# Cruise Control Scheduling of Heavy Haul Trains

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Abstract—An optimal offline scheduling of the cruise control of heavy haul trains is introduced. The objective of the scheduling is to minimize the in-train forces, which are the most important for the safety driving and maintenance cost of excessively long trains. The constraints of throttling and braking are taken into consideration. With the gradual progresses of implementing electronically controlled pneumatic (ECP) braking systems in practice, three control strategies are proposed. Simulation results of these strategies on different types of trains are given. The ECP braking systems demonstrate superb performance compared with pneumatic braking systems.

*Index Terms*—Electronically controlled pneumatic (ECP) brake system, heavy haul trian, linear-quadratic control, optimal control.

## I. INTRODUCTION

**R** AILWAY is believed to be the most economical among all transportation means, especially for the transportation of mineral resources. In South Africa, most of mines are situated inland, so heavy haul trains are required to transport these resources to harbours. It is presumed that the cost is less with larger quantity of load per car or per train concerning the schedule and the number of people involved. This has resulted in long trains with multilocomotives.

Traditionally, the operation of such multilocomotive trains with pneumatic braking system is in essence a simple one. The brake control signal is transmitted throughout the train wagons, which results in the same effort command of all the wagons. All the locomotives also have the same efforts, for the remote locomotives (groups) are operated in tandem with the leading one. In this operation there are only two control signals, one for locomotives, and the other one for wagons. There are some drawbacks with the operation method as follows:

- locomotives are distributed, but the power is not distributed independently;
- 2) wagons' brake system is pneumatic and the braking control signal is propagated to each wagon through the air pressure change in the air pipe running throughout the train, which leads to different time delays in braking the wagons.

These drawbacks result in slow running speed, the possibility of derailment, and the limit on the train length. To improve the train performance, the Association of American Railroads (AAR) developed a new brake system—the electronically controlled pneumatic (ECP) brake system, in which the brake command signals are electronic and are received by all the wagons

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simultaneously although the pneumatics is still used as the brake power. Spoornet, one of the train operators in South Africa, is the first railway in the world to roll-out the ECP brake system (on its COALink line) on a large scale. Operation advantages follow the application of ECP brake systems [1].

- 1) Wheel and brake shoe wear can be reduced with the appropriate distribution of braking and pressure control.
- 2) Energy-efficient operation can be reached with the use of graduated release capability to eliminate power braking.
- 3) The safety level can be increased with the accurate control of the whole train and decreased stopping distance.
- 4) The in-train forces can be reduced owing to the complete brake control of every car of the train.

It is exactly due to these advantages that extremely long trains (up to 10 km in length) are considered in the business plan of Spoornet of South Africa on its COALink. This increase in train length has posed unprecedented technical challenges.

According to [2], train handling includes the start phase of train, speed maintenance phase, and stop phase of train. Since the railway track is long and the train is running in the speed maintenance phase during most of the running time, the train scheduling of the speed maintenance phase is the focus of the study of the paper. In realizing an optimal management of in-train forces, it is justifiable to assume that a steady state of the train motion is reached and held. In this paper, we borrow the term "scheduling" from the railway industry for train operation and handling, where it refers to the decision of a driving sequence in terms of locomotives' power notches and wagons' braking pressure along a specific railway track. In the context of control systems, this "scheduling" activity is interpreted as an open-loop control design which brings the train to an expected motion trajectory.

For heavy haul trains, energy consumption, running time, and in-train forces between the neighboring cars are of much concern to transportation corporations. The energy consumption is related to the direct economic profit while the running time determines the quality of the service. The in-train force control is attributed to the safe running of the train and limiting the maintenance cost. The larger the in-train force is, the higher the maintenance cost. For long trains, the latter is even more important. It is also more difficult to control the in-train forces of a long heavy haul train.

For the specific problems of train handling, some studies have been tried out. Basically, there are two types of train models. One is to take the whole train as a mass point [3], [4]. In [3], the optimal objective is to minimize the fuel (energy) consumption with the train travelling a given distance in a given time period. The locomotives are supposed to have three discrete control settings: power, coast, and brake. The finite sequence of the locomotive settings is predetermined, and then the problem is to find the switching points where the control setting will be

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changed. The key equations are used to decide switching track points. In [4], with the similar model as in [3], the maximum principle is applied to decide the sequence of optimal controls and switching speed points at which the control setting will be changed. The model in these two papers is enough to consider minimizing the fuel (energy) consumption, but not to consider the in-train forces. The other model is to take a train as cascade mass points connected with nonlinear couplers [5]-[9]. In [5], a linear quadratic regulation (LQR) optimal algorithm is employed to minimize the coupler forces and/or velocity deviations from the reference values. It is assumed that at the nominal point, the nominal input vector consisting of throttling and braking forces to maintain the nominal speed, is equal to the sum of the resistance and gravity forces. Then a linearized model is used to calculate the control law. Considering the large number and the constraints of the variables, the train model is simplified. This paper offers an excellent setup to deal with the in-train forces and various calculations of optimal closed loop control. While the closed loop control is to optimize the interplay between in-train forces and speed holding, the scheduling of the desired holding speed is typically determined through an open loop controller design. It is noted however, the offline open-loop scheduling in [5] is a rather heuristic one, without optimally making use of the control redundancies.

It is quite interesting to note that the early study of [5] takes into consideration of the practical aspects of ECP and independent distributed power (iDP) even though the ECP/iDP technology is not implemented in practice on a visible scale. Some later studies, without mentioning the practical constraints of ECP/iDP, investigated the application of some of the advanced control techniques to high speed passenger trains. In [6] and [7], the  $H_2/H_\infty$  method is employed to deal with the cruising of high speed trains. The objective is to maintain the train speed as expected. The cruise control is proposed for two types of highspeed trains, the distributed driving with each car having its own driving force, and the push-pull driving only with driving forces at the first and the last car. A calculation method of equilibrium point for distributed driving is given. Even though the push-pull driving is also taken as a way to operate heavy haul trains, again, this paper does not present an optimal scheduling of equilibrium points. In [8], similar to [6], different input/output decoupling problems for high speed trains are studied. To get the equilibrium point, it is assumed that one of the in-train forces is zero or the driving force is averagely distributed to the locomotives. This assumption leads to a heuristic trim point as obtained in [5], too. In [9], with a cascade mass point model of the heavy haul train validated in [10], an LQR approach is employed to optimize the in-train forces, energy-consumption, and velocity tracking. In offline scheduling, the trim point is calculated under the assumption that the driving force is averagely distributed to the locomotives while all the braking forces are zeros.

For the heavy haul trains, a natural choice of model is a model of cascade mass points connected with nonlinear couplers. It is shown in this paper that there are redundancies in designing an open-loop controller. An optimization procedure can be applied to schedule a cruise control by taking the in-train forces into initial design consideration. Hopefully, an optimal open-loop controller design presents a better starting point for a closed-loop



Fig. 1. Longitudinal model of car.

controller design. To demonstrate the open-loop control design, the throttling and braking are constrained and three different operation strategies of heavy haul trains are distinguished.

In this paper, the train model is set up first. Then, three train handling types are proposed from the control point of view. An optimal control algorithm is derived for one case and the other two are similar. At last the simulation results are given.

# II. TRAIN MODEL

A heavy haul train, composed of locomotives and wagons (both referred to as cars), can be modeled as cascade mass points connected with couplers. In the following model, only longitudinal dynamics of the train is analyzed.

# A. Car Model

A car is running on the track while it is subjected to the aerodynamic force, the adjoining cars' internal-forces, the gravity force and its own traction or brake force. The forces experienced by a car in the longitudinal direction are shown in Fig. 1.

The aerodynamics of train can be divided into two parts [11], [12]: mechanical drag and aerodynamic drag. The former includes the sliding forces between the train's wheels and the track, and the rolling forces of wheels. The aerodynamic drag is dependent on the cross-sectional area of train body, train length, shape of train fore- and after-bodies, surface roughness of the train body, and geographical conditions around the proceeding train.

It has been reasonably assumed that the aerodynamic drag is proportional to the square of the speed, while the mechanical drag is proportional to the speed. Compared with the mechanical drag, the portion of the aerodynamic drag becomes larger as the train speed and length increase (see details from [11] and [12]).

In the open air without any crosswind effects, the total drag on a traveling car can be expressed by a sum of the aerodynamic and mechanical ones

$$f_{\text{aero}} = D_M + D_A = mc_0 + mc_1v + mc_2v^2 \qquad (1)$$

where  $D_A$  and  $D_M$  are the aerodynamic and mechanical drags, respectively,  $c_0$ ,  $c_1$ , and  $c_2$  are constants determined by experiments, v is the car speed, and m is the car mass under discussion.

The variables  $f_{in_i}$  and  $f_{in_{i+1}}$  are the in-train forces between the neighboring cars. Only one in-train force is experienced by the front and rear car. The variable  $u_i$  is the car's traction or brake force. For a wagon, it refers to the brake force which must be no more than zero while for a locomotive it refers to traction force or brake force whose quantity depends on the locomotive's power notch and speed, dynamic brake capacity. The dynamic brake power is also called regenerative brake power, which can be fed back to the system and saved.

In Fig. 1, the resistance force related to the position  $f_p = f_g + f_c$  is composed of the gravity force  $f_g = mg\sin\theta \approx mg\theta$ 



Fig. 2. Longitudinal model of coupler.



Fig. 3. Coupler force versus displacement.



Fig. 4. Longitudinal model of train.

in longitudinal direction and the curvature resistance force  $f_c$  [13]. In the following simulation study, the curvature resistance force is ignored.

## B. Coupler Model

The coupler between two cars is modeled as Fig. 2. When the draft gear is in its natural length, the in-train force is zero. Considering the coupler's slack length, the coupler can be regarded as a composition of the two gears plus the slack length. Assuming the sum of length of two gears is  $L_0 - (1/2)L_{\text{slack}}$ while the in-train force is zero, the displacement of the coupler is defined as  $x = L - L_0$  in which L is coupler length. The variable  $f_{\text{in}_i}$  is the in-train force between the *i*th and (i + 1)th cars, which is a function of  $x_i$ , the relative displacement between the two neighboring cars, and the difference of the neighboring cars' velocities (damping effect). A typical relationship between the static in-train force  $f_{\text{in}}$  (without damping) and x is depicted in Fig. 3, which is simplified from the data of Spoornet.

# C. Train Model

Fig. 4 is a sketch of the longitudinal motion of a train. Assuming the train consists of n cars and the locomotives are located at positions  $l_i$ , i = 1, 2, ..., k, where k is the number of locomotives. The train model is described by following equations:

$$m_s \dot{v}_s = u_s + f_{in_{s-1}} - f_{in_s} - f_{a_s}, \quad s = 1, 2, \dots, n$$
 (2)

$$\dot{x}_s = v_s - v_{s+1}, \quad s = 1, 2, \dots, n-1$$
 (3)

where the variable  $m_i$  is the *i*th car's mass; the variable  $v_i$  is the speed of the *i*th car; the variables  $f_{a_i} = f_{aero_i} + f_{p_i}$ , i = 1, 2, ..., n; the variable  $f_{aero_i} = m_i (c_{0_i} + c_{1_i}v_i + c_{2_i}v_i^2)$  are the cars' aerodynamic force; the variable  $f_{p_i} = f_{g_i} + f_{c_i}$  is the force due to the tracking slope and curvature where the *i*th car is running; and the variable  $f_{in_i}$  is the in-train force between the *i*th and (i + 1)th cars. In (2), one has  $f_{in_0} = 0$ ,  $f_{in_n} = 0$ .

#### **III. CONTROL STRATEGIES**

A traditional heavy haul train with the pneumatic controlled braking system is controlled by drivers in the leading locomotive. Single air pipe connects throughout the whole train, responsible for supplying pressure to the braking system in each wagon as well as transmitting the braking control. The driver controls the leading locomotive effort while other locomotives efforts follow the effort of the leading one. Because of the pressure wave propagation speed, the front wagons are responsible for most of the braking due to the signal propagation delay and the pressure gradient. From a control point of view, there are only two control signals in this kind of strategy, one for locomotive effort and the other one for wagon brake.

When the locomotives efforts are controlled independently and separately, it is referred to as multipowered [5] or distributed powered. In this strategy, every locomotive or every locomotive consist (some locomotives connected with rigid drawbar) has an independent control signal.

While the train is equipped with an ECP braking system, the braking control signal is transmitted electronically. There is nearly no time delay for the braking signal transmission. When the above two control strategies are implemented with an ECP system, the braking signals are not delayed.

An ECP braking system adds a new dimension to control strategy: it allows individual wagon braking. So in a fully ECP/iDP mode, every car, including locomotives and wagons, has its own independent control signal.

Summarizing the above, there are three major types of control discussed in this paper.

## A. 1-1 Strategy

In this strategy, there is one control signal for all the locomotives and one braking control signal for all the wagons. Without ECP, there are time delays for the braking control signals, and with ECP, there are no time delays.

## B. 2-1 Strategy

In this strategy, the control signal of every locomotive effort may be different, and the braking control signals of all the wagons are identical. This is an iDP-only strategy. Without ECP, there are time delays for the braking control signals, and with ECP, there are no time delays.

# C. 2-2 Strategy

This control strategy can only be used in the trains equipped with ECP brake systems. There is an independent control command for every car, including locomotives and wagons. This is also a fully ECP/iDP mode. There are no time delays for all the control signals.

TABLE I ACCELERATION PROFILE

distance		1,000	$s_1$	5,000	$s_2$	
a	0	$a_r$	0	$-a_r$	0	

#### IV. FORMULATION OF THE OPTIMAL PROBLEM

# A. Transient Control

In the following open-loop scheduling, the velocity accelerations and decelerations are considered. An acceleration profile is calculated according to the velocity profile with a parameter, the acceleration limit,  $a_r$ . For example, at the travel distance 1000 m, the reference velocity is changed from 12 to 15 m/s and at the distance 5000 m, it is changed to 10 m/s, then the acceleration profile is as in Table I, where  $s_1, s_2$  are calculated as  $s_1 =$  $1,000 + (15^2 - 12^2)/(2a_r), s_2 = 5,000 + (15^2 - 10^2)/(2a_r)$ . Thus, from the point 1000 m to the point  $s_1$  and from the point 5000 m to the point  $s_2$ , the open-loop scheduling should maintain the accelerations.

## B. Performance of Optimal Control

The objectives of a train control project are that with the optimal control: 1) the train can travel a given distance within a given period; 2) the energy consumption is reduced; and 3) the range of in-train forces is in the admission range of the train couplers. At the equilibrium point, where the speeds of the cars and the displacements of couplers are constant, that is,  $\dot{v}_i = 0$ ,  $\dot{x}_j = 0$ , i = 1, 2, ..., n, j = 1, 2, ..., n - 1, the energy consumption of all control strategies is nearly equal, for most of the energy is to conquer the resistance of drag forces which is determined by the speed profile, track profile and the train. So the second objective can be ignored in scheduling the open loop controller. The first objective is more related to the speed profile and speed holding. In scheduling the open-loop controller, it is assumed that the desired speed is reached and held. In this paper, the objective, therefore, is taken as

$$J = \sum_{i=0}^{n-1} f_{\text{in}_i}^2$$
(4)

where n is the number of cars in the train. That is, the purpose of the scheduling is to optimize the in-train forces.

In the following analysis, the train is assumed to consist of n cars, in which there are k locomotives. The cars are numbered from the front to the end with 1 to n. The locomotives' number is from  $l_1$  to  $l_k$ .

## C. Constraints of the Optimal Problem

For the open loop control, the dynamic process in the train is ignored and the reference velocity is reached or the acceleration is maintained, that is

$$\frac{dv_i}{dt} = a, \quad i = 1, 2, \dots, n$$
$$\frac{dx_j}{dt} = 0, \quad j = 1, 2, \dots, n-1$$
(5)

where a is the acceleration, which is zero when the train is cruising and is  $a_r(-a_r)$  when the train is running within a scheduled acceleration (deceleration) period.

Applying (5) to (2) and (3), one has

$$u_s + f_{in_{s-1}} - f_{in_s} - f_{a_s} - m_s a = 0, \quad s = 1, 2, \dots, n.$$
(6)

From the first (n-1) of (6), the in-train forces can be calculated as

$$f_{in_s} = \sum_{i=1}^{s} u_i - \sum_{i=1}^{s} (f_{a_i} + m_i a), \quad s = 1, 2, \dots, n-1.$$
(7)

From the last equation of (6), one has

$$\sum_{i=1}^{n} u_i - \sum_{i=1}^{n} (f_{a_i} + m_i a) = 0.$$
 (8)

In train operations, the inputs and the in-train forces have some constraints

$$\underline{U}_i \le u_i \le \overline{U}_i, \quad i = 1, 2, \dots, n$$
  

$$\underline{F}_{in_j} \le f_{in_j} \le \overline{F}_{in_j}, \quad j = 1, 2, \dots, n-1$$
(9)

where  $\underline{U}_i$ ,  $\overline{U}_i$  are the up constraint and low constraint for the *i*th input, and  $\underline{F}_{in_j}$ ,  $\overline{F}_{in_j}$  are the up and low constraints for the *j*th in-train force, respectively. For a wagon,  $\overline{U}_i = 0$  and the value of  $\underline{U}_i$  depends on the capacity of the wagon's brake. For a locomotive, the constraints  $\underline{U}_i$ ,  $\overline{U}_i$  depend on the locomotive's capacity of traction effort. The notch should be changed step by step, and every notch should be kept for longer than a fixed time interval before it is changed. The constraints  $\underline{F}_{in_j}$ ,  $\overline{F}_{in_j}$  are limited because of the requirement of safety operation and limiting maintenance cost.

Thus, the open-loop scheduling is an optimization problem of the objective function (4) with the equality constraints (6) and inequality constraints (9).

# V. OPTIMIZATION ALGORITHM

#### A. 1-1 Strategy

With this strategy, all the locomotives equally share the drag forces and the brake forces of all the wagons are equal. This imposes additional constraints to the optimization problem.

$$u_{l_1} = u_{l_2} = \dots = u_{l_{m-1}} = \dots = u_{l_k} \stackrel{\scriptscriptstyle \Delta}{=} u_t$$
$$u_i \stackrel{\scriptscriptstyle \Delta}{=} u_b, \quad i = 1, \dots, n; \quad i \neq l_j, \quad j = 1, \dots, k.$$
(10)

It is distinguished between two cases.

1) The last locomotive is not at the rear of the train. In this case,  $l_k < n$ . Combining (10) with (7), one has

$$\int iu_b - \sum_{j=1}^i (f_{a_j} + m_j a), \qquad 1 \le i < l_1$$

$$f_{in_i} = \begin{cases} (i-1)u_b + u_t - \sum_{j=1}^{i} (f_{a_j} + m_j a), & l_1 \le i < l_2 \\ \dots \\ (i-k)u_b + ku_t - \sum_{j=1}^{i} (f_{a_j} + m_j a), & l_k \le i < n \end{cases}$$

and

$$ku_t + (n-k)u_b = \sum_{i=1}^n (f_{a_i} + m_i a).$$

The objective function is rewritten as

$$J = \sum_{i=1}^{n-1} f_{in_i}^2$$
  
=  $\sum_{i=1}^{l_1-1} \left( iu_b - \sum_{j=1}^i (f_{a_j} + m_j a) \right)^2$   
+  $\dots + \sum_{i=l_{k-1}}^{l_k-1} \left( (i - k + 1)u_b + (k - 1)u_t - \sum_{j=1}^i (f_{a_j} + m_j a) \right)^2$   
+  $\sum_{i=l_k}^{n-1} \left( (i - k)u_b + ku_t - \sum_{j=1}^i (f_{a_j} + m_j a) \right)^2$ .

The optimization with equality and inequality constraints can be solved with the Lagrange multiplier approach [14]. The equality constraint can be taken care of with the following extended objective function with a Lagrange multiplier:

$$\bar{J} = J + 2\lambda \left( ku_t + (n-k)u_b - \sum_{j=1}^n (f_{a_j} + m_j a) \right).$$
(11)

First, one calculates

$$\frac{1}{2}\frac{\partial J}{\partial u_t} = \sum_{j=1}^k \left( \sum_{i=l_j}^{l_{j+1}-1} jQ_j \right)$$
$$\frac{1}{2}\frac{\partial J}{\partial u_b} = \sum_{j=0}^k \left( \sum_{i=l_j}^{l_{j+1}-1} (i-j)Q_j \right)$$
(12)

where  $Q_s = (i-s)u_b + su_t - \sum_{j=1}^{i} (f_{a_j} + m_j a), s = 0, \dots, k,$  $l_0 = 1, l_{k+1} = n$  and, denotes them as

$$\frac{1}{2}\frac{\partial J}{\partial u_t} = T_b u_b + T_t u_t + \sum_{i=1}^{n-1} T_i (f_{a_i} + m_i a)$$
$$\frac{1}{2}\frac{\partial J}{\partial u_b} = B_b u_b + B_t u_t + \sum_{i=1}^{n-1} B_i (f_{a_i} + m_i a).$$
(13)

The necessary condition for extremality of  $\overline{J}$  is

$$\frac{1}{2}\frac{\partial \bar{J}}{\partial u_t} = \frac{1}{2}\frac{\partial J}{\partial u_t} + \lambda k = 0$$
$$\frac{1}{2}\frac{\partial \bar{J}}{\partial u_b} = \frac{1}{2}\frac{\partial J}{\partial u_b} + \lambda(n-k) = 0$$
$$ku_t + (n-k)u_b - \sum_{i=1}^n (f_{a_i} + m_i a) = 0.$$
(14)

From them, one can get the following equations:

$$P_{bb}u_{b} + P_{tt}u_{t} = \sum_{i=1}^{n-1} (kB_{i} - (n-k)T_{i})F_{i}$$

$$P_{bb} = (n-k)T_{b} - kB_{b}$$

$$P_{tt} = (n-k)T_{t} - kB_{t}$$

$$(n-k)u_{b} + ku_{t} = \sum_{i=1}^{n}F_{i}$$
(15)

where  $F_i = f_{a_i} + m_i a$  and from which one can get the solutions of  $u_b, u_t$ . Applying this solution to (9), if no constraint is violated, this solution is the optimal value. If some constraints are violated, then one takes these inequality constraints as equality constraints, and resolves the optimization problem. For example, to minimize J(x) subject to  $f(y) \leq 0$ , where f, J, and x are vectors of different dimensions. Suppose that x has p components and that n components of the inequality constraint are violated, that is,  $f_i(x) > 0, i = 1, 2, \ldots, n$ . The other constraints,  $f_i(x) \leq 0, i = n + 1, \ldots$ , may be disregarded. Define a new function,  $\overline{J} = J + \lambda^T F$ , where  $\lambda^T = [\lambda_1 \cdots \lambda_n], F = [f_1(x) \cdots f_n(x)]^T$  to replace J. Solving this minimization problem, one can get a new solution which is more admissible. The above process is repeated if necessary. This procedure of solving a constrained optimization problem is described in detail in [14].

2) The last locomotive is at the rear of the train. In this case,  $l_k = n$ .

In the above calculation  $(l_k < n)$ , one could consider  $l_{k+1} = n$ . So one can replace k in the above case with k - 1 in this case; also see the equation at the bottom of the next page. One can get similar results.

For instance,  $n = 52, l_1 = 1, l_2 = 52$ , then

$$u_{t} = \frac{1}{2262} \sum_{i=1}^{52} \frac{-3888 - 25i + 5i^{2}}{2} (f_{a_{i}} + m_{i}a)$$
$$u_{b} = \frac{1}{2262} \sum_{i=1}^{52} \frac{1230 + 5i - i^{2}}{10} (f_{a_{i}} + m_{i}a).$$
(16)

The mathematic developments for other two control strategies are similar to the above one.

#### VI. SIMULATION

In simulation, one assumes that the train consists of 200 wagons. Every four wagons (a rake) are linked with rigid drawbars of which the in-train forces are not considered and regarded as one unit. There are two locomotives at the front and two at the rear, respectively. The neighboring locomotives are linked with rigid drawbars and regarded as one unit too. So the train can be regarded as consisting of 50 wagons between two locomotives.

The parameters of the train are given in Tables II and III [9].

In the tables,  $F_b$  represents the capacity of brake force, and L is the longitudinal length of a locomotive or wagon group. Fig. 5 shows the locomotive (group) effort (7E1) corresponding to a particular notch level and velocity. These data, including the track profile are based on the COALink trains operated in South

m(ton)	$c_0(m/s^2)$	$c_1(1/s)$	$c_2(1/m)$	<i>L</i> (m)
252	7.6685e-3	1.08e-4	2.06e-5	40.94

TABLE II Locomotive Group Parameters

TABLE III
WACON GROUD DADAMETERS

m(ton)	$c_0(m/s^2)$	$c_1(1/s)$	$c_2(1/m)$	L(m)	$F_b(KN)$
417	6.3625e-3	1.08e-4	1.492e-5	48.28	720



Fig. 5. Locomotive efforts of different notches versus velocity.

Africa by Spoornet. The relation between the displacement and the static force (without damping) of the coupler is shown in Fig. 3. Since the damping coefficient is not available and can be as high as 1/34 of the spring coefficient according to [5], it is taken as 1/100 of the spring coefficient in the train model, and ignored in the control design.

In (9),  $\overline{F_{in_i}} = -\underline{F_{in_i}} = 1600$  KN. There are some constraints with the locomotive notch operation. First, the notch could only be changed stepwise; second, the locomotive engine should stay at a notch for at least 10 s, and when the locomotive's effort changes from traction to dynamic braking or the other way, the first notch should last at least 20 s. The acceleration limit  $a_r$  is  $0.07 \text{ m/s}^2$ . The reference velocity is 36 km/h from the simulation starting point -2 km to 3 km and then it is 43.2 km/h. At the

point 6 km, it is changed to 54 km/h, while it is changed to 43.2 km/h again at the point 8 km. Some distances are negative because the reference point is chosen in the middle of the track and the distance values are relative.

The initial state is that the train is in its steady state with all the cars' velocities 10.5 m/s and all the in-train forces zeros. For a traditional train, the time delay for a wagon's braking force is calculated with the wagon's distance to the first locomotives divided by the velocity of sound.

The simulation is proceeded with MATLAB. The train model is running continuously and the control signal is updated every second. Simulation results are shown in the following figures.

Figs. 6 and 7 show the applications to traditional heavy haul trains (with pneumatic brake system) of 1-1 strategy controller and 2-1 strategy controller, respectively. The train is not equipped with an ECP brake system and, therefore, there are time delays for the wagons' control signal transmission.

The applications to a heavy haul train with an ECP brake system installed of 1-1 strategy controller, 2-1 strategy controller and 2-2 strategy controller are shown in Figs. 8–10, respectively. The control inputs in Fig. 8 are the same as in Fig. 6, and these in Fig. 9 are the same as in Fig. 7. However, because of the installation of an ECP brake system, there is no time delay for the wagon control signal transmission.

In these figures, the first subplot is the front locomotive group speed, rear locomotive group speed, and the mean speed of all the cars with respect to the distance from the starting point. The second subplot is maximum, minimum in-train forces and the mean value of the absolute values of all the in-train forces in a specific time with respect to the distance. The third one is the energy consumption, where Energy is calculated with the dynamic braking power lost and Energy1 is calculated with the dynamic braking power fed back to the power system and saved. The fourth and fifth subplots are the notches and efforts of the front and rear locomotive groups. The sixth is the steady in-train forces, which are calculated with applying the efforts of the cars into the train model with the reference speed (and the acceleration) maintained and the dynamic process ignored. The seventh of Fig. 6 is the track profile. All the simulations are proceeded on this track profile and is omitted in the subsequence figures.

The mean value of the absolute value of the difference between the reference velocity and the mean value of all the cars' velocities in Figs. 6–10 are, respectively, 0.9179, 0.7032,

$$f_{\text{in}_{i}} = \begin{cases} iu_{b} - \sum_{j=1}^{i} (f_{a_{j}} + m_{j}a), & 1 \le i < l_{1} \\ (i-1)u_{b} + u_{t} - \sum_{j=1}^{i} (f_{a_{j}} + m_{j}a), & l_{1} \le i < l_{2} \\ \dots \\ (i-k+1)u_{b} + (k-1)u_{t} - \sum_{j=1}^{i} (f_{a_{j}} + m_{j}a), & l_{k-1} \le i < m \end{cases}$$

$$ku_{t} + (n-k)u_{b} = \sum_{i=1}^{n} (f_{a_{i}} + m_{i}a).$$



Fig. 6. 1-1 strategy without ECP.



Fig. 7. 2-1 strategy without ECP.



Fig. 8. 1-1 strategy with ECP.



Fig. 9. 2-1 strategy with ECP.

0.9650, 0.7401, 0.6510 m/s, while the mean value of the absolute values of all the couplers' in-train forces are 173.1741, 151.1971, 146.7035, 113.5059, 111.1791 KN, respectively.

When comparing Fig. 6 with Fig. 7, it can be seen the locomotive speed error is smaller in the 2-1 strategy than in the 1-1 strategy. The absolute values of the maximum and the minimum in-train forces are smaller with the 2-1 strategy when it comes to steadily running. However, the energy consumption with the 1-1 strategy is a little less than with the 2-1 strategy. This is because some energy is used to overcome the in-train forces fluctuation and larger brake forces are applied. The same result can be seen when comparing Fig. 8 with Fig. 9.

When comparing Fig. 9 with Fig. 10, the locomotive speed fluctuation and error with the 2-2 strategy are smaller than those with the 2-1 strategy. The absolute values of maximum and minimum in-train forces with the 2-2 strategy are also a little smaller



Fig. 10. 2-2 strategy with ECP/iDP.

than with the 2-1 strategy. The energy consumption with the 2-1 strategy is a little greater than with the 2-2 strategy. The steady in-train forces with 2-2 strategy are nearly zeros when the train is running in its steady state without velocity accelerations or decelerations.

When comparing Fig. 6 with Fig. 8, both with 1-1 strategy, the former train is equipped with a pneumatic braking system and the latter is equipped with an ECP braking system. The speed fluctuation in Fig. 6 is greater than in Fig. 8. The absolute values of maximum and minimum in-train forces and the mean in-train forces are greater in Fig. 6 than in Fig. 8. The energy consumption is equal in both two figures.

When comparing Fig. 7 with Fig. 9, one can get similar conclusions as comparing Fig. 6 with Fig. 8.

It can be seen that the velocity error exists in all the results with the open-loop scheduling. When comparing the steady in-train forces, which is the reference value for the closed loop control, the performance of 2-2 strategy is the best among the three control strategies and the performance of the train equipped with an ECP brake system is better than that of a traditional train. With the introduction of the acceleration profile, the speed variations lead to larger in-train forces, especially within the speed acceleration periods. However, the accelerations decrease the speed tracking error. The transient control is a "tradeoff" between the two aspects.

From the simulation results, the following conclusions can be drawn.

- 1) The scheduling with the averagely distributed power among the locomotives is not optimal for the train performance.
- 2) The more the number of the controllable inputs are, the better the train performance.

- 3) The ECP brake system has demonstrated a superb performance compared with pneumatic brake system.
- 4) The 2-2 strategy is the best among the strategies for heavy haul trains equipped with ECP braking systems.
- 5) The open-loop scheduling cannot reach a satisfied performance, but may give a good reference for the closed-loop control, which is the purpose of this paper.

## VII. CONCLUSION

This paper emphasizes on an open-loop scheduling for the handling of heavy haul trains, which constitutes a basic problem about the trim point. In this paper, the cascade mass point model is set up for a long heavy haul train. Then three control strategies are proposed and followed by the open-loop optimal scheduling algorithms for them. Simulation results of these control strategies to a traditional heavy haul train and a train equipped with an ECP brake system are shown.

The study in this paper is a first step toward a closed-loop cruise control of a heavy haul train. It is under current investigation to combine the closed-loop controller in [9] with the optimal open-loop schedule in this paper.

### REFERENCES

- R. C. Kull, "WABTEC ECP system update," in *Proc. IEEE Vehicular Technol. Soc. Land Transportation Division ASME Rail Transportation Division Joint Railroad Conf.*, Toronto, ON, Canada, Apr. 2001, pp. 129–134.
- [2] Management of Train Operation and Train Handling: Air Brake Association, 1972, pp. 9–9.
- [3] P. Howlett, "Optimal strategies for the control of a train," *Automatica*, vol. 32, no. 4, pp. 519–532, Apr. 1996.
- [4] R. Liu and I. M. Golovitcher, "Enery-efficient operation of rail vehicles," *Transportation Res.*, pt. A, vol. 37, pp. 917–932, 2003.

- [5] P. Gruber and M. Bayoumi, "Suboptimal control strategies for multilo-comotive powered trains," *IEEE Trans. Autom. Control*, vol. 27, no. 3, pp. 536–546, Jun. 1982.
  [6] C. Yang and Y. Sun, "Robust cruise control of high speed train with a first strategies."
- [6] C. Yang and Y. Sun, "Robust cruise control of high speed train with hardening/softening nonlinear coupler," in *Proc. Amer. Control Conf.*, vol. 3, Jun. 2–4, 1999, pp. 2200–2204.
- [7] —, "Mixed  $H_2/H_{\infty}$  cuise controller design for high speed train," *Int. J. Control*, vol. 74, no. 9, pp. 905–920, 2001.
- [8] A. Astolfi and L. Menini, "Input/output decoupling problem for high speed trains," in *Proc. Amer. Control Conf.*, Anchorage, AK, May 2002, pp. 549–554.
- [9] M. Chou and X. Xia, "Optimal cruise control of heavy haul train equipped with electronic controlled pneumatic brake systems," in *Proc.* 16th IFAC World Congr., Prague, The Czech Republic, Jul. 2005.
- [10] M. Chou, X. Xia, and C. Kayser, Modeling and Model Validation of Heavy-Haul Trains Equipped With Electronic Controlled Pneumatic Brake Systems, 2005, submitted for publication.
- [11] R. S. Raghunathan, H. D. Kim, and T. Setoguchi, "Aerodynamics of high-speed railway train," *Progress Aerospace Sci.*, vol. 38, no. 6–7, pp. 469–514, Aug.-Oct. 2002.
- [12] J. A. Scetz, "Aerodynamics of high-speed trains," Annu. Rev. Fluid Mech., pp. 371–414, 2001.
- [13] V. K. Garg and R. V. Dukkipati, *Dynamics of Railway Vehicle System*, XIII ed. Toronto, ON, Canada: Academic, 1984.
- [14] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization*. London, U.K.: Academic, 1981.