(16), this, together with (18), implies that there is some $c > 0$ such that $J(z(t_1^*), x^-(t_1^*), \tilde{u}) \leq \bar{J}_r(\mathcal{X}) - c$ and this contradicts the fact that $(z(t_1^*), x^-(t_1^*)) \in \partial\mathcal{I}_r^{ad}(\mathcal{X})$ since the latter would imply, according to (6) that $J(z(t_1^*), x^-(t_1^*), \tilde{u}) \geq \bar{J}_r(\mathcal{X})$. This ends the proof of Proposition 2. Indeed, the global aspects are immediate consequences of Proposition 1. $\diamond$

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Global Frequency Estimation Using Adaptive Identifiers

X. Xia

***Abstract*—Online estimation of the frequencies of a signal which is the sum of n sinusoidals with unknown amplitudes, frequencies and phases is made through yet another well-known and simple system theoretical tool—adaptive identifiers. Convergence of the proposed estimator is proved. The new frequency estimator is of $3n$ order, as compared to the order $5n - 1$ resulting from Marino–Tomei observers. Results are demonstrated via simulation.**

***Index Terms*—Adaptive filter, adaptive identifier, frequency estimation, observer.**

I. INTRODUCTION

Consider the problem of online estimation of the frequencies $\omega_i > 0$, $i = 1, \ldots, n, \omega_i \neq \omega_j$, for $i \neq j$, of a signal of the following form:

$$y(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i) \tag{1}$$

where $y(t)$ is measurable, the amplitudes, $A_i \neq 0$, the phase angles, φ_i, are constant but also unknown. For simplicity, the signal in (1) is unbiasd. However, the technique to be developed can also be applied to a signal with an unknown constant bias.

Though this estimation problem is an important one in systems theory with applications in diverse fields [2], most of the existing solutions have been sought from the perspective of signal processing and/or telecommunication: line enhancers [14], finite impulse response filters [13], infinite impulse response filters or notch filters [7], [10], [11], and frequency locked loop [6]. They are also local. The first globally convergent estimator was proposed only recently in [3] for the case of a single frequency. This global estimator is based on the adaptive notch filter (ANF) and takes the following form:

$$\begin{aligned}
\ddot{\xi} + 2\rho\hat{\omega}\dot{\xi} + \hat{\omega}^2\xi &= \hat{\omega}^2 y \\
\dot{\hat{\omega}} &= g\left(2\rho\dot{\xi} - \hat{\omega}y\right)\xi\hat{\omega} \\
g &= \frac{\epsilon}{\left\{1 + N\left[\xi^2 + \left(\frac{\dot{\xi}}{\hat{\omega}}\right)^2\right]\right\}(1 + \mu|\hat{\omega}|^\alpha)}
\end{aligned} \tag{2}$$

with $\alpha > 1$ and ϵ, N and ρ positive reals.

The paper [3] has stimulated several responses from the control theoretical community. First, it was found in [15] that a simple fourth order estimator can be designed through the so-called Marino–Tomei observers for the case of a single frequency. Though the estimator is one order higher than the one given in [3], it has a simpler and more of a control system theoretical structure, as well as a more elegant global stability proof. Independently, [5] obtained the same result via designing an adaptive observer for the case of a single frequency and generalized the method to multiple frequencies with an unknown constant bias. It is noted that the order of this estimator is $5n - 1$ for the case of n frequencies. Another solution was provided by the application of a linear tracking differentiator [1].

Manuscript received September 25, 2001; revised November 28, 2001. Recommended by Associate Editor P. Tomei.

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Publisher Item Identifier 10.1109/TAC.2002.800670.

In this note, a new solution is proposed by using yet another well-known and simple system theoretical tool—adaptive identifiers. Convergence of the proposed estimator is proved. The new frequency estimator is of $3n$ order. Results are demonstrated via simulation.

In Section II, the case of a single frequency is considered. The Marino–Tomei observer designed in [15] is reviewed for comparison purposes, followed by a third order estimator designed using adaptive identifiers. Note that the stability condition for the Marino–Tomei observer is slightly different from those given in [4] and [5]. The multiple frequency case is discussed in Section III. Section IV shows the simulation results and conclusions are drawn in Section V.

II. Global Estimator of a Single Frequency

Note that when $n = 1$ the sinusoidal signal in (1) satisfies

$$\ddot{y}(t) + \omega^2 y(t) = 0 \tag{3}$$

which has the following state-space realization:

$$\begin{aligned} \dot{x}_1 &= -\theta x_2 \\ \dot{x}_2 &= x_1 \\ y &= x_2 \end{aligned} \tag{4}$$

in which $\theta = \omega^2$. Perform the following filtered transformation, $\eta_1 = x_1 - \xi\theta$. $\eta_2 = x_2$, in which:

$$\dot{\xi} = -b\xi - y \tag{5}$$

and b is a positive real, then the system (4) is transformed into

$$\begin{aligned} \dot{\eta}_1 &= b\xi\theta \\ \dot{\eta}_2 &= \eta_1 + \xi\theta \\ y &= \eta_2. \end{aligned} \tag{6}$$

The system (6) is in the so-called adaptive observer form [4], thus admits a global adaptive observer

$$\begin{aligned} \dot{z}_1 &= b\xi\hat{\theta} + k_1 (y - z_2) \\ \dot{z}_2 &= z_1 + \xi\hat{\theta} + k_2 (y - z_2) \\ \dot{\hat{\theta}} &= \gamma\xi (y - z_2) \end{aligned} \tag{7}$$

in which γ is a positive real and k_1 and k_2 are chosen as [4], $k_1 = \lambda b$, $k_2 = \lambda + b$, with a positive λ.

A slightly more general result can be stated as: when $k_2 > b > 0$, $k_1 > 0$, $\gamma > 0$, the system (7) and (5) is a global adaptive observer of (4) with global parameter exponential convergence, i.e., as $t \to \infty$, $\|\hat{\theta}(t) - \theta\| \to 0$.

To prove this conclusion, defining

$$A_0 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b \\ 1 \end{bmatrix} \quad c_0 = [0 \quad 1].$$

Note that $c_0(sI - A_0 - kc_0)^{-1}\bar{b} = (s + b)/(s^2 + k_2 s + k_1)$ and this transfer function is positively real if and only if $k_1 > 0$, $k_2 > b$.

The rest of the proof follows exactly the same line as in the proof of [4, Th. 5.3.2]. To prove $\|\hat{\theta}(t) - \theta\| \to 0$, note that the persistency of excitation condition now reads as

$$\int_t^{t+T} \xi(\tau)^2 d\tau \geq k_0 > 0$$

which, by (5), holds, thus [4, Th. 5.3.3] applies and the convergence of the parameter is guaranteed.

Having the estimation $\hat{\theta}$ of θ, the frequency estimation can be obtained as $\hat{\omega} = \sqrt{\hat{\theta}}$.

Note the estimator given by (5) and (7) is of order 4. A third-order estimator is given as follows by making use of the technique of adaptive identifiers [12].

To develop such an estimator, first reparameterize (3) through formal Laplace transform of the both sides, ignoring the terms with initial conditions

$$s^2 y(s) = -\theta y(s).$$

Let λ_1 and λ_2 be two positive-real numbers, then

$$\left(s^2 + \lambda_1 s + \lambda_2\right) y(s) = \lambda_1 s y(s) + (\lambda_2 - \theta)\, y(s).$$

Denote $\phi = \lambda_2 - \theta$ and $\delta(s) = s^2 + \lambda_1 s + \lambda_2$, then

$$y(s) = \frac{\lambda_1 s}{\delta(s)} y(s) + \frac{\phi}{\delta(s)} y(s).$$

This relationship has the following state-space realization:

$$\begin{aligned} \dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= -\lambda_2\xi_1 - \lambda_1\xi_2 + y(t) \\ (t) &= \lambda_1\xi_2(t) + \phi\xi_1(t) \\ &= \lambda_1\xi_2(t) + \lambda_2\xi_1(t) - \theta\xi_1(t). \end{aligned} \tag{8}$$

Note that this last equality holds only when (8) is properly initialized: in general, the right-hand side of the equality differs from $y(t)$ by terms exponentially vanishing due to initial conditions ignored in the above derivation. Note also that (8) gives a parameterization containing one parameter for the unknown frequency. In this sense, it is a certain simplification of the external identifiers of [8] and [9] where a parameterization with two parameters per frequency was introduced.

Equation (8) is referred to as *the identifier structure* [12]. The identifier output

$$y_i(t) = \lambda_1\xi_2(t) + \lambda_2\xi_1(t) - \hat{\theta}\xi_1(t)$$

differs from the signal $y(t)$ by *an identifier error*

$$e(t) = y_i(t) - y(t)$$

due to different initialization of (8) and estimation $\hat{\theta}$ of θ.

Now the parameter update law can be generated in a number of ways as demonstrated in [12]. In this note, the standard *gradient algorithm*

$$\dot{\theta} = ge(t)\xi_1(t) \tag{9}$$

in which $g > 0$, or the *normalized gradient algorithm*

$$\dot{\theta} = g\frac{e(t)\xi_1(t)}{1 + \gamma\xi_1^2} \tag{10}$$

in which $\gamma > 0$, are used.

Equations (8)–(9) or (8)–(10) give a third-order estimator for θ.

The convergence of the parameter estimate $\hat{\theta}$ is guaranteed by the persistency of excitation condition of ξ_1 [12, Th. 2.5.3], i.e.,

$$\int_t^{t+T} \xi_1^2(\tau) d\tau \geq k > 0$$

is satisfied for some $T > 0$ and every $t \geq 0$. To see that ξ_1 is persistently excited, note from (8) that

$$\frac{\xi_1(s)}{y(s)} = H(s) = \frac{1}{\delta(s)}.$$

Since the sinusoidal signal $y(t)$ in (3) is sufficiently rich of order 2 with spectrum points at ω and $-\omega$ and the transfer function $H(s)$ is proper and stable and $H(j\omega) \neq 0$, then from the second half of [12, Th. 2.7.2], ξ_1 is persistently excited.

For a comparison with the adaptive notch filter (2) obtained in [3], a second-order differential equation in terms of ξ_1 is derived from (8)

$$\ddot{\xi}_1 + \lambda_1\dot{\xi}_1 + \lambda_2\xi_1 = y \tag{11}$$

and expanded form of (10) is written as

$$\dot{\theta} = g\frac{\left(\lambda_1\dot{\xi}_1 + \lambda_2\xi_1 - \hat{\theta}\xi_1\right)\xi_1}{1+\gamma\xi_1^2}. \tag{12}$$

It is noted that the basic structure of the adaptive notch filter and the adaptive identifier is similar. The third equation in (2) gives a special structure of parameter tuning, therefore, a different gain.

III. Global Estimator of n Frequencies

Since

$$y = \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i)$$

one has

$$\ddot{y} = \sum_{i=1}^{n} \kappa_i A_i \sin(\omega_i t + \varphi_i)$$

in which $\kappa_i = -\omega_i^2$. Similarly

$$\begin{aligned} y^{(4)} &= \sum_{i=1}^{n} \kappa_i^2 A_i \sin(\ omega_i t + \varphi_i), \\ &\vdots \\ y^{(2n)} &= \sum_{i=1}^{n} \kappa_i^n A_i \sin(\omega_i t + \varphi_i). \end{aligned} \tag{13}$$

Rewriting the previous first n equations

$$\begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(2n-2)} \end{bmatrix} = V \begin{bmatrix} A_1\sin(\omega_1 t + \varphi_1) \\ A_2 \sin(\omega_2 t + \varphi_2) \\ \vdots \\ A_n \sin(\omega_n t + \varphi_n)\end{bmatrix}$$

in which V is the Vandermonde matrix

$$V = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \kappa_1 & \kappa_2 & \cdots & \kappa_n \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_1^{n-1} & \kappa_2^{n-1} & \cdots & \kappa_n^{n-1}\end{bmatrix}.$$

Since $\omega_i \neq \omega_j$ for $i \neq j$, $\kappa_i \neq \kappa_j$ for $i \neq j$, the Vandermonde matrix V is nonsingular. Thus, one solves

$$\begin{bmatrix} A_1\sin(\omega_1 t + \varphi_1) \\ A_2 \sin(\omega_2 t + \varphi_2) \\ \vdots \\ A_n \sin(\omega_n t + \varphi_n)\end{bmatrix} = V^{-1}\begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(2n-2)} \end{bmatrix}.$$

Substituting this into (13), we have

$$\begin{aligned} y^{(2n)} &= (\kappa_1^n, \kappa_2^n, \ldots, \kappa_n^n)\, V^{-1}\left(y, \dot{y}, \ldots, y^{(2n-2)}\right)^T \\ &= -\theta_1 y^{(2n-2)} - \cdots - \theta_{n-1}\ddot{y} - \theta_n y. \end{aligned} \tag{14}$$

It can be verified that

$$\prod_{i=1}^{n}\left(s^2 + \omega_i^2\right) = s^{2n} + \theta_1 s^{2n-2} + \cdots + \theta_{n-1}s^2 + \theta_n.$$

So, $(\theta_1, \ldots, \theta_n)$ is an invertible reparameterization of the original n unknown frequencies $(\omega_1, \ldots, \omega_n)$. The estimation of $(\omega_1, \ldots, \omega_n)$ can then by obtained by first estimating $(\theta_1, \ldots, \theta_n)$.

To estimate $(\theta_1, \ldots, \theta_n)$, the technique of the adaptive identifiers is used.

Rewrite (14) in the s-domain by taking Laplace transform of both sides of the equation, ignoring the terms depending on initial conditions

$$s^{2n}y(s) = -\theta_1 s^{2(n-1)}y(s) - \cdots - \theta_{n-1}s^2 y(s) - \theta_n y(s).$$

Let $\delta(s) = s^{2n} + \lambda_{2n}s^{2n-1} + \cdots + \lambda_2 s + \lambda_1$ be a Hurwitz polynomial and denote

$$\phi_k = \lambda_{2k+1} - \theta_{n-k}$$

for $k = 0, \ldots, n-1$, then

$$\delta(s)y(s) = \sum_{k=1}^{n}\lambda_{2k}s^{2k-1}y(s) + \sum_{k=1}^{n}\phi_{k-1}s^{2(k-1)}y(s)$$

and thus

$$y(s) = \sum_{k=1}^{n}\left(\lambda_{2k}\frac{s^{2k-1}}{\delta(s)} + \phi_{k-1}\frac{s^{2k-2}}{\delta(s)}\right)y(s). \tag{15}$$

Denote

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\lambda_1 & -\lambda_2 & -\lambda_3 & \cdots & -\lambda_{2n}\end{bmatrix}, b_\lambda = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1\end{bmatrix}$$

and define

$$\dot{w} = \Lambda w + b_\lambda y \tag{16}$$

then the signal has the following time-domain realization, if (16) is properly initialized:

$$y(t) = \sum_{k=1}^{n}\lambda_{2k}w_{2k}(t) + \sum_{k=1}^{n}\phi_{k-1}w_{2k-1}(t).$$

If the identifier structure (16) is initialized differently and an estimate $\hat{\phi}_i$ is made for ϕ_i, then *the identifier output*

$$y_i(t) = \sum_{k=1}^{n}\lambda_{2k}w_{2k}(t) + \sum_{k=1}^{n}\hat{\phi}_{k-1}w_{2k-1}(t)$$

differs from the signal $y(t)$ by *an identifier error*

$$e(t) = y_i(t) - y(t).$$

Now, the parameter update law can be given as the following gradient algorithm:

$$\dot{\Theta} = ge(t)W(t) \tag{17}$$

where $\hat{\Theta} = (\hat{\theta}_1, \hat{\theta}_3, \ldots, \hat{\theta}_n)^T$, $W(t) = (w_1(t), w_3(t), \ldots, w_{2n-1}(t))^T$ and $g > 0$, or the normalized gradient algorithm

$$\dot{\Theta} = ge(t)\frac{W(t)}{1+\gamma W^T(t)W(t)} \tag{18}$$

where $\gamma > 0$.

Equations (16)–(17) or (16)–(18) give a $3n$th-order estimator for Θ. The convergence of the parameter estimation is guaranteed by the persistency of excitation of $W(t)$. To see that $W(t)$ is persistently excited, note from the construction of Λ and b_λ that

$$(sI - \Lambda)^{-1}b_\lambda = \frac{1}{\delta(s)}\left(1, s, \ldots, s^{2n-1}\right)^T$$

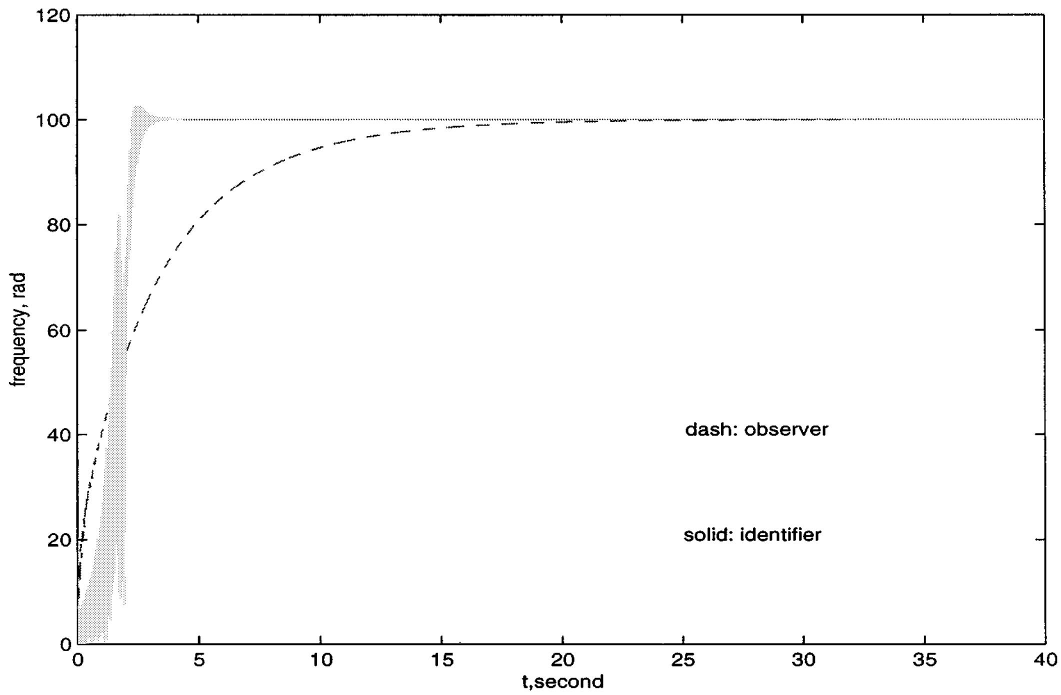

Fig. 1. Estimation of large frequency.

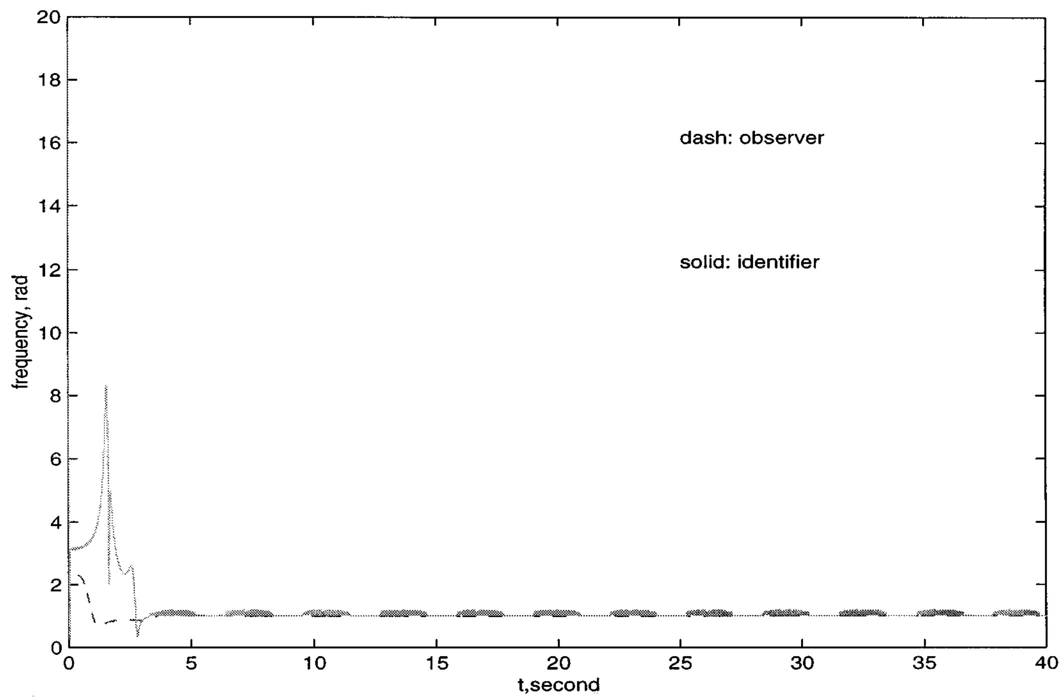

Fig. 2. Estimation of small frequency.

and the transfer function between $y(t)$ and $W(t)$ is

$$\frac{W(s)}{y(s)} = \frac{1}{\delta(s)}\left(1, s^2, \ldots, s^{2(n-1)}\right)^T = H_W(s)$$

since the sinusoidal signal $y(t)$ in (3) is sufficiently rich of order $2n$ with spectrum points at ω_i and $-\omega_i$, for $i = 1, \ldots, n$ and the transfer function $H_W(s)$ is proper and stable and $H_W(j\omega_i)$ are linearly independent for $i = 1, \ldots, n$, then from the second half of [12, Th. 2.7.2], $W(t)$ is persistently excited.

Since $\hat{\theta}_i$ is exponentially convergent, estimation of $-\omega_i^2$ (and therefore ω_i) can be computed as the zeros of the polynomial $s^n + \hat{\theta}_1 s^{n-1} + \cdots \theta_{n-1} s + \hat{\theta}_n$.

IV. SIMULATION

Simulation is carried out in the Matlab/Simulink environment.

First of all, the fourth-order observer is simulated against large and small frequencies. The parameters are tuned as $b = 1$, $\gamma = 1000$, $k_1 = 100$, $k_2 = 300$ and all initial conditions for the observer are set to 1.

For comparison, simulations of the third-order identifier are also carried out for the same signals. In this case, the identifer parameters are tuned as $\lambda_1 = 100, \lambda_2 = 300, g = 9\,000\,000, \gamma = 1000$ and all initial conditions are set to 1.

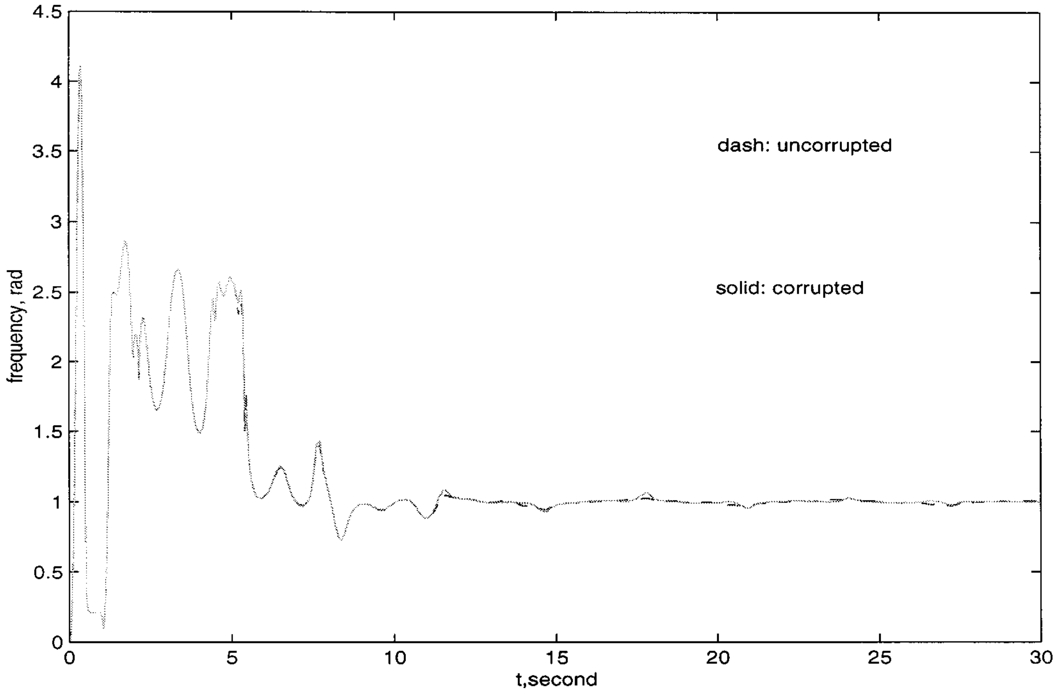

Fig. 3. Estimation of $\boldsymbol{\omega = 1}$.

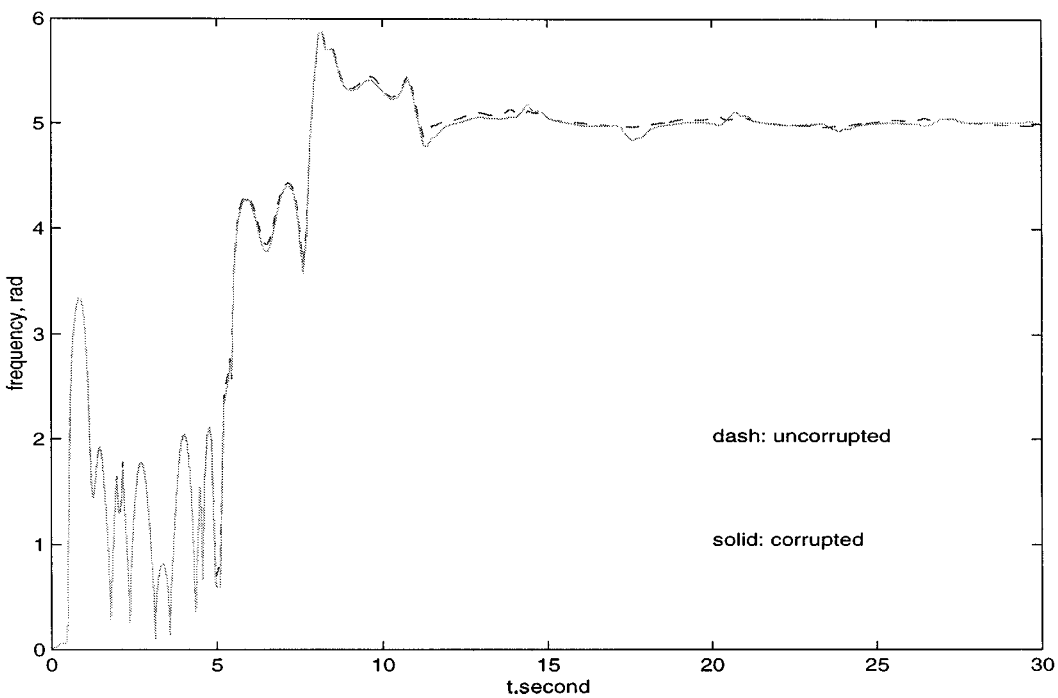

Fig. 4. Estimation of $\boldsymbol{\omega = 5}$.

Fig. 1 shows the simulation results when the signal is $y(t) = 40\sin(100t + 229.18°)$.

It is observed that a quicker estimation can be given by the adaptive identifier than by the adaptive observer. However, the initial response of the adaptive identifier undergoes very abrupt fluctuations.

Fig. 2 shows the simulation results when the signal is $y(t) = 3\sin(t + 229.18°)$.

It is observed that periodic steady state errors/fluctuations exist for the estimation given by the adaptive identifier. The response of the adaptive observer is also quicker. In practical situations, a further low pass filter might need to be cascaded with the adaptive identifier.

Next, it is assumed that the following signal with two frequencies is available for measurement, $y(t) = \sin(t) + 1.35\sin(5t)$.

Choose $\lambda_1 = 2.5$, $\lambda_2 = 5$, $\lambda_3 = 10$, $\lambda_4 = 3$, and therefore, the identifier structure is

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ \dot{\xi}_3 &= \xi_4 \\ \dot{\xi}_4 &= -3\xi_1 - 10\xi_2 - 5\xi_3 - 2.5\xi_4 + y(t) \end{aligned} \tag{19}$$

and the identifier output is $y_i(t) = 2.5\xi_4 + 10\xi_2 + (5 - \hat{\theta}_1)\xi_3 + (3 - \hat{\theta}_2)\xi_1$. The parameter update law is defined by the standard gradient algorithm in which $g_1 = g_2 = 7500$

$$\begin{aligned} \dot{\theta}_1 &= 7500(y_i(t) - y(t))\xi_3 \\ \dot{\theta}_2 &= 7500(y_i(t) - y(t))\xi_1. \end{aligned} \tag{20}$$

The estimations of ω_1 and ω_2 are then given by

$$\hat{\omega}_{1,2} = \sqrt{\frac{-\hat{\theta}_1 \pm \sqrt{\hat{\theta}_1^2 - 4\hat{\theta}_2}}{2}}.$$

The estimator consisting of (19) and (20) is a sixth-order one. In simulation, all initial conditions are set to be zero.

A simulation is also done where $y(t)$ is corrupted by a uniform random noise between -0.01 and 0.01.

Fig. 3 shows the convergence of the first estimated frequencies for both uncorrupted and corrupted version of $y(t)$. Fig. 4 shows the convergence of the second estimated frequencies for both uncorrupted and corrupted version of $y(t)$.

It can be observed that the estimations are accurate for both uncorrupted and corrupted signals. Simulation is also done for large corruptions, it is found that when corruptions are larger in magnitude, the steady state errors are bigger.

V. Conclusion

A design of adaptive identifiers to globally estimate the frequencies of a signal composed of n sinosuoidal components was shown. Convergence of the proposed estimator is proven. The new frequency estimator is of $3n$ order, comparing with the order $5n - 1$ of the estimator through Marino–Tomei observers. Results are demonstrated via simulation.

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On Semiglobal Stabilizability of Antistable Systems by Saturated Linear Feedback

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***Abstract*—It was recently established that a second-order antistable linear system can be semiglobally stabilized on its null controllable region by saturated linear feedback and a higher order linear system with two or more antistable poles can be semiglobally stabilized on its null controllable region by more general bounded feedback laws. We will show in this note that a system with three real-valued antistable poles cannot be semiglobally stabilized on its null controllable region by the simple saturated linear feedback.**

***Index Terms*—Actuator saturation, antistable systems, semiglobal stabilizability.**

I. Introduction

There has been a long history of exploring global or semiglobal stabilizability for linear systems with saturating actuators. In 1969, Fuller [1] studied global stabilizability of a chain of integrators of length greater than two by saturated linear feedback and obtained a negative result. This important problem also attracted the attention of Sussmann and Yang [9]. They obtained similar results independently in 1991. Because of the negative result on global stabilizability with saturated linear feedback, the only choice is to use general nonlinear feedback. In 1992, Teel [11] proposed a nested feedback design technique for designing nonlinear globally asymptotically stabilizing feedback laws for a chain of integrators. This technique was fully generalized by Sussman, Sontag and Yang [8] in 1994. Alternative solutions to global stabilization problem consisting of scheduling a parameter in an algebraic Riccati equation according to the size of the state vector were later proposed in [7], [10], and [12].

Another trend in the development, motivated by the objective of designing simple controllers, is semiglobal stabilizability with saturated linear feedback laws. The notion of semiglobal asymptotic stabilization for linear systems subject to actuator saturation was introduced in [5] and [6]. The semiglobal framework for stabilization requires feedback laws that yield a closed-loop system which has an asymptotically stable equilibrium whose domain of attraction includes an *a priori* given (arbitrarily large) bounded subset of the state space. In [5] and [6], it was shown that, a linear system can be semiglobally stabilized by saturated

Manuscript received September 28, 2001; revised January 13, 2002. Recommended by Associate Editor J. Huang. This work was supported in part by the US Office of Naval Research Young Investigator Program under Grant N00014-99-1-0670.

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Publisher Item Identifier 10.1109/TAC.2002.800671.