and the Specialist Editor (Control) of the SAIEE Africa Research Journal. His research interests include: non-linear feedback control, observer design, time-delay systems, hybrid systems, modelling and control of HIV/AIDS, and control and handling of heavy-haul trains. He is supported as a leading scientist by the National Research Foundation of South Africa, and elected a fellow of the South African Academy of Engineering.