Optimal Scheduling and Control of Heavy Haul Trains Equipped With Electronically Controlled Pneumatic Braking Systems

X. Zhuan and Xiaohua Xia, *Senior Member, IEEE*

***Abstract*—This brief compares the performances of different operation strategies based on optimal scheduling and heuristic scheduling for heavy haul trains equipped with electronically controlled pneumatic braking systems. Train scheduling here refers to an open-loop control design that brings the train to a desired (steady-state) motion trajectory. A (closed-loop) cruise control is used to maintain a steady-state motion of a train. In train handling, energy consumption, speed tracking, and in-train force are concerns for transportation corporations. The last is particularly important for safe train running. An optimal train scheduling as well as an optimal cruise control can take these factors into consideration. A speed profile is assumed first. The objective of the study is to find optimal driving methodologies for an implementation of a desired speed profile with energy consumption and in-train forces considered. Simulation results show that optimal scheduling can improve the performance of the closed-loop controller, and that the 2-2 strategy, the electronically controlled pneumatic/independent distributed power mode, is the best of all strategies.**

***Index Terms*—Cruise controller, electronically controlled pneumatic (ECP) braking system, heavy haul train, linear quadratic regulator (LQR), quadratic programming.**

I. Introduction

LONG HEAVY haul trains with multilocomotives are used to transport the inland mineral resources to harbours in South Africa. The cost is less for a larger load per car or per train in terms of the schedule and the number of people involved. Traditionally, the operation of such multilocomotive trains with pneumatic braking systems has, in essence, been naive [12], which results in slow running speed, the possibility of derailment, and a limit on the length of the train. A new braking system, electronically controlled pneumatic (ECP) braking system, has been developed to improve the train's performance [1]. In an ECP braking system, the brake command signals are electronic and received by all wagons simultaneously. The braking effort can be different although the pneumatics are still used to supply the brake power. This braking system was first rolled out by Spoornet, one of the train operators in South Africa, on its COALink line on a large scale. According to [2], train handling includes the start phase of a train, speed maintenance phase, and stop phase of a train. The phases of speed cruise (speed maintenance) and speed acceleration/deceleration are studied in this brief.

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The authors are with the Department of Electrical, Electronic, and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa, and also with the Department of Automation, Wuhan University, Wuhan 430072, China (e-mail: zhuan.xiangtao@up.ac.za; xxia@postino.up.ac.za).

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For heavy haul trains, energy consumption, running time (speed tracking), and the in-train forces of the couplers are of much concern to the transportation corporations [12]. For long trains, in-train forces are even more important and more difficult to control. It is noted that the in-train forces depend both on the driving speed and on the power/brake distribution along the train. This is why the independent distributed power (iDP) operation and ECP braking systems have been introduced into practice.

In the study of train handling, there are two types of train model. One is to treat the whole train as a point mass, such as in [3] and [4], where the objective is to minimize the energy consumption during travelling on a given track within a given time period. The other models a train as a cascade of point masses connected with couplers, such as in [5]–[9]. In these papers, a desired speed profile along a given track is assumed first. The subject of the studies is to design controllers to maintain the desired speed with some objectives considered. In [6]–[8], high-speed trains (passenger trains) are studied, where the speed tracking is emphasized and the in-train forces of the couplers are not considered, since the problem of in-train forces is not so important in short trains. The calculation of the equilibria in these papers is based on different assumptions, which are heuristic. A linear quadratic regulator (LQR) controller was designed to optimize the in-train forces and/or speed deviation from the reference speed in [5], which considered the practical aspects of ECP and iDP, even though the ECP/iDP technology was not implemented in practice on a visible scale. The equilibria in [5] are calculated according to the predetermined or tuned control sequence, which is heuristic, too.

In [9] and [10], based on a cascade point-mass model, which is validated in [11] with the operation data from Spoornet, an LQR approach is employed to optimize the in-train forces, energy consumption, and velocity tracking of a heavy haul train equipped with an ECP braking system. In offline scheduling, the equilibria are calculated under the assumption that the driving force is equally distributed to the locomotives while all the braking forces of wagons are zeros and the braking force is equally distributed to the locomotives and wagons. This open loop scheduling is heuristic, too. With the discrete quantities of the locomotives' efforts considered, the efforts of the locomotives are almost always equal. This may lead to irrational power distribution, especially when one locomotive group is climbing uphill and the other one is driving downhill. In [12], an optimization procedure is applied to schedule cruise control by taking the in-train forces into initial design consideration for three different operation strategies (1-1 strategy, 2-1 strategy, and 2-2 strategy, described later). With this open-loop scheduling, the running error always exists and it sometimes leads to oscillation, which should be avoided in train handling. As mentioned in [12], it is expected that optimal scheduling will lead to better performance of the closed-loop controllers than that based on heuristic scheduling in [10]. In this brief, the optimal scheduling in [12] and the closed-loop controller in [10] are integrated. A speed profile is assumed first. The objective of the study is to find optimal driving methodologies for an implementation of the desired speed profile with energy consumption and in-train forces considered. Simulation results of the closed-loop controllers based on heuristic scheduling and optimal scheduling are shown with different operation strategies. In comparison to heuristic scheduling, simulation results show that optimal scheduling can improve the performance of the closed-loop controller, and that the 2-2 strategy, the ECP/iDP mode, is the best of all strategies. Based on this observation of the 2-2 strategy, another "suboptimal" controller design is proposed in [13] for the fully ECP/iDP mode subject to the assumption that only speed measurement of locomotives is available: while optimality is kept in the open-loop control design, the closed-loop control is done by employing a nonlinear system regulator theory. Both [13] and this brief favor an optimal open-loop scheduling, and therefore, the same track topology and train composition are used in simulation to compare the results on different closed-loop controllers. It is quite interesting to find out that energy consumption and in-train forces with the approach in [13] are generally better than the corresponding results of this brief in the case of the 2-2 strategy, although the controller in this brief has a more rapid response to the change of reference speed, or a better capability of speed tracking. Assuming full state feedback, the other advantages of the approach of this brief include an application of a well-known linear, optimal design methodology as well as the applicability to all cases besides the 2-2 strategy.

In this brief, Section II describes the train model. The design of open-loop scheduling and closed-loop controllers is given in Section III. Simulation results of the closed-loop strategies for heavy haul trains are shown in Section IV.

II. Train Model

A heavy haul train, composed of locomotives and wagons (both referred to as cars), can be modelled as a cascade of mass points connected with couplers. The detailed model description can be found in [10] and [12].

Assuming the train consists of n cars and the locomotives are located at positions l_i, $i = 1, 2, \ldots, k$, where k is the number of locomotives, the train model is described by the following equations:

$$\begin{aligned} m_i \dot{v}_i &= u_i + f_{\mathrm{in}_{i-1}} - f_{\mathrm{in}_i} - f_{a_i}, \quad i = 1, \ldots, n \\ \dot{x}_i &= v_i - v_{i+1}, \quad i = 1, \ldots, n-1 \end{aligned} \tag{1}$$

where the variable m_i is the ith car's mass and the variables v_i and u_i are the speed and effort of the ith car. The variable $f_{a_i} = f_{\mathrm{aero}_i} + f_{p_i}$,, in which $f_{\mathrm{aero}_i} = m_i(c_{0_i} + c_{1_i} v_i + c_{2_i} v_i^2)$ is the ith car's aerodynamic force, the variable $f_{p_i} = f_{g_i} + f_{c_i}$ is the force due to the track slope and curvature, where the ith car is running. The variable f_{in_i} is the in-train force between the ith and $(i+1)$th cars, which is a function of x_i, the relative displacement between the two neighbouring cars, and the difference of the neighbouring cars' velocities (damping effect). The variables $c_{0_i}, c_{1_i}, c_{2_i}$ are constants determined by experiments. In (1), one has $f_{\mathrm{in}_0} = 0, f_{\mathrm{in}_n} = 0$.

A. Model Input Constraint

For a heavy haul train, the control inputs are the efforts of the locomotives and the wagons. The efforts of locomotives can be traction forces or dynamic braking forces and the efforts of wagons are braking forces. The dynamic brake power is also called regenerative brake power, which can be fed back to the system and could conceivably be saved. All these inputs are constrained. For a locomotive, the effort is governed by the current velocity and the current notch setting, which can be referred to [12] for the 7E1 locomotive used in the COALink trains. The locomotives in this brief are assumed to be electric and it is also convenient to formulate the problem of a train with diesel–electric locomotives in a similar way.

In practical operation of 7E1 locomotives, any notch change requires an interval delay for the field changes. When it changes from dynamic braking to traction or the other way, the time delay requires a longer interval.

The braking forces of the wagons are also limited by the braking capacities of the wagons.

B. In-Train Forces Constraints

The quantity of the in-train forces is related to the safe running of the train. In practice, the safety range of the in-train forces for COALink trains are ± 2000 kN.

III. Controller Design

A. Operation Strategies

When a heavy haul train is equipped with an ECP braking system, it allows individual wagon braking. The following three major types of operation strategies are discussed in this brief.

- 1-1 strategy: There is one control signal for all the locomotives and one braking control signal for all the wagons.
- 2-1 strategy: The control signal of every locomotive effort may be different, and the braking control signal of all the wagons is the same. This is an iDP-only strategy.
- 2-2 strategy: There is an independent control command for every car, including locomotives and wagons. This is a fully ECP/iDP mode.

B. Open-Loop Controller

An open-loop controller is used to calculate the inputs when a train is running in its steady state with the reference velocity and acceleration maintained. The input constraints are not considered. When the inputs are applied, an anti-windup technique is used.

In [5], the offline schedule for the throttling and braking inputs is chosen in such a way that the train is in its steady state with the reference velocity maintained. The settings do not contribute to additional accelerations/decelerations of the train. The schedule determines the sequencing and the amplitudes of the inputs in case there are continuous input variations and no power limits. The applied inputs ${}^0u(t)$ are nonlinear functions of the schedule parameters p (grade of the track, velocity profile and train data) and the travelled distance z of

TABLE I
ACCELERATION PROFILE

distance	$\cdots$	1,000	s_1	5,000	s_2	$\cdots$
a	0	a_{rr}	0	$-a_{rr}$	0	$\cdots$

the train: ${}^0u = f_u(z, p)$. The inputs are approximated by step functions of variable amplitudes. The sequence of the steps are predetermined and tuned, and the time instants of the step functions at which the steps are applied, are decided on line. It is obvious that this offline schedule is heuristic and subjected to the predetermined control sequence, so it will not be discussed in this brief.

1) Transient Control: The inputs in (1) are insensitive to the change of the reference speed. To get a rapid response to the reference speed change, a transient control is designed through an acceleration profile in the following open-loop scheduling. When a closed-loop controller is considered, this step is unnecessary. An acceleration profile is calculated according to the velocity profile with a parameter, the acceleration limit, a_{rr}. For example, at the travel distance $\mathrm{dis} = 1000$ m, the reference velocity is changed from 12 to 15 m/s and at the distance $\mathrm{dis} = 5000$ m, it is changed to 10 m/s, then the acceleration profile is as shown in Table I, where s_1, s_2 are calculated as $s_1 = 1000 + (15^2 - 12^2)/(2a_{\mathrm{rr}})$, $s_2 = 5000 + (15^2 - 10^2)/(2a_{\mathrm{rr}})$. Thus, from the point 1000 m to the point s_1 and from the point 5000 m to the point s_2, the open-loop scheduling should maintain the accelerations.

2) Heuristic Scheduling: According to [9], the open-loop control is chosen as follows, with $\mathrm{Beq} = \sum_{i=0}^{n}(f_{a_i} + m_i a)$

$$\begin{aligned} &u_l = \mathrm{Beq}/k, \quad u_b = 0, \quad \mathrm{Beq} \geq 0 \\ &u_l = \mathrm{Beq}/n, \qquad u_b = \mathrm{Beq}/n, \qquad \mathrm{Beq} < 0 \end{aligned} \tag{2}$$

where u_l is the locomotives' effort and u_b is the wagons' effort, and the variables k and n are the respective total numbers of locomotives and cars. The acceleration $a = 0$ in the cruising period, while $a = \pm a_{\mathrm{rr}}$ in the scheduled acceleration/deceleration periods. The power distribution is heuristic, so one calls it heuristic scheduling.

3) Optimal Scheduling: According to the three operation strategies described in Section III-A, there are three corresponding optimal open-loop controllers for the train. In designing the controllers, the performance is a function of the in-train forces and the energy, which can be written as

$$J = \sum_{i=0}^{n-1} K_f f_{\mathrm{in}_i}^2 + \sum_{i=0}^{n} K_e u_i^2 \tag{3}$$

where the weights of the in-train force and energy consumption are K_f and K_e, respectively. Optimal power distribution is characteristic of this scheduling, so one calls it optimal scheduling.

For open-loop control, the dynamic process in the train is ignored and the system is assumed to be in its steady state with the acceleration maintained, that is

$$\begin{aligned} &\frac{dv_i}{dt} = a, \quad \frac{dx_j}{dt} = 0, \\ &\quad i = 1, \ldots, n, \quad j = 1, \ldots, n-1. \end{aligned} \tag{4}$$

Applying (4) to (1), and assuming $f_{\mathrm{in}_0} = 0$, one has

$$u_s + f_{\mathrm{in}_{s-1}} - f_{\mathrm{in}_s} - f_{a_s} - m_s a = 0, \quad s = 1, \ldots, n. \tag{5}$$

In train operations, the inputs, $u_i, i = 1, \ldots, n$ and the in-train forces f_{in_i} have some constraints

$$\begin{aligned} &\underline{U}_i \leq u_i \leq \bar{U}_i, \quad i = 1, \ldots, n \\ &\underline{F}_{\mathrm{in}_j} \leq f_{\mathrm{in}_j} \leq \bar{F}_{\mathrm{in}_j}, \quad j = 1, \ldots, n-1 \end{aligned} \tag{6}$$

where $\underline{U}_i, \bar{U}_i$ are the upper constraint and lower constraint for the ith input and $\underline{F}_{\mathrm{in}_j}, \bar{F}_{\mathrm{in}_j}$ are the upper and lower constraints for the jth in-train force, respectively. For wagons, $\bar{U}_i = 0$ and the values of $\underline{U}_i$ depend on the braking capacities of the wagons. For locomotives, the constraints $\underline{U}_i, \bar{U}_i$ depend on the locomotives' capacities of traction efforts and the running states. The constraints $\underline{F}_{\mathrm{in}_j}, \bar{F}_{\mathrm{in}_j}$ are limited because of the requirement of safe operation and maintenance cost.

Thus, optimal scheduling is a standard quadratic programming (QP) problem with objective function (3), equality constraints (5), inequality constraints (6), and some additional equality constraints. With 2-2 strategy, there is no additional constraint. With 1-1 strategy, the additional constraints imposed on the optimization problem are

$$\begin{aligned} &u_{l_j} \triangleq u_t, \quad u_i \triangleq u_b, \\ &\quad i = 1, \ldots, n, \quad i \neq l_j, \quad j = 1, \ldots, k. \end{aligned} \tag{7}$$

With 2-1 strategy, the additional constraints are the following:

$$u_i \triangleq u_b, \qquad i = 1, \ldots, n, \quad i \neq l_j, \quad j = 1, \ldots, k. \tag{8}$$

A QP problem can be solved by the active set method [15] and the numerical method is referred to in the Appendix.

C. Closed-Loop Controller

With the calculation of an open-loop scheduling, the steady state and input of the train can be denoted as $f_{\mathrm{in}_j}^0(x_j^0), v_i^0(v_r), u_i^0, j = 1, \ldots, n-1, i = 1, \ldots, n$, which are the in-train forces (static displacement of coupler), the velocities and the traction forces or braking forces of the cars. The static displacement x_j^0 is interpolated from $f_{\mathrm{in}_j}^0$. Then one can rewrite the train model with the following equations:

$$\begin{aligned} &\delta\dot{v}_s = \left(\delta u_s + \delta f_{\mathrm{in}_{s-1}} - \delta f_{\mathrm{in}_s} - \delta f a_s\right)/m_s, \quad s = 1, \ldots, n \\ &\delta\dot{x}_j = \delta v_j - \delta v_{j+1}, \quad j = 1, \ldots, n-1 \end{aligned} \tag{9}$$

where $\delta v_s = v_s - v_s^0 = v_s - v_r, \delta u_s = u_s - u_s^0, \delta f_{\mathrm{in}_s} = f_{\mathrm{in}_s} - f_{\mathrm{in}_s}^0, \delta x_j = x_j - x_j^0$. The variable v_r is the reference speed. When the damping of the coupler is ignored, this model can be linearized as follows:

$$\begin{aligned} &\delta\dot{v}_s = (\delta u_s + k_{s-1}\delta x_{s-1} - k_s \delta x_s)/m_s - (c_{1_s} + c_{2_s} v_r)\delta v_s, \\ &\qquad\qquad\qquad\qquad\qquad\qquad s = 1, \ldots, n \\ &\delta\dot{x}_j = \delta v_j - \delta v_{j+1}, \quad j = 1, \ldots, n-1 \end{aligned} \tag{10}$$

where k_s is the linearized coefficient of the coupler with the assumption $k_0 = 0$. The model can be written as

$$\dot{X} = AX + BU$$

where

$$X = \mathrm{col}(\delta v_1, \ldots, \delta v_n, \delta x_1, \ldots, \delta x_{n-1})$$
$$U = \mathrm{col}(\delta u_1, \ldots, \delta u_n)$$
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$
$$A_{11} = -\mathrm{diag}(c_{1_1} + c_{2_1} v_r, \ldots, c_{1_n} + c_{2_n} v_r)$$
$$A_{22} = 0_{(n-1)\times(n-1)}$$
$$B = \mathrm{diag}((1)/(m_1), \ldots, (1)/(m_n))$$
$$A_{12} = \begin{bmatrix} -\frac{k_1}{m_1} & 0 & \cdots & 0 & 0 \\ \frac{k_1}{m_2} & -\frac{k_2}{m_2} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \frac{k_{n-2}}{m_{n-1}} & -\frac{k_{n-1}}{m_{n-1}} \\ 0 & \cdots & 0 & 0 & \frac{k_{n-1}}{m_n} \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}.$$

The variables $k_i,\ i = 1, \ldots, n-1$ are chosen to be constant. Although different scheduling has different equilibria, the coefficients in the linearized model (10) are identical.

In simulation, however, the original nonlinear model has been used for the train model.

When an LQR controller is to be designed with the approach in [10], the performance function is chosen as

$$\delta J = \int (X'QX + U'RU)\, dt = \int \left(\sum_{i=0}^{n-1} K_f^o \delta x_i^2 + \sum_{i=0}^{n} K_e \delta u_i^2 + \sum_{i=0}^{n} K_v^o \delta v_i^2 \right) dt \tag{11}$$

where K_f^0, K_e, K_v^0 are the weights for in-train forces, energy consumption, and velocity tracking, respectively. When the coefficients K_f^0, K_e, K_v^0 are chosen such that the first item of (11) dominates, the controller is an in-train force emphasized one. When the second item of (11) dominates, the controller is an energy emphasized one. It is a speed emphasized control if the third item dominates.

Based on the optimization approach [16], one can get a feedback control $U = -KX$, and the complete closed-loop control is

$$u = U + u^0. \tag{12}$$

IV. Simulation

A. Simulation Parameters

In simulation, the model parameters of the train, the track profile, and the reference speed profile are the same as those in [12] as well as the locomotive notch operation constraints.

The track profile shown in Fig. 1 is from the COALink line, which is typically downhill when the train is loaded. In this section of track, there are also two uphill segments, which make it difficult to drive a long train that may extend over several different gradient sections at any given time. The largest incline degree is 0.09152 and the largest decline is -0.1, which are very similar to the slope degree (± 0.1) in [3].

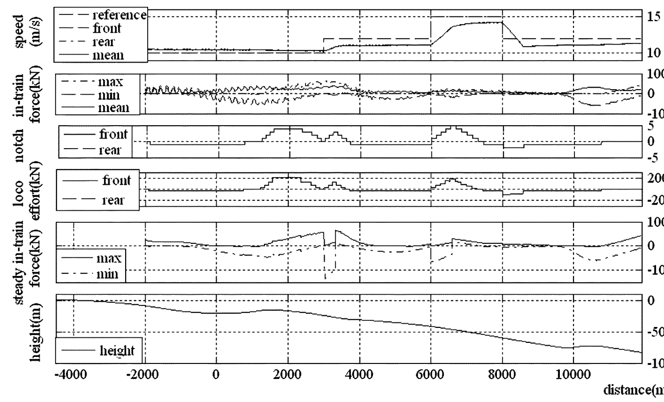

Fig. 1. Heuristic scheduling.

A piecewise linear function is used to approximate the nonlinear function of a coupler. In the controllers one chooses a greater value as 3×10^7 N/m for all the couplers' linearized coefficients in (10).

A safe-operation requirement for a train on the COALink is that the in-train forces should be less than ± 2000 kN. In simulation, $\bar{F}_{\mathrm{in}_i}, \underline{F}_{\mathrm{in}_i}$ are chosen as 1200 kN considering the redundancy for a longer train with 800 wagons.

In the simulation, the weights for in-train forces, energy, and velocity are K_f, K_e, K_v, respectively, and $K_f^0 = 3 \times 10^8 K_f, K_v^0 = 5 \times 10^6 K_v$, which gives the same value for the in-train forces, speed deviation, and input in (11) as would be obtained when $\delta x = 0.01$ m, $\delta v = 0.1$ m/s^2, $\delta u = 200$ N with the weights $K_f = K_e = K_v$. The acceleration limit a_{rr} is 0.07 m/s^2. This value is calculated with the assumption that the train is running on a flat track and all the traction power of the locomotives is used to accelerate. The maximum acceleration can be $760 \times 2/(252 \times 2 + 417 \times 50) = 0.07118$ m/s^2. The maximum deceleration is more than the maximum acceleration, but in the simulation they are assumed to be the same for the sake of simplicity.

The initial state of the train is that the train is in its steady state with all the cars' velocities 10.5 m/s and all the in-train forces equal to zero.

B. Simulation Results

The simulation is performed in MATLAB, and for the quadratic programming and LQR, the standard programs in MATLAB are adopted for the open-loop scheduling and closed-loop controllers. The results for the three operation strategies, based on different open-loop scheduling, are shown in the following figures and tables.

1) Heuristic Scheduling Versus Optimal Scheduling: There is only one open-loop operation strategy for the heuristic scheduling as shown in Fig. 1. Figs. 2–4 are the optimal scheduling of 1-1 strategy, 2-1 strategy, and 2-2 strategy, respectively, with $K_e = 1,\ K_f = 1$. The first subplot of these figures shows

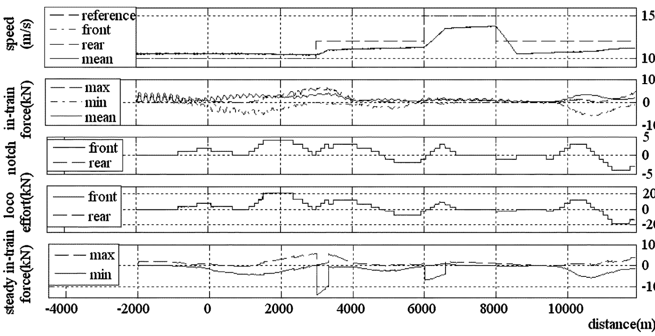

Fig. 2. 1-1 strategy optimal scheduling.

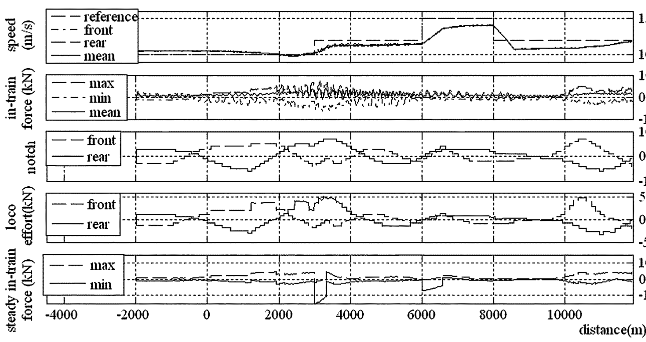

Fig. 3. 2-1 strategy optimal scheduling.

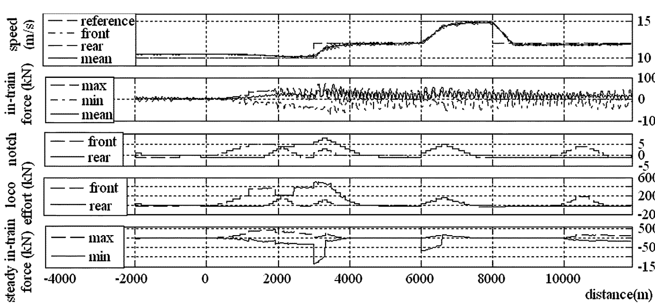

Fig. 4. 2-2 strategy optimal scheduling.

the front locomotive group speed, rear locomotive group speed, and the mean speed of all the cars with respect to the distance from the starting point. The second subplot shows maximum and minimum in-train forces and the mean value of the absolute values of all the in-train forces at a specific time with respect to the distance. The third and fourth ones show front and rear locomotive groups' notches and efforts. The fifth one shows the maximum and minimum values of the steady in-train forces which is calculated from (5) with the scheduling efforts of the cars when the train dynamic process is ignored. The sixth subplot of Fig. 1 shows the track profile, which is identical in all the simulations and omitted in all subsequent figures. The energy consumptions in Figs. 1–4 are 8517, 11 380, 23 250, and 16 450 MJ, respectively. The running results of the open-loop scheduling show that the velocity tracking error exists with all the scheduling. The performance of the heuristic scheduling and 1-1 strategy optimal scheduling are similar. The performance of in-train force of 2-2 strategy optimal scheduling is the worst because of oscillation, while its speed tracking error is the

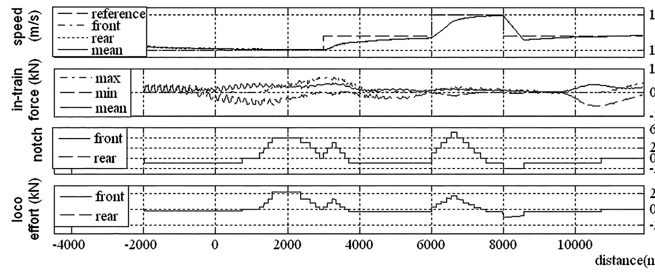

Fig. 5. 1-1 strategy closed-loop control based on heuristic scheduling.

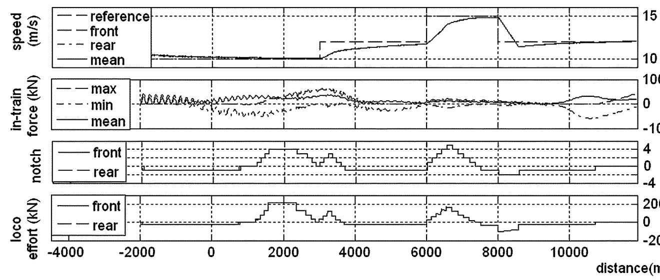

Fig. 6. 2-1 strategy closed-loop control based on heuristic scheduling.

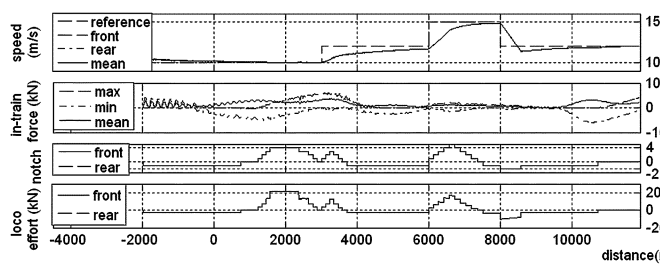

Fig. 7. 2-2 strategy closed-loop control based on heuristic scheduling.

smallest. The velocity tracking error and the possibility of oscillation are the drawbacks of an open-loop controller.

However, the performance of the steady-state in-train force of the 2-2 strategy optimal scheduling is best. The performance of the steady-state in-train force of 1-1 strategy optimal scheduling is similar to that of heuristic scheduling. The performance of the steady-state in-train force of 2-1 strategy optimal scheduling is also better than that of 1-1 strategy optimal scheduling, except within the acceleration/deceleration periods, where the states of the train changes abruptly. Actually, all the state of the train should change continuously, which leads to smoother change. The open-loop scheduling does not consider the real running state, and it is difficult to say which scheduling is best. However, the open-loop scheduling gives a reference to the closed-loop controller, so the steady state calculated by the scheduling is more important than the real running result. From this point of view, one can see that the performance of the 1-1 strategy optimal scheduling is similar to that based on heuristic scheduling, and the performance of 2-2 strategy optimal scheduling is best.

2) Closed-Loop Controller: Simulation results of the three different strategies' closed-loop controllers based on the heuristic scheduling and optimal scheduling are shown from Figs. 5–10, where the weights are $K_f = 1,\ K_e = 1,\ K_v = 1$. The energy consumptions in these figures are shown in Table II.

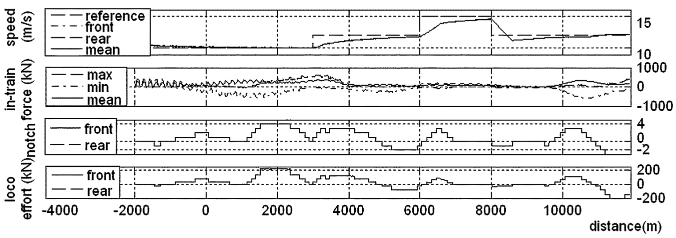

Fig. 8. 1-1 strategy closed-loop control based on optimal scheduling.

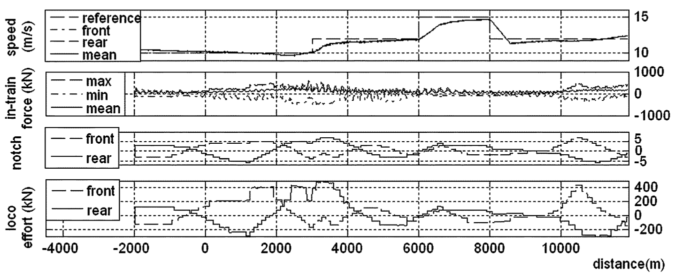

Fig. 9. 2-1 strategy closed-loop control based on optimal scheduling.

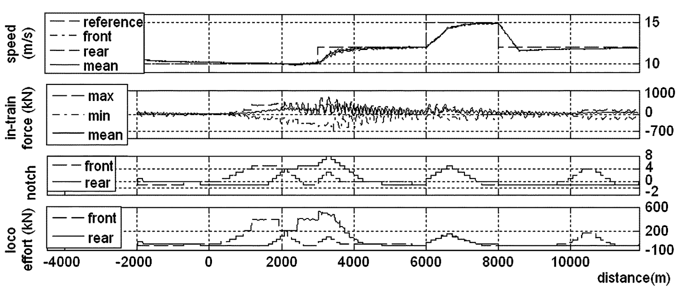

Fig. 10. 2-2 strategy closed-loop control based on optimal scheduling.

When comparing the figures of the closed-loop controllers with those of the open-loop scheduling, it is obvious that the steady velocity error is much smaller and better in closed-loop controllers than in open-loop scheduling. For heuristic scheduling, the performances of the in-train force and the energy consumption of the open-loop scheduling are similar to those of closed-loop controllers. For optimal scheduling, the performances of the energy consumption with the closed-loop controllers of the three strategies are similar to these of the corresponding open-loop scheduling. The 2-1 strategy and 2-2 strategy closed-loop control give better in-train force performances than the corresponding scheduling.

When comparing the three different strategies' closed-loop controllers based on the heuristic scheduling, the performances are very similar.

When comparing the three different strategies' closed-loop controllers based on corresponding optimal scheduling, the performances of the velocity and in-train force with 2-2 strategy are best and those with 1-1 strategy are worst. The performance of the energy consumption with 1-1 is a little better than that with the 2-2 strategy, which is also a little better than that with 2-1 strategy.

When comparing the corresponding strategy closed-loop controllers based on optimal scheduling and heuristic scheduling, the energy consumption with the three different strategies

TABLE II
PERFORMANCE WITH $K_e = 1, K_f = 1, K_v = 1$

	$\lvert\delta\bar{v}\rvert$ (m/s)			$\overline{\lvert f_{in}\rvert}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
$C01$	3.3241	0.4573	0.58	386.94	145.82	100.27	8700
$C02$	3.3244	0.4539	0.57	376.78	145.30	99.32	8610
$C03$	3.3241	0.4613	0.58	373.60	144.45	97.50	8470
$C1$	3.2274	0.4992	0.56	387.04	147.52	102.65	11760
$C2$	3.1405	0.4585	0.53	318.97	106.16	59.35	22100
$C3$	3.0182	0.3166	0.48	454.50	97.40	86.44	16500

based on heuristic scheduling is less than that based on the corresponding optimal scheduling. The performances of the velocity and the in-train force with 2-2 strategy-based optimal scheduling are better than those based on heuristic scheduling. The performance of the velocity with 1-1 strategy based on optimal scheduling is worse than that based on heuristic scheduling while the performances of the in-train force based on the two schedulings are similar. The performances of the velocity with the 2-1 strategy based on the two schedulings are similar and the performance of in-train force with the 2-1 strategy based on optimal scheduling is better than that based on heuristic scheduling.

From the previous comparison, it can be seen that the performances of the in-train force and the velocity with the 2-2 strategy based on optimal scheduling are best. In this strategy, it is very interesting to see, as depicted in Fig. 10, the variation of the traction efforts of the front and rear locomotives (groups) when the train travels from the distance 0 to 4000 m and from 9500 to 11 000 m; those sections are hills in the track. When the front locomotives (groups) are climbing uphill and the rear ones are driving downhill, the front locomotives make increasing traction efforts and the rear ones are braking. When more and more cars are climbing uphill, the rear locomotives begin to gradually increase traction efforts. When the front locomotives pass the top of the hill and begin to drive down, their efforts begin to decrease and the rear ones gradually increase their efforts. When the front locomotives are driving downhill and the rear ones are climbing uphill, the front ones are braking and the rear ones make traction efforts. At 3000 m, the train begins to accelerate from 10 to 12 m/s. The front and rear locomotives begin to increase their traction efforts simultaneously, which can also be seen from distance points 6000 and 8000 m. That is consistent with common sense.

Tables II–V are the simulation results of the six closed-loop controllers with different weights in the performance function for the in-train force, the energy consumption, and the velocity tracking. Table II is the performance comparison of Figs. 5–10. In these tables, C01, C02, and C03 are 1-1 strategy, 2-1 strategy, and 2-2 strategy closed-loop controllers based on heuristic scheduling, and C1, C2, and C3 are 1-1 strategy, 2-1 strategy, and 2-2 strategy closed-loop controllers based on optimal scheduling. $\lvert\delta\bar{v}\rvert$ is the absolute value of the difference between the reference velocity and the mean value of all the cars' velocities at a specific point. $\lvert\overline{f_{\rm in}}\rvert$ is the mean value of the absolute values of all the couplers' in-train forces at a specific point. The item E is the energy consumed during travel. The

TABLE III
PERFORMANCE WITH $K_e = 1, K_f = 1, K_v = 10$

	$\|\delta\bar{v}\|$ (m/s)			$\overline{\|f_{in}\|}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
$C01$	3.0412	0.3062	0.55	394.39	145.72	99.57	8620
$C02$	3.0413	0.3080	0.55	394.50	144.54	100.07	8550
$C03$	3.0412	0.3085	0.55	369.24	144.61	96.63	8586
$C1$	3.0070	0.3372	0.57	382.57	147.38	102.40	11100
$C2$	2.9891	0.3629	0.53	344.95	103.57	67.20	21800
$C3$	3.0225	0.2443	0.50	408.70	74.07	76.34	16500

TABLE IV
PERFORMANCE WITH $K_e = 1, K_f = 10, K_v = 1$

	$\|\delta\bar{v}\|$ (m/s)			$\overline{\|f_{in}\|}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
$C01$	3.3251	0.4611	0.57	385.83	146.17	100.43	8790
$C02$	3.3234	0.6609	0.57	377.25	145.96	98.68	8610
$C03$	3.3243	0.4604	0.58	368.28	144.61	96.40	8460
$C1$	3.2663	0.5312	0.58	384.58	147.53	101.76	11460
$C2$	3.1379	0.4542	0.52	331.48	105.82	62.57	22300
$C3$	3.0090	0.367	0.47	405.70	70.77	78.04	15000

TABLE V
PERFORMANCE WITH $K_e = 100, K_f = 1, K_v = 1$

	$\|\delta\bar{v}\|$ (m/s)			$\overline{\|f_{in}\|}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
$C01$	3.8039	0.7383	0.56	392.08	145.01	99.50	8795
$C02$	3.8056	0.7361	0.56	389.83	146.27	99.24	8600
$C03$	3.8041	0.7415	0.56	386.49	144.19	99.08	8560
$C1$	3.6744	0.6889	0.57	390.46	143.61	101.44	9550
$C2$	3.4603	0.6388	0.53	300.06	110.47	61.90	16800
$C3$	3.247	0.4918	0.47	297.27	78.90	63.27	13400

items max, mean, and std are the maximum value, mean value, and standard deviation of the statistical variable, respectively.

From these tables, it can be seen that the three strategies based on heuristic scheduling have similar performances. This is because their scheduling are the same. However, based on optimal scheduling, the 2-2 strategy has a better performance of velocity, in-train force, and energy consumption than the 2-1 strategy with the same parameters. The performance of velocity in Table III, which is with the velocity emphasized optimal parameters, is the best compared with the corresponding operation strategy of the other tables. The performance of the in-train force based on heuristic scheduling of the corresponding operation strategy is approximate in the four tables, while the performance of the in-train force based on optimal scheduling is best in Table IV, which is with the in-train force emphasized parameters, but only the improvement of 2-2 strategy is obvious and those of the other two are only approximate. In Table V, with the energy consumption emphasized parameters, the performance of energy consumption with all the corresponding controllers based on optimal scheduling, is best among the four tables. On the whole, local optimization does work and leads to global optimization in some degree.

From simulation, it is shown that the train tracks the reference speed quickly when the reference speed changes and tracks the reference speed very well when the train is cruising. So the track length is enough for simulation of the driving profile. However, it should be pointed out that when the objective is to test the optimization combination of a driving profile and a reference speed profile, a longer track might be necessary.

Based on the observation of the 2-2 strategy, another approach of controller design is proposed in [13] by assuming only the locomotives' speed measurement. Performance comparisons are further detailed in [13].

V. CONCLUSION

For train handling, the optimal objective is to minimize energy consumption, maintain the presetting velocity, and decrease the in-train forces. The last is the most important for safe running. An optimal train scheduling as well as an optimal cruise control can take these factors into consideration. A speed profile is first assumed in this brief. The objective of the study is to find optimal driving methodologies for an approximate implementation of a desired speed profile with energy consumption and in-train forces considered. Simulation results show that optimal scheduling can improve the performance of the closed-loop controller, and that the 2-2 strategy, the ECP/iDP mode, is best of all strategies. With different optimization parameters, the controller based on optimal scheduling can realize the corresponding objective.

APPENDIX

A. Active Set Method for QP [14]

Considering the following QP

$$\begin{aligned} \text{(QP)} \quad \min \quad & Q(x) \equiv \frac{1}{2}x^T G x \\ \text{s.t.} \quad & a_i^T x = b_i, \quad i \in \mathcal{E} \\ & a_i^T x \le b_i, \quad i \in \mathcal{I}. \end{aligned} \tag{13}$$

Suppose that x_k is a feasible point of (QP) and $A(x_k)$ is called *active set* if the constraints $i \in A(x_k)$ are active constraints if $A(x_k) \equiv \{i \in \mathcal{E} \cup \mathcal{I} : a_i^T x = b_i\}$. Then the constraints are classified into two at point x_k: $a_i^T x = b_i, i \in A(x_k)$ and $a_i^T x < b_i, i \notin A(x_k)$.

Step 1) The active set method looks for a correction d_k from x_k by solving the following problem:

$$\begin{aligned} \text{(QP)}_k \quad \min \quad & Q(d) \equiv \frac{1}{2}(x_k + d)^T G(x_k + d_k) \\ \text{s.t.} \quad & a_i^T(x_k + d) = b_i, i \in A(x_k). \end{aligned} \tag{14}$$

If the solution $d_k \neq 0$ and $x_k + d_k$ is feasible, then $x_{k+1} = x_k + d_k$ and the process is repeated.

Step 2) If the solution $d_k \neq 0$ and $x_k + d_k$ is infeasible, then $x_{k+1} = x_k + a_k d_k$, where

$$a_k = \min\left(1, \min_{i \notin A(x_k), a_i^T d_k > 0} \frac{b_i - a_i^T x_k}{a_i^T d_k}\right).$$

If $a_k = (b_q - a_q^T x_k)/(a_q^T d_k) < 1$, then a new constraint with the index q is active and the index q is added to $A(x_{k+1})$. Otherwise, $A(x_{k+1}) = A(x_k)$. Go to Step 1).

Step 3) If $d_k = 0$, the Lagrangian method is used to solve the following quadratic programming problem with equality constraints:

$$L(x, \lambda) = Q(x) + \lambda^T(A^T x - b)$$

if one writes the equality constraints $a_i^T x = b_i, i \in A(x_k)$ in the form of $A^T x = b$. The Kuhn–Tucker (KT) condition states that (x^*, λ^*) is a KT-pair of the previous problem if and only if it solves $\Delta_x L(x, \lambda) = 0,\ \Delta_\lambda L(x, \lambda) = 0$. Suppose the solution of the Lagrange multipliers is $\lambda_i^k, i \in A(x_k)$. If $\lambda_i^k > 0, \forall i \in A(x_k) \cap \mathcal{I}$, then the KT-conditions are satisfied at x_k and x_k is the solution.
Otherwise, there exists an index q, such that $\lambda_q = \min_{i \in A(x_k) \cap \mathcal{I}} \lambda_i^k$ and $\lambda_q < 0$. The index q should be removed from $A(x_k)$. Then go to Step 1).

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