Development of Efficient Model Predictive Control Strategy for Cost-Optimal Operation of a Water Pumping Station

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***Abstract*— Considering time-of-use electricity pricing, the optimal scheduling problem of a pumping station is reformulated into a control sequence (CS) optimal scheduling problem, for which a reduced dynamic programming algorithm (RDPA) is proposed to obtain the solution. It is shown that the RDPA allows a reduction of the operational cost by about 60% compared to a basic conventional control strategy, in the example investigated. The fast computation feature of the RDPA facilitates the implementation of a model predictive control (MPC) strategy. In the simulations, RDPA within the MPC structure is found to provide robust control and a marginally increased operational cost, given a ±10% inflow rate uncertainty and a modest stochastic rainfall variability (up to 20%).**

***Index Terms*— Cost efficiency, dynamic programming (DP), model predictive control (MPC), pumping station.**

I. Introduction

An important task in pump operation is maintaining the level of the tank/reservoir within the safety range. Another important objective is to reduce electricity consumption and its cost. Since energy scarcity is getting very serious, energy efficiency or better energy usage in pumping operation is becoming more of a concern. Energy usage improvement could be achieved by optimal scheduling of the pumping (station) operation.

There are still many pumping stations driven by fixed-speed motors in use in unfavorable conditions under which only ON/OFF scheduling is possible. However, considering the time-of-use (TOU) electricity tariff, it is possible to reduce energy costs with load shifted from the peak time period to the off-peak time period [1]. This results in more economical energy usage.

Much work has been done to improve the energy/cost efficiency of the pumping (station) operation. The optimal scheduling problem of a pumping station network could be split into several subproblems, each of which is designed to optimize the operation of one pumping station, as done in [2] and [3].

Manuscript received February 14, 2011; revised April 8, 2012; accepted June 5, 2012. Manuscript received in final form June 14, 2012. Date of publication July 19, 2012; date of current version June 14, 2013. This work was supported in part by the South Africa-China Joint Research Project CS05-L07. Recommended by Associate Editor G. E. Stewart.

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Digital Object Identifier 10.1109/TCST.2012.2205253

The problem of fixed-speed-motored pump operation scheduling is intrinsically a binary, integer, or mixed programming problem (linear or nonlinear), depending on the mathematical models of the hydraulic structures, networks, etc. For such kinds of optimal scheduling problems, various techniques have been employed in load-shifting for different processes, such as linear programming in [4] for a wind/hydro hybrid water supply system, dynamic programming (DP) in [5] for a water supply system and in [6] and [7] for pump (station) operation, stochastic DP in [8] for a water supply system, and mixed integer linear programming in [9] for a water pumping station. Considering the complexity of the underlying models and the curse of dimensionality of the DP or the interminable branch and bound of integer programming, only a few of these methods have found limited use in practice.

To reach the global optimal solution for a programming problem, some modern optimization methods, such as genetic algorithm [3], [10], simulated annealing [11], particle swarm optimization [12], and ant colony optimization [13], have been adopted. Those approaches improve the possibility of obtaining a global optimization solution, but the computational time is sometimes very long and the algorithms are sometimes too complex, again limiting their application.

The above approaches intrinsically use open-loop control. Usually, a closed-loop control strategy is expected to better handle uncertainties and disturbances. In [1] and [14], model predictive control (MPC) is employed to design closed-loop controllers. In [1], a near-optimal switching scheme in an MPC framework, incorporated with a binary integer programming (BIP) method at each step, is proposed to reduce the TOU and maximum-demand-based electricity costs. The branch-and-bound algorithm for the BIP is recursively applied to obtain an optimal solution at every step of the MPC. Similarly, in [15], an MPC approach (formulated as a BIP problem at every step and solved with an adapted branch-and-bound algorithm) is proposed to provide closed-loop feedback control for the optimal operation of a twin-rock winder system. A common feature of the above-mentioned optimal scheduling problems is the periodically repeated manner in which the optimal solutions are implemented along a horizon of time. These are summarized in [16] as a special class of optimal dynamic resource allocation problems. An MPC approach is proposed in [16], with proven convergence and robustness.

In the above MPC controllers, fast optimization was not addressed, since the focus was on the disturbance rejection capabilities of the MPC controllers. However, a feasible implementation of the MPC as a closed-loop controller relies on fast optimization. Practical application of the MPC requires a computationally efficient algorithm. One possibility is the

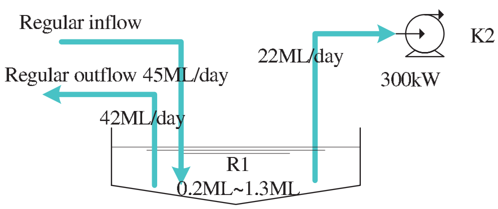

Fig. 1. Schematic of the water purification plant.

use of multiparametric programming [17]. In this brief, an alternative approach is proposed in the optimal operational scheduling of pumping stations.

The main contributions of this brief are as follows: 1) the optimal scheduling problem of pump operation is reformulated into a control sequence (CS) optimal scheduling problem (CS problem) and 2) a fast computation algorithm—reduced dynamic programming algorithm (RDPA)—is proposed for a class of BIP problems and is incorporated with an MPC strategy for the optimal operation of a pumping station.

This brief is organized as follows. The background and the problem formulation are given in Section II, while the solution is given in Section III. A case study is presented in Section IV, and conclusions are drawn in Section V.

II. Problem Formulation

A water purification plant in South Africa has been chosen for a case study, as shown in Fig. 1. This plant is also used here as a baseline for algorithm comparison.

A detailed description of the plant can be found in [1] and a brief one for the operation is given below.

The plant could be simplified as a single-reservoir single-pump system, where the operation of the pump K2 in [1] is required to be scheduled. The fixed-speed pump's flow capacity is 22 Ml/day and the rating power is 300 kW. The regular inflow rate to R1 and the outflow rate from R1 are 45 and 42 Ml/day, respectively.

The level of the reservoir R1, owing to the beneficial reservoir capacity, should be below 1.3 Ml for safety considerations. The level should be above 0.2 Ml, which is determined by the dead reservoir capacity.

The TOU pricing structure[1] is employed. The tariff is 82.05 cents per kWH during the peak time period (7:00–10:00, 18:00–22:00), 14.11 cents per kWH during the standard time period (6:00–7:00, 10:00–18:00), and 11.87 cents per kWH during the off-peak time period (0:00–6:00, 22:00–24:00).

The electricity cost within the time period $[t_0, t_f]$ is

$$J = \sum_{s=1}^{S} P u_s C_s dt \tag{1}$$

where $P = 300$ kW is the pump's rating power, and $C(t)$ and $u(t)$ are the electricity price and the pump's state at time t, respectively [when the pump is on (off), $u(t) = 1$ $(u(t) = 0)$.]

According to the water mass balance of R1, there exists

$$\frac{dv(t)}{dt} = \frac{45-42}{24} - \frac{22}{24}u(t) = \frac{1}{8} - \frac{11}{12}u(t) \tag{2}$$

[1]The winter pricing system in [1] is used for illustration purposes. The rand is the currency unit of South Africa, and 1 rand = 100 cents.

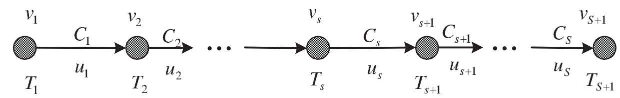

Fig. 2. Process schematic of the CS problem.

where $v(t)$ is the water volume in R1 at time t and

$$B_l \le v(t) \le B_u \qquad \forall t \tag{3}$$

with $B_l = 0.2$ Ml and $B_u = 1.3$ Ml.

Thus the optimal scheduling problem for the pump is as in [18], i.e., to find a scheduling ON/OFF CS $\{u_1, \ldots, u_N\}$ and the corresponding switching time sequence $\{t_1, \ldots, t_{N-1}\}$ within the period $[t_0, t_f]$, such that the electricity cost is minimized while the level of R1 is constrained with the bounds.

This problem is further reformulated into a CS problem, in which the time sequence $\{t_1, \ldots, t_{N-1}\}$ is given, and the objective is to optimize the CS $\{u_1, \ldots, u_N\}$.

According to the TOU pricing structure, one day (24 h) is equally divided into $S = 24/DT$ sampling periods where DT is the sampling period, and the water volume at sampling time $T_s = sDT, s = 1, \ldots, S$ is v_s. The operational variables in 24 h could be depicted as shown in Fig. 2.

Assuming that within the period $t \in [T_s, T_{s+1})$, the electricity price is a constant C_s and the pump control is a constant u_s, the state transition equation (4) is obtained by (2) discretized with the sampling period DT

$$v_{s+1} = v_s + b_0 + b_1 u_s, \qquad s = 1, 2, \ldots, S \tag{4}$$

where $b_0 = (1/8)DT$, and $b_1 = -(11/12)DT$. The energy consumption within $t \in [T_s, T_{s+1})$ is Pu_s and the corresponding electricity cost is $Pu_s C_s$.

Then the electricity cost within the 24 h is

$$J = \sum_{s=1}^{S} P u_s C_s. \tag{5}$$

The cost-efficient scheduling of the pump operation could be rewritten in the following form:

$$\begin{aligned} &\min J \\ &\text{subject to } u_s \in \{0,\ 1\}, \quad v_{s+1} \in [B_l, B_u] \end{aligned}$$

and (4).

III. Solutions

A. BIP

With the TOU tariff pricing system and the binary integer domain of control considered, the BIP approach is a natural choice for the CS problem of the pump operation.

Considering the binary property of $u_s \in \{0, 1\}$, the CS problem is a BIP problem which could be solved with the branch-and-bound algorithm [19], for instance, in [1], the bintprog in MATLAB with the branch-and-bound algorithm is employed for such a problem.

With this algorithm, when DT is large, the load during the peak time interval sometimes fails to be shifted to the off-peak time interval (shown in the following section). This is because of the stringent constraint on the control, i.e., the control should be kept on within the time interval $t \in [T_s, T_s + DT]$. When the time interval DT is decreased, it is hoped that load-shifting could be achieved. However, this will increase the dimensionality of the BIP problem, which would lead to much more computational time.

Remark 1: From the point of view of optimization, the smaller the DT, the better the optimization. However, when DT is too small, the operational frequency may be large, which will shorten the life duty cycle of the pump and increase the starting cost. The penalties for the life duty cycle and the starting cost are not considered in this brief, because they can be ignored when DT is sufficiently large.

B. RDPA

The branch-and-bound algorithm is employed in solving a BIP problem. In the algorithm, a search tree is created by repeatedly adding constraints to the problem, which is called "branching." At a branching step, the algorithm chooses a variable x_j whose current value is not an integer and adds the constraint $x_j = 0$ to form one branch and the constraint $x_j = 1$ to form the other branch. This process can be represented by a binary tree, in which the branch nodes represent the added constraints. The algorithm could potentially search all 2^n binary integer vectors, with n the number of variables. A complete search might take a very long time.

Fortunately, for a class of BIP problems, a RDPA can be employed to replace the branch-and-bound algorithm such that the stage values and the searching tree can be significantly reduced.

Consider the state transition equation

$$x_{s+1} = x_s + b_0 + b_1 u_s, \qquad s = 1, \ldots, S \tag{6}$$

where $x_s \in [B_l, B_u] \subset R$ is the state variable, $u_s \in \{0, 1\}$ is the control variable, and b_0, b_1 are constants. Equation (6) is rewritten as

$$x_{s+1} = (x_1 + sb_0) + b_1 \left(\sum_{i=1}^{s} u_i \right), \qquad s = 1, \ldots, S. \tag{7}$$

When $u_i \in \{0, 1\}$, $(\sum_{i=1}^{s} u(i)) \in \{0,\ 1, \ldots, s\}$, which implies that, at the $(s+1)$th sampling time point, the possible values of x_{s+1} are

$$x_{s+1}^1, x_{s+1}^2, \ldots, x_{s+1}^{s+1}$$

with $x_{s+1}^m = (x_1^1 + sb_0) + (m-1)b_1,\ m = 1, 2, \ldots, s+1$, constituting $(s+1)$ nodes.

It can be seen that the number of nodes at stage s is reduced from 2^s (BIP with branch-and-bound algorithm) to s.

Furthermore, from sampling time s to sampling time $s+1$, the possible control is and can only be zero or 1, which implies that for a node $x_{s+1}^{m+1} = (x_1^1 + sb_0) + mb_1, m = 1, \ldots, s-1$, at the sampling time $s+1$, can only be transferred through (7) from $x_s^{m+1} = x_1^1 + (s-1)b_0 + mb_1$ at sampling time s with the control $u_s^{m+1} = 0$, or from $x_s^m = x_1^1 + (s-1)b_0 + (m-1)b_1$ at sampling time s with the control $u_s^m = 1$. The first possible node $x_{s+1}^1 = x_1^1 + sb_0$ at the sampling time $s+1$ can only be transferred from $x_s^1 = x_1^1 + (s-1)b_0$ with $u_s^1 = 0$, while the last possible node $x_{s+1}^{s+1} = (x_1^1 + sb_0) + sb_1$ can only be transferred from $x_s^s = x_1^1 + (s-1)b_0 + (s-1)b_1$ with $u_s^s = 1$.

Algorithm 1 RDPA

Require: x_0, B_l, B_u, S
  for $s = 1, \ldots, S+1$ **do**
    for $m = 1, \ldots, s$ **do**
      Calculate cost from initial state to state x_s^m: J_s^m according to (8)
      if $x_s^m \notin [B_l, B_u]$ **then**
        $J_s^m = \infty$
      end if
      Record CS and state transition for state x_s^m: D_s^m, X_s^m
    end for
  end for
  for $m = 1, \ldots, S+1$ **do**
    Calculate $J_{S+1}^m = J_{S+1}^m + f_1(x_{S+1}^m)$
  end for
  Find $k = arg\ min J_{S+1}^m$
  $J^* = J_{S+1}^k, D^* = D_{S+1}^k, X^* = X_{S+1}^k$

At the sampling time $s+1$, the cost function

$$J_{s+1}^m = \min\{f(x_s^m, x_{s+1}^m, 0) + J_s^m, f(x_s^{m-1}, x_{s+1}^m, 1) + J_s^{m-1}\} \tag{8}$$

where J_{s+1}^m is the cost function value from x_1^1 to x_{s+1}^m and $f(x1, x2, u)$ is the increase in the cost function within the transformation from $x1$ to $x2$ with the control $u \in \{0, 1\}$.

When the number of nodes at stage s and $s+1$ is s and $s+1$, respectively, with the above observations, the number of possible routes from stage s to stage $s+1$ is only $2(s-1)+2 = 2s$ and the compared times of the cost function values will be only $2(s-1)$, while in a conventional DP algorithm the number of the corresponding possible routes is $s(s+1)$ and the compared times of the cost function values are $s(s+1)$. Then the computation could be much less than that with the branch-and-bound algorithm and that with the conventional DP algorithm.

Remark 2: For a class of integral systems, the BIP problem is reformulated as a DP problem, in which the number of nodes at intermediate stages is reduced. Furthermore, the number of the possible state transformation routines from a stage to the consecutive stage is reduced, too.

Based on these observations, an RDPA is formulated for the CS problem with the state transition equation in (6) and the cost function to be minimized as follows:

$$\begin{aligned} J(x_1, x_2, \ldots, x_S, x_{S+1}, u_1, u_2, \ldots, u_S) \\ = f_0(x_1) + \sum_{s=1}^{S} f(x_s, x_{s+1}, u_s) + f_1(x_{S+1}) \end{aligned} \tag{9}$$

where $f_0(x_1)$ is the cost determined by the initial state, $f_1(x_S)$ is the cost determined by the final state, and $f(x_s, x_{s+1}, u_s)$ is the operational cost within the time interval $t_0 + sDT$ from the state x_s to x_{s+1}.

Remark 3: The above algorithm is intrinsically a DP one and just takes advantage of options that the constraints impose. The convergence and optimality are the same as those of DP.

Algorithm 2 MPC Algorithm

Initialization ($x(1) = x(t_0)$, $s = 1$).

1) Measure $x(s)$, and apply RDPA to find the optimal scheduling CS $u^s_{opt} = \{u^s_s, u^s_{s+1}, \ldots, u^s_{s+TL}\}$, where TL is the optimization time length.
2) Implement u^s_s at the time interval s in (6)

$$x_{s+1|s+1} = x(s) + b_0 + b_1 u^s_s.$$

Let $s = s + 1$, and go to step (1).

However, the solution is constrained to only search over the practical control domain, which could significantly reduce the searching time.

Remark 4: In the above scheduling solutions for CS problem, it can be seen from (2) that the water volume within a sampling interval $t \in [T_s,\ T_{s+1}]$ is monotonic if there is no disturbance or unmodeled dynamics

$$\min\{\upsilon(T_s),\ \upsilon(T_{s+1})\} \le \upsilon(t) \le \max\{\upsilon(T_s),\ \upsilon(T_{s+1})\}. \quad (10)$$

In the scheduling, the values of $\upsilon(T_s),\ \upsilon(T_{s+1})$ are limited within the range of constraints. So $\upsilon(t), t \in (T_s, T_{s+1})$ is also limited within the range of the constraints. Thus there is no violation of the level constraints.

In practice, violations of the constraints are generally not permitted. If the volume reaches the lower bound, the pump cannot work and stops. If the volume reaches the upper bound, the pump will start in this emergency circumstance. Furthermore, if the operation of the pump cannot independently release the surplus water in the reservoir, some operations of other structures, such as sluice gates, will be executed. In the following simulation, the operations of other structures are not considered and thus the water is assumed to be stored in R1.

In the above solutions, disturbances are not considered. It is expected that closed-loop control will be applied to attenuate the disturbances. To improve the attenuation capacity of disturbances, a class of closed-loop control incorporating the RDPA is introduced.

C. RDPA for CS Problem in a Closed-Loop Strategy

The proposed RDPA optimization algorithm can be adopted in an MPC strategy and thus an optimal closed-loop control can be reached. In the MPC strategy, the RDPA optimization is periodically executed for fixed time intervals as the following algorithm given in [16] for (6).

A reoptimization using the RDPA improves the stability and the safety during an external disturbance and it could compensate for the inaccurate or unmodeled dynamics in the model owing to its intrinsic characteristic of error feedback control [16]. The fast computation of RDPA facilitates the practice of the MPC strategy.

As described in the above sections, the optimization time length is 24 h, which is preset as the predictive time length of the MPC control. The time interval of the MPC strategy is the same as the sampling period DT.

Remark 5: If the predictive time length is set as 24 h, the above process is a perfect choice for optimizing the pump (station) operation. For such kind of problems, the convergence and robustness of the periodic reoptimization in an MPC strategy are proved in [16].

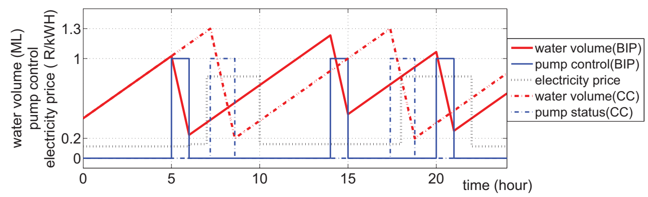

Fig. 3. Simulation results with CC and BIP ($DT = 1$ h).

With the optimal scheduling in MPC strategies (closed-loop control), the disturbance and unmodeled dynamics can be attenuated. If the pump is operated just according to the periodic scheduling, a violation of the water volume constraints may occur. To ensure a reservoir level that satisfies the constraints, the level is checked online. Once the water volume reaches the bounds, optimal scheduling will be triggered and a new optimal control will be scheduled and applied (this is a kind of emergency operation policy). With this policy, the optimal scheduling process is triggered not only periodically but also by the violation of the water volume. Thus the constraint violation is limited.

IV. Simulation Experiments and Results

A. Conventional Control (CC)

Traditionally and practically, CC based on real-time water volume is applied. Such a kind of closed-loop control is very simple and convenient for implementation. When $\upsilon(t) = B_u$, the pump is switched on; when $\upsilon(t) = B_l$, the pump is switched off; and when $B_l < \upsilon(t) < B_u$, the pump's present ON/OFF status is kept.

With the initial state of 0.4 Ml, a fraction (24 h) of 30-day simulation with CC is given in Fig. 3, where the dash and dotted lines represent the water volume and pump control, respectively.

B. BIP for the CS Problem

With $DT = 1$, the optimization scheduling problem is solved by MATLAB with a branch-and-bound algorithm.[2] The initial water volume is 0.4 and the result is shown in Fig. 3, where the solid lines represent the water volume and pump control. The computational time is 2.3423 s.

From Fig. 3, it is seen that the pump is switched on within the peak time period, and the load during the peak time period fails to be shifted to the off-peak time period. This is because of the stringent constraint on the control, i.e., the control should be kept on within the time period $t \in [T_s, T_s + DT]$. When the time interval DT is decreased, it is hoped that load shifting will be accomplished. However, this will increase the dimensionality of the BIP problem, which leads to much more computational time. When DT is decreased to 0.5 h, the algorithm is terminated after the computational time has

[2] All the algorithms in this brief are processed with MATLAB in the same computer: Dual core, 1.6 GHz CPU frequency, and 1 GB RAM.

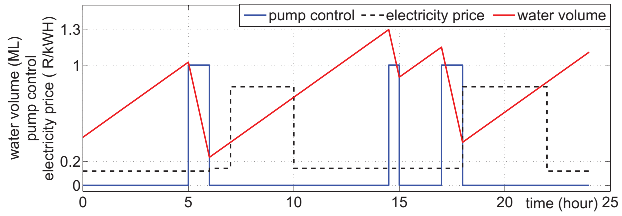

Fig. 4. RDPA result with $DT = 0.5$ h (48 stages).

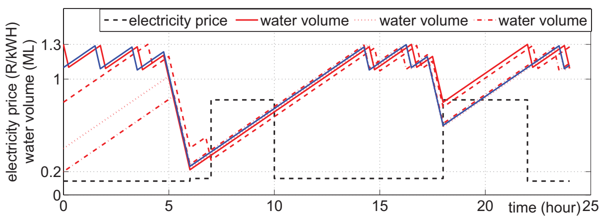

Fig. 5. RDPA result with different initial water volumes.

reached 3494 s because the maximum nodes were reached without converging.

C. RDPA

With the above-mentioned RDPA, a 24-h optimization result is the same as the BIP results shown in Fig. 3. However, the computational time is 0.053 s, which is $1/44$ of the time in the direct BIP algorithm.

When $DT = 0.5$ h, the simulation result is shown in Fig. 4. The computational time is 0.305 s.

Remark 6: With different initial water volumes (0.2, 0.4, 0.8, 1.1, and 1.3 Ml), the simulation results are given in Fig. 5. It can be seen that, although the initial water volumes are different, the operations between 6:00 and 22:00 (peak time periods) are very similar, and the final values of the water volumes are close to 1.1 Ml. This implies that the initial value of the water volume has no impact on the optimization results. In this brief, all the simulations are done with an initial water volume of 0.4 Ml to demonstrate the feasibility of the proposed algorithm.

A comparison of the algorithms is summarized in Table I, where the data with all algorithms are calculated with the 30-day values. The energy consumption with different algorithms is not equal, because the final water volumes with the algorithms are different. When the final water volumes are equal, the energy consumption is the same.

From Table I, it is obvious that the optimization algorithms have shifted the load from the peak time period to off-peak time and thus the energy cost is reduced considerably although the energy consumption is almost the same. This is the reason for the study of optimal scheduling of pump operation under a TOU electricity pricing structure.

From the comparison and considering the computational time and cost efficiency, the RDPA for the CS problem is a good choice for pump operation scheduling, although the computational time increases as the time interval DT decreases.

TABLE I
COMPARISON OF THE ALGORITHMS OVER A 30 DAY PERIOD

Algorithm	CC	BIP ($DT = 0.5$)	RDPA ($DT = 0.5$)	RDPA ($DT = 0.25$)
Energy consumption (kWH)	29 190	–	29 250	29 175
Energy cost (Rand)	971 213	–	379 987	374 490
Average price (R/kWH)	33.27	–	12.99	12.84
Computational time per round	<0.01 s	infeasible within 3494 s	0.31 s	3.21 s

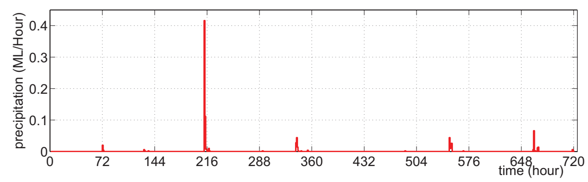

Fig. 6. Hourly precipitation record of the reservoir.

D. RDPA in MPC Framework

There are two classes of uncertainties in a reservoir. One results from the precipitation. The hourly precipitation record around the reservoir during the period from 0:00 on 9/21/2009 to 24:00 on 10/19/2009 is shown in Fig. 6 (provided by the South African Weather Service). The other one is the uncertainty of the inflow rate (low-frequency disturbance), which results from the inaccuracy of the modeling. This inaccuracy could be positive or negative. Two cases (-10% and $+10\%$) are considered in the following simulations. The rain disturbance (high frequency) mainly results in the emergency operation policy, while the inflow rate uncertainty mainly affects the long-term electricity cost of the control strategy.

In the following part, the inflow rate uncertainties vary from -10% to $+10\%$ of the net inflow for the pump K2. Simulations are done with and without rain disturbance. The results are given in Fig. 7. In the first subplot of Fig. 7, the inflow rate uncertainties exist while the rain disturbances do not. In the second subplot of Fig. 7, both the inflow rate uncertainties and the rain disturbances shown in Fig. 6 exist. In that figure, all data are obtained with 30-day simulations. The average price is the measure of the average energy price, which is equal to the quotient of energy cost and energy consumed.

From Fig. 7, the average energy cost prices in a control strategy under different circumstances are very similar, i.e., the average prices are around 33 cents/kWH with the CC strategy and about 13 cents/kWH with the RDPA control strategy. In those results, the constraint violations do not occur. All control strategies show good robustness. Even with the rain disturbances, the robustness of the three control strategies is shown from the observations of Fig. 7.

Remark 7: With the above MPC strategy incorporated into the RDPA, together with the emergency operation policy, the

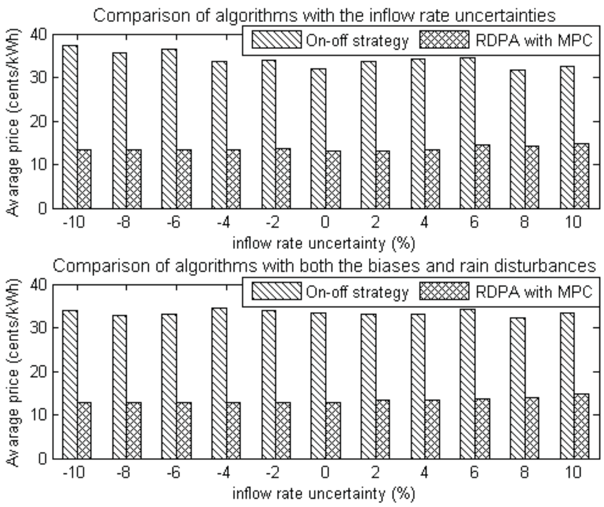

Fig. 7. Comparison of algorithms with disturbances.

constraint violations are limited and the robustness of control is improved.

However, if the disturbance is so large that the inflow of R1 is larger than the pump's capacity, constraint violations may occur with CC and RDPA in the MPC strategy. In such cases, the occurrence of the violations is dependent on the magnitude of the disturbance and the water volume when the disturbance occurs.

If the disturbances can be predicted, corresponding operations can be scheduled such that constraint violations might be avoided and operational performance can be improved.

V. Conclusion

The optimal scheduling problem of a kind of pumping station was reformulated into a CS problem, for which an RDPA was proposed to solve it, and the results were compared with those obtained with the direct BIP approach. The latter are not suitable because the computational time increases significantly when the number of stages increases, and sometimes a feasible solution cannot be reached even within a long time period.

The RDPA significantly reduces the computational time for the BIP problem, which was shown by the application in the optimal scheduling of the pump operation. Incorporated with the MPC strategy, the proposed RDPA can be applied to develop online closed-loop control for the pump operation such that the disturbances can be compensated for. The closed-loop behavior of the RDPA was investigated by means of simulation. The closed-loop control strategy was found to provide robust control and marginally increased operational cost, given a $\pm 10\%$ inflow rate uncertainty and a modest stochastic rainfall variability.

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