Brief paper

Speed regulation with measured output feedback in the control of heavy haul trains

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Abstract

An approach of output regulation with measurement feedback is proposed for the control of heavy haul trains. The objective is to regulate all cars’ speeds to a prescribed speed profile. The output regulation problem of nonlinear systems with measurement feedback is formulated and solved for the first time in this paper. Its application to train control is detailed. Simulation result shows the feasibility of the approach, in terms of its simplicity, cost effectiveness and its implementation convenience.

Keywords: Output regulation; Measured output feedback; Quadratic programming; Heavy haul trains; ECP braking system

1. Introduction

Trains play an important role in the transportation of mineral resources such as in South Africa. In terms of the schedule and the number of people involved, the cost is less for a larger load per car or per train. This has resulted in long heavy haul trains with multi-locomotives. The application of electronically controlled pneumatic (ECP) braking systems in the 1990s makes it possible to have very long trains stretching over 10 km. An introduction of ECP is seen in Kull (2001). This braking system was firstly rolled out on a large scale by Spoornet, a train operator in South Africa, on its COALink line.

Energy consumption, running time (speed tracking) and in-train force are of much concern to transportation corporations (Zhuan & Xia, 2007). Various studies have tried to achieve different objectives. Some studies have been done in Cheng and Howlett (1992, 1993), Howlett, Milroy, and Pudney (1994), Howlett (1996), Khmelnitsky (2000), Liu and Golovitcher (2003) to schedule a train to travel from one station to the next one in a given time with minimal energy consumption. “To schedule” means to determine a driving sequence in terms of locomotives’ power notches and wagons’ braking pressure along a specific railway track. In those papers, a train is modelled as a mass point, and the dynamics within a train are ignored. Those approaches are essentially open loop control.

The other models a train as a cascade of mass points, for example, in Yang and Sun (2001), Gruber and Bayoumi (1982), where a speed profile is assumed first, and the aims are to drive a train according to a speed profile with some objectives considered. For the study of high speed (passenger) trains in Yang and Sun (2001), Astolfi and Menini (2002), speed tracking is emphasized without considering in-train forces because in-train forces are not so important for such short trains. An early study for in-train forces can be seen in Gruber and Bayoumi (1982), where a linear quadratic regulator (LQR) approach is employed to optimize in-train forces and/or speed deviation from a reference speed with a largely simplified model. Recently, an LQR approach is employed in Chou and Xia (2007) to optimize in-train force, energy consumption and velocity tracking based on a validated model in Chou, Xia, and Kayser (2007) with the operation data from Spoornet. The methods in those papers are all within a linear system theory. The off-line scheduling
is based on some heuristic assumptions. In Rao (2005), based
on an infinite dimensional linear model, flatness based meth-
ods are used to design open loop control for a train. In Zhuan
and Xia (2006), an approach is applied to get an equilibrium
optimizing in-train force. The paper (Zhuan & Xia, 2007) inte-
grates the ideas of optimal scheduling in Zhuan and Xia (2006)
and LQR closed-loop control in Chou and Xia (2007). How-
ever, the closed-loop controller is designed based on full state
feedback, which is not practical since not all the states can be
measured.

An observer could be designed to supplement the LQR con-
troller if partial states are measured. This is, however, not the ap-
proach taken in this paper. Instead, output regulation is adopted
on the assumption that only speed measurement of locomotives
is available: while optimality is retained in open loop control,
closed-loop control is done by employing an output regulation
theory. This approach is practically feasible and is easily inte-
grable with human drivers who generally drive a train accord-
ing to the train’s speed. Instead of a linear system theory, a
nonlinear system theory is adopted such that without a linear
approximation philosophy, the control is closer to reality. An-
other advantage is the assumption that only locomotives’ speeds
are measurable.

Output regulation problem with state/error feedback in the
local version has been well studied. A seminal contribution
was made in Isidori and Byrnes (1990). Recent studies include
Huang and Chen (2004), Chen and Huang (2005) in which
general frameworks for global version have been proposed.
However, all those studies, by their nature, consider state/error
feedback, which is not practical since not all the states can be
measured.

The structure of this paper is: the output regulation problem
of nonlinear systems with measurement feedback is first formu-
lated and solved in this paper. Its application to train control is also
detailed. To make applicable the nonlinear regulator theory, the reference speed
trajectories are carefully redesigned according to the prescribed speed profile. Simulation result shows the feasibility of such
a controller in terms of its simplicity, cost-effectiveness and implementation convenience.

The state of this paper is: the output regulation problem
of nonlinear systems with measurement feedback is formulated
and solved in Section 2. A train model is briefly described in
Section 3 while Section 4 discusses the application issues of
output regulation to train control. Simulation result is shown in
Section 5.

2. Measurement feedback output regulation

\[
\dot{x} = f(x) + g(x)u + p(x)w, \\
e = h(x) + q(w), \\
y_m = h_m(x, w).
\]  

Consider a nonlinear system (1), where the first equation defines
a plant with state \( x \in X \subset \mathbb{R}^n \) and input \( u \in U \subset \mathbb{R}^m \), \( e \in \mathbb{R}^p \)
is the output to be regulated, and \( y_m \in \mathbb{R}^{pm} \) is the measured
output. The variable \( w \) is the state of the exosystem defined by

\[
\dot{w} = s(w).
\]

The mappings \( f(x), g(x), p(x), h(x), q(w), h_m(x, w), s(w) \)
am are assumed to be smooth satisfying \( f(0) = 0, h(0) + q(0) =
0, h_m(0, 0) = 0, s(0) = 0 \) and the composite system of (1) and
(2) has an equilibrium at the origin \( \text{col}(x, w) = [x^T, w^T]^T =
\text{col}(0, 0) \) with the input \( u = 0 \).

The class of controllers considered here is in the form of
measured output feedback

\[
\dot{z} = \eta(z, y_m), \\
u = k(z, y_m), \\
k(0, 0) = 0.
\]

As a result, the closed-loop system can be written as (4), of
which the first two are denoted as \( \dot{x} = f_c(x, z) \).

\[
\dot{x} = f(x) + g(x)k(z, y_m) + p(x)w, \\
\dot{z} = \eta(z, y_m), \\
\dot{w} = s(w), \\
e = h(x) + q(w), \\
y_m = h_m(x, w).
\]

The output regulation problem is to find a controller of the form
(3) such that the closed-loop system (4) satisfies

\[ P1. (\partial f_c)/(_\partial x, z) \text{ is Hurwitz at the origin } x_z = 0. \]

\[ P2. \text{ For all sufficiently small } x_{z0}, w_0, \]

\[
\lim_{t \to \infty} e(x(t), w(t)) = 0.
\]

The exosystem is assumed to be neutrally stable, i.e., \( S =
\partial x(0)/\partial w \) has all its eigenvalues on the imaginary axis. One
also has the following notations.

\[
A = \frac{\partial f(0)}{\partial x}, \\
Q_m = \frac{\partial h_m(0, 0)}{\partial w}, \\
C_m = \frac{\partial h_m(0, 0)}{\partial x}, \\
G = \frac{\partial \eta(0, 0)}{\partial y_m}, \\
F = \frac{\partial \eta(0, 0)}{\partial z}, \\
H_1 = \frac{\partial k(0, 0)}{\partial y_m}, \\
B = g(0), \\
P = p(0), \\
H = \frac{\partial k(0, 0)}{\partial z}.
\]

Theorem 1. For the nonlinear system (1), assuming the ex-
osystem is neutrally stable, \((A, B)\) is controllable and the pair of

\[
\left[ \begin{array}{c}
A \\
P \\
0 \\
S
\end{array} \right], [C_m, Q_m]
\]

is detectable, the local dynamic measurement feedback output
regulation problem is solvable if and only if there exist smooth
mappings \( x = \pi(w), u = c(w) \) with \( \pi(0) = 0, c(0) = 0 \) satisfying

\[
\frac{\partial \pi(w)}{\partial w} s(w) = f(\pi(w)) + g(\pi(w))c(w) + p(\pi(w))w, \\
0 = h(\pi(w)) + q(w).
\]

Proof. The necessity is as follows. The closed-loop system of
(4) has the form

\[
\dot{x} = (A + BH_1C_m)x + BH_2z + (P + BH_1Q_m)w + \phi(x, z, w), \\
\dot{z} = GC_m x + Fz + GQ_m w + \psi(x, z, w), \\
\dot{w} = Sw + \psi(w),
\]
The property P2 is satisfied. □

Consider the following control law,
\[ \ddot{z} = f(\pi(w)) + g(\pi(w))c(w) + p(\pi(w))w, \]
\[ \ddot{\sigma} = \eta(\sigma(w), h_m(\pi(w), w)). \]

The first equation of (5) is satisfied.

Suppose the second equation of (5) is not true at some \((\pi(w^0), w^0)\) sufficiently close to \((0, 0)\). Then, \(M = \|h(\pi(w^0)) + q(w^0)\| > 0\) and there exists a neighborhood \(V\) of \((\pi(w^0), w^0)\) such that \(\|h(\pi(w)) + q(w)\| > M/2\) at each \((\pi(w), w) \in V\).

If condition P2 holds for a trajectory starting at \((\pi(w^0), w^0)\), there exists \(T\) such that \(\|h(\pi(w)) + q(w)\| < M/2\) for all \(t > T\). However, if \((\pi(w^0), w^0)\) is Poisson stable, then for some \(t' > T\), \((\pi(w(t')), w(t')) \in V\) and this contradicts the previous inequality. So the second equation of (5) must be true, that is, P2 can hold only if this center manifold is annihilated by the error map \(e = h(x) + q(w)\).

The proof of sufficiency is constructive. In Theorem 1, assuming the equations (5) are satisfied with some \(\pi(w)\) and \(c(w)\), consider the following control law,
\[ u = \theta(z), \]
\[ \dot{z} = \eta(z, y_m), \]

with \(z = col(z_1, z_2)\) and \(\theta(z)\) and \(\eta(z)\) defined by,
\[ \theta(z) = c(z_2) + H(z_1 - \pi(z_2)), \]
\[ \eta(z, y_m) = \text{col}(\eta_1(z, y_m), \eta_2(z, y_m)), \]
\[ \eta_1(z, y_m) = f(z_1) + p(z_1)z_2 + g(z)c(z_2) + H(z_1 - \pi(z_2)) - G_1(h_m(z_1, z_2) - y_m), \]
\[ \eta_2(z, y_m) = s(z_2) - G_2(h_m(z_1, z_2) - y_m), \]
where \(H, G_1\) and \(G_2\), with the assumptions of Theorem 1, can be chosen such that \(A + BH\) and
\[ \begin{bmatrix} A - G_1C_m & P - G_1Q_m \\ -G_2C_m & S - G_2Q_m \end{bmatrix} \]

have all their eigenvalues with negative real parts, which guarantees the Jacobian matrix of (4) is Hurwitz. Thus P1 is satisfied.

Because of the continuity of \(e = h(x) + q(w)\) and the limit of \(w\), it is easy to reach
\[ \lim_{t \to \infty} e(x(t), w(t)) = \lim_{x \to \pi(w)} e(x, w) = 0. \]

The property P2 is satisfied. □

Remark 1. The above theorem is very similar to the one in Isidori and Byrnes (1990) for error feedback as well as its proof. However, the controller in Theorem 1 is in the form of measurement feedback. The measurement generally differs from the output to regulate. For example in this study of train handling, the outputs to be regulated are all the cars’ speeds while only part of the speeds can be practically measured. On the other hand, measurement feedback covers the form of error or state feedback.

Remark 2. In particular, when \(w\) is known, for example, \(y_m = col(y_m, w)\), the problem can be solved by
\[ \dot{z} = f(z) + g(z)u + p(z)w + G_1(y_m - h_m(z, w)), \]
\[ u = c(w) + H(z - \pi(w)), \]
where \(G_1, H\) are chosen such that \(A + BH\) and \(A - G_1C_m\) are Hurwitz. \((C_m^1 = \dot{c}y_m(0, 0)/\dot{c}x.)\)

Remark 3. The parameters \(K, G\) can be determined with different kinds of methods, such as with pole placement. Although \(K, G\) are chosen with a linear system theory, the output regulation problem is solved with them.

3. Train model

A heavy haul train is composed of \(k\) locomotives and \(n - k\) wagons (both referred to as cars). The longitudinal dynamics of a train can be modeled as
\[ m_i\dot{v}_i = u_i + f_{ini} - f_{fin_i} - f_{fin}, \]
\[ x_j = v_j - v_{j+1}, \]

where \(m_i\) is the \(i\)th car’s mass, \(v_i, u_i\) are the speed and the effort of the \(i\)th car. The variable \(f_{ini}\) is the force undertaken by the \(i\)th car from the environment. \(f_{aero} = m_i c_0 + m_i c_1 v_i + m_i c_2 v_i^2\), where \(c_0\), \(c_1\), and \(c_2\), are constants determined by experiments. The variable \(f_{fin}\), the gravity force in longitudinal direction, and \(f_{fin}\) are references determined by experiments. The variable \(f_{fin}\) is the in-train force between the \(j\)th and \((j + 1)\)th cars, which is a function of \(x_j\), the relative displacement between the two neighboring cars.

A more detailed description of the model is referred to Chou and Xia (2007) and Zhuan and Xia (2006).

4. Output regulation of heavy haul trains

In this paper, the 2–2 control strategy (Zhuan & Xia, 2006), an ECP/idP-only strategy is assumed, i.e., all the inputs may be different. For (9), some changes are required to be done for the application of output regulation. On the one hand, the origin is not an equilibrium. On the other hand, there are a lot of trajectories to regulate cars’ speeds to the reference speed. However, for train handling, the choice of the trajectories involves the balance between energy consumption and in-train forces. In this paper, optimal scheduling in Zhuan and Xia (2007) is employed. With this optimal scheduling, the equilibrium can be denoted as \(f_{in}^0(x_j^0, y_j^0, u_j^0)\), which are in-train forces (coupler displacements), speeds (reference speed) and efforts.
of cars. The train model is rewritten as:
\[
\begin{align*}
\dot{v}_i &= (\delta u_i + \delta f_i_{n+1}) - \delta f_i_{m+1}, \\
\dot{x}_j &= \delta v_j - \delta v_{j+1}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n - 1,
\end{align*}
\] (10)
where \(\delta v_i = v_i - v_i^0\), \(\delta u_i = u_i - u_i^0\), \(\delta f_i_{m+1} = f_i_{m+1} - f_i_{m+1}^0\), \(\delta x_j = x_j - x_j^0\). With \(X = [\delta v_1, \ldots, \delta v_n, \delta x_1, \ldots, \delta x_{n-1}]^T\) and \(U = [\delta u_1, \ldots, \delta u_n]^T\), the system (10) is also denoted in the form
\[
\dot{X} = f(X) + g(X)U.
\] (11)

The outputs to be regulated are the cars’ speeds, i.e., assuming the reference speed is \(w_1\), \(v_i = v_i - w_1 = X_i + v_i - w_1\). The measurement is part of the cars’ speeds, i.e., \(y_m = C_m(X + v_r)\), where \(C_m = (C_{ij})_{n \times (2n-1)}\) and all the entries of the row vectors of \(C_m\) are zeros except one of the first \(n\) ones which is one. For example,
\[
C_m = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \end{bmatrix}
\]
if only the first two cars’ speeds are measured. Notice that the measurement is different from the error output.

The linearized system of (11) has system matrices
\[
A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} \text{diag} \left( \frac{1}{m_1}, \ldots, \frac{1}{m_n} \right) \end{bmatrix},
\]
\[
A_{11} = -\text{diag}(c_{11}, c_{21}, v_r, \ldots, c_{1n}, c_{2n}, v_r),
\]
\[
A_{12} = \begin{bmatrix} -k_1 & 0 & 0 & 0 \\ -m_2 & -k_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -k_n \end{bmatrix},
\]
\[
A_{21} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}, \quad A_{22} = 0_{(n-1) \times (n-1)}.
\]

It can be verified that \((A, B)\) is controllable and \((A, C_m)\) is observable with the PBO criterion in Kailath (1980).

\[
[\lambda I - A | B] = \begin{bmatrix} \lambda I_{n \times n} - A_{11} & -A_{12} \text{diag}(1/m_1) \\ A_{21} & 0_{n \times (n-1)} \end{bmatrix},
\]
from which one can get rank \((\lambda I - A | B) = 2n - 1\). If the first or last car’s speed is measured, the pair \((A, C_m)\) is observable. With the first car’s speed available, one has
\[
\Gamma = \begin{bmatrix} A - \lambda I \\ C_m \end{bmatrix} = \begin{bmatrix} A_{11} & -I_{n \times n} \\ A_{21} & 0_{n \times (n-1)} \end{bmatrix},
\]
from which one knows that rank \((\Gamma) = 2n - 1\), and the pair \((A, C_m)\) is observable according to the PBO criterion. Actually the first car of a train is usually a locomotive, whose speed is available. So the above assumption does not lose generality.

The speed maintenance phase, speed acceleration and speed deceleration phases are discussed in this paper. The cars’ speeds are the subject of regulation. According to the reference speed profile, the trajectory of the reference speed can be redesigned as \(w_1\) of
\[
\dot{w}_1 = aw_2, \quad \dot{w}_2 = -aw_1 - w_3, \quad w_3 = 0,
\] (12)
whose solution is \(w_1 = (w_3(0) + A \sin(\alpha t + \phi_0))\), \(w_2 = A \cos(\alpha t + \phi_0)\), \(w_3 = w_3(0)\), where \(A\) and \(\phi_0\) are determined by the initial conditions \((w_1(0), w_2(0), w_3(0))\). Within the cruise phase, the initial conditions are chosen as \((w_1(0), w_2(0), w_3(0)) = (v_r, 0, 0)\), where \(v_r\) is the cruise speed. Assuming the reference speed before acceleration is \(v_r\) and the reference speed after acceleration is \(v_r\), then the initial conditions are chosen such that \(w_3(0) = v_r, \phi_0 = 0, A = \sqrt{2(v_r - v_r)}\). The variable \(a\) in (12) is chosen with considering the acceleration limit \(a_i\) or deceleration limit \(a_c\) of the train, which is determined by the effort capacity of the train. In simulation, \(a = a_i / A\) within an acceleration phase and \(a = a_c / A\) within a deceleration phase. For example, one chooses \(a_i = 0.07 \text{ m/s}^2, a_c = -0.2 \text{ m/s}^2, \phi_0 = 0\), and the time interval \(T_1 = \pi/4a\) as acceleration/deceleration phase. The modified speed file according to the speed profile is shown in Fig. 1.


\[
\begin{align*}
\text{Fig. 1. Modified speed profile.}
\end{align*}
\]
In simulation, $K$, $G$ is chosen with a linear quadratic algorithm (Chou & Xia, 2007). These choices of $K$ and $G$ are consistent with Remark 3.

Since the throttle of the locomotives takes discrete values and the braking capacities of the wagons are constrained, the input $u$ in the complete controller may violate these constraints. When this happens, an anti-windup technique is employed in simulation. For wagons, the application of the anti-windup technique is very simple. For locomotives, inputs are discrete with some operation constraints. Similar methods as described in Slotine and Li (1991) are used to smooth continuous control inputs. For the speed tracking, the optimal controller with state feedback is formulated and solved for the first time, extending control regulation problem of nonlinear systems with measurement feedback, and for in-train forces, the former is a little better than the output regulating controller with measurement feedback, no matter what the optimal parameters are. This is because for the state feedback controllers are sensitive to the state deviation from the equilibrium and the energy optimization is local, the locomotives’ traction efforts and the cars’ braking change more frequently, which leads to consume more energy.

For the speed tracking, the optimal controller with state feedback is a little better than the optimal regulating controller with measurement feedback, and for in-train forces, the former is worse than the latter.

### 6. Conclusion

An approach of output regulation with measurement feedback is proposed for the control of heavy haul trains. The output regulation problem of nonlinear systems with measurement feedback is formulated and solved for the first time, extending
the existing results. Based on a cascade-mass-point model, the application of output regulation to train control is detailed. The conditions of the application are verified. Optimal scheduling is integrated in the controller of output regulation. Simulation shows the feasibility of the output regulating controller with only measurement of the locomotive speeds, in terms of its simplicity, cost-effectiveness and its implementation convenience.

References


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