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A semi-discrete approach to modelling and control of the continuous casting process

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Starting from a partial differential equation describing the heat transfer in the continuous casting of steel, the Kirchhoff transformation is used to obtain a more simplified governing heat equation. A boundary state and control transformation is then introduced to obtain a non-linear heat equation with boundary control. The semi-discrete approximation is applied in the investigation to obtain an ordinary differential equation model. The derived lumped parameter control model can be used to get an approximate solution for the system as well as for finite dimensional system control studies. Finally, local controllability is proved for the system using a linearization technique.

Ein Ansatz zur Modellierung und Steuerung des Stranggießprozesses. Ausgehend von einer partiellen Differentialgleichung, die den Wärmeübergang beim Stranggießen von Stahl beschreibt, wird die Kirchhoff-Transformation herangezogen, um eine wesentlich einfachere, bestimmende Gleichung für die Wärme aufzustellen. Dann werden die Randbedingungen definiert und eine Steuerungstransformation eingeführt, um eine nicht-lineare Wärmegleichung mit Steuerung der Randbedingungen zu erhalten. Der halbdiskrete Ansatz wird auf die Untersuchung angewendet, um ein einfaches Differentialgleichungsmodell zu erhalten. Das hergeleitete Steuerungsmodell für die einzelnen Parameter eignet sich für eine Näherungslösung des Systems als auch für Steuerungsstudien. Schließlich wird die örtliche Regelbarkeit des Systems mit einem Linearisierungsverfahren nachgewiesen.

Continuous casting is widely used in the steel industry for the casting of different grades of steel. In the continuous casting process, the aim is to solidify molten steel into a solid structure with as few defects as possible. A brief description of the process is as follows. The molten steel arrives at the continuous caster in a ladle, **figure 1**. The ladle feeds the tundish which acts as a reservoir of molten steel. The tundish feeds the mould with liquid steel through a stopper rod and submerged entry nozzle (SEN) system. The primary extraction of heat occurs in the water-cooled copper mould. Below the mould, heat is extracted from the strand by means of water sprays. This region is known as the secondary cooling zone (SCZ). In the SCZ, the strand is supported by rollers. After the secondary cooling zone, the strand cools off naturally in air in the radiation zone. After the radiation zone the strand is cut and sent for further processing such as rolling into sheet metal.

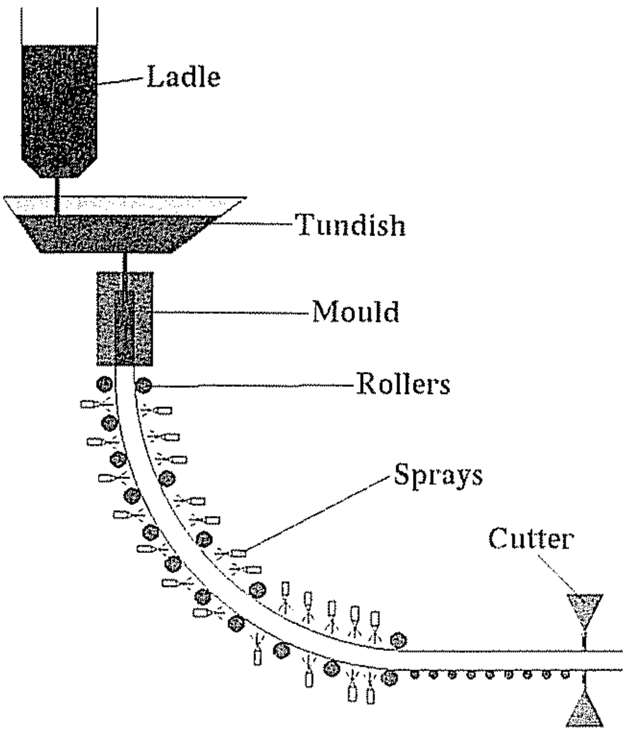

Figure 1 The continuous casting process

The primary control problem in the casting process is that of level control in the mould [2]. However, it was observed in [1] that improper cooling such as excessive reheating in the secondary cooling zone severely contributes to crack formation on the surface and in the interior of the strand. Therefore, control of the temperature profile in the secondary cooling zone can contribute to improving the quality of the cast product. This control problem was first investigated by people outside the control field. Besides [1], it is worthwhile mentioning two earlier papers [3; 4]. In these papers, the effect of the water spray profile on the cooling of billets, blooms and slabs was studied by numerical simulations of the governing partial differential equations. Some recent works along the same direction can be found in [5 7].

It should be noted that the numerical analysis dominates the research methods for this problem due to the highly non-linear nature of the PDE model and its boundary control. More recently, [8] dealt with the problem from a control point of view where an optional steady-state water spray profile was obtained from an open-loop control design problem. Moreover, in [9], a simple difference scheme was adopted for the investigation of the controllability of the linearized system which utilises insight into the axial heat transfer due to the water spray in the secondary cooling zone.

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Generally speaking, the mathematical models describing the continuous casting process, whether dynamic or steady, are non-linear heat equations with non-linear boundary controls in the SCZ. Because of the highly non-linear and partial differential equation nature of these equations, very few theoretical results for their solution are available in literature. Discrete solutions, in which co-ordinate variables are discretized, result in very large system matrices for which often only numerical solutions are possible.

The object of this paper is to obtain a lumped control system describing the continuous casting process of steel by applying the semi-discrete approximation to a non-linear heat equation with boundary control. The Kirchhoff transformation is used to transform the original governing equation into a simplified form and then boundary state and control transformations are applied to obtain a non-linear heat equation with boundary control. A semi-discrete approximation is applied to obtain an ordinary differential equation model. The derived lumped parameter control model can be used to easily get the approximate solution of the system using many different numerical methods for ordinary differential equations, as shown in [10]. In addition, because this model is in the form of a typical non-linear lumped control system, the fruitful results on non-linear control systems can be used to theoretically study important problems in continuous casting. As an example, local controllability is presented here for the ordinary differential equation system using a linearization technique.

Non-linear state and control transform

For simplicity, it is assumed that the strand of steel is a rectangular parallel piped although the approach used here is certainly applicable to other shapes. The general model describing the temperature distribution of the steady-state continuous casting process can be written as the following non-linear heat transfer equation with boundary control [5]:

$$\begin{cases} \rho(W)[c(W)+Lf(W)]\dfrac{\partial W}{\partial t} = div(K(W)\nabla W), \\ (x,y)\in\Omega, \quad 0\le t\le t^*, \\ -K(W)\dfrac{\partial W}{\partial n} = Q(x,y,t,W), \quad (x,y)\in\Gamma, \\ W(x,y,0) = W_m \end{cases} \tag{1}$$

where $z = vt$ is the position which the steel with cross-section $\Omega = \{0 < x < a,\ 0 < y < b\}$ is moving at the time-invariant speed v. $W(x,y,t)$ is the temperature at a point (x,y) $\Omega\cap\Gamma$ and time t, where Γ is the boundary of Ω. $c(W)$ denotes the specific heat, $\rho(W)$ the density and $K(W)$ the thermal conductivity. W_m is the initial temperature of the mould, n the outward normal unity vector of Γ and $t^* = t_1^* + t_2^* + t_3^*$ where vt_i^*, $i = 1, 2, 3$ denote the length of the mould, secondary cooling zone and the radiation zone, respectively. L is the latent heat and $f(W)$ is a function which describes the solid-phase fraction variation with temperature. Q is the heat flux on the boundary:

$$Q(x,y,t,W) = \begin{cases} Q_t\left[-\dfrac{3(1-c_w)}{2+c_w}\left(\dfrac{r_d - r}{r_d}\right)^2 + \dfrac{3}{2+c_w}\right] \\ \text{in the mould;} \\ h(W - W_{H_2O}) + \sigma\varepsilon(W^4 - W_{ext}^4), \text{ in the SCZ;} \\ \sigma\varepsilon(W^4 - W_{ext}^4), \quad \text{in the radiation zone} \end{cases} \tag{2}$$

where W_{H_2O} is the spray-water temperature, W_{ext} the ambient spray zone temperature, σ the Stefan-Boltzmann constant, ε the emission factor and h the heat-transfer coefficients which, in the secondary cooling cone, is actually determined by water sprays and hence the real control variable is assumed to be (see e.g. [1]):

$$Q_t = \frac{6(\alpha - \beta\sqrt{t_c})}{1+2c_a}\left(1 + \sqrt{\frac{t}{t_c}}(c_a - 1)\right)$$

where α and β are constants, and t_c is the dwell time in the mould. C_w is a constant representing the ratio of the heat flux in the corner of the mould relative to the heat flux at the middle surface. r_d is half of the width of the mould. c_a is the ratio of the heat flux at the mould exit to the heat flux at the top level of liquid steel (meniscus) in the mould. $r = x$ at the $(x,0,t)$ or (x,b,t) and $r = y$ at the $(0,y,t)$ or (a,y,t).

Applying the Kirchhoff transformation (see [5]):

$$T = g(W) = \int_{W_0}^{W} K(\rho)d\rho \tag{3}$$

which is an invertible transformation for any given W_0, one obtains:

$$T_t = K(W)W_t, \quad \nabla T = K(W)\nabla W, \qquad \frac{\partial T}{\partial n} = K(W)\frac{\partial W}{\partial n} \tag{4}$$

Equation (1) is then transformed into

$$\begin{cases} T_t = F(T)\Delta T, \quad (x,y)\in\Omega, \quad 0\le t\le t^*, \\ -\dfrac{\partial T}{\partial n} = Q(x,y,t,g^-(T)), \quad (x,y)\in\Gamma, \\ T(x,y,0) = g(W_m) \end{cases} \tag{5}$$

where:

$$F(T) = \frac{K(g^-(T))}{\rho(g^-(T))[c(g^-(T)) + Lf(g^-(T))]} = \frac{K(g^-(T))}{H'(g^-(T))} \tag{6}$$

where $H = H(W)$ is the enthalpy-temperature relationship of the steel in question. The function $F(T)$ is constructed using the enthalpy-temperature relationship (derived from the enthalpy-temperature relation) and the thermal conductivity-temperature relationship. An example of these four graphs for a low carbon steel is depicted in **figure 2**.

The enthalpy temperature relationship clearly shows the included latent heat in the mushy zone, and the thermal conductivity-temperature relationship shows the effect of convective mixing in the mushy zone. It is because of these sharp gradients that such a rapid increase occurs in the function $F(T)$. However, to accurately define the physical meaning of $F(T)$ is complex and it should only be seen as a mathematical tool in the solution of the system.

Because of symmetry, only one quarter region $\Omega = \{(x,y)|0 < x < a/2,\ 0 < y < b/2\}$ is considered, that is

$$\begin{cases} \dfrac{\partial T}{\partial t} = F(T)\Delta T, & 0 < x < a/2, 0 < y < b/2, \\ T_x(a/2, y, t) = T_y(x, b/2, t) = 0, & \\ T_y(x, 0, t) = u_1(x, t), \quad T_x(0, y, t) = u_2(y, t), & \\ T(x, y, 0) = g(W_m) & \end{cases} \qquad (7)$$

where u_1 and u_2 are obtained from the feedback control transformation:

$$u_1(x,t) = \begin{cases} Q_t\left[-\dfrac{3(1-c_W)}{2+c_W}\left(\dfrac{a/2-x}{a/2}\right)^2 + \dfrac{3}{2+c_W}\right], & \text{in the mould;} \\ h(g^-(T) - T_{H_2O}) + \sigma\varepsilon(g^-(T)^4 - W_{ext}^4), & \text{in the SCZ;} \\ \sigma\varepsilon(g^-(T)^4 - W_{ext}^4), & \text{in the radiation zone,} \end{cases}$$

$$u_2(x,t) = \begin{cases} Q_t\left[-\dfrac{3(1-c_W)}{2+c_W}\left(\dfrac{b/2-y}{b/2}\right)^2 + \dfrac{3}{2+c_W}\right], & \text{in the mould;} \\ h(g^-(T) - T_{H_2O}) + \sigma\varepsilon(g^-(T)^4 - W_{ext}^4), & \text{in the SCZ;} \\ \sigma\varepsilon(g^-(T)^4 - W_{ext}^4), & \text{in the radiation zone.} \end{cases} \qquad (8)$$

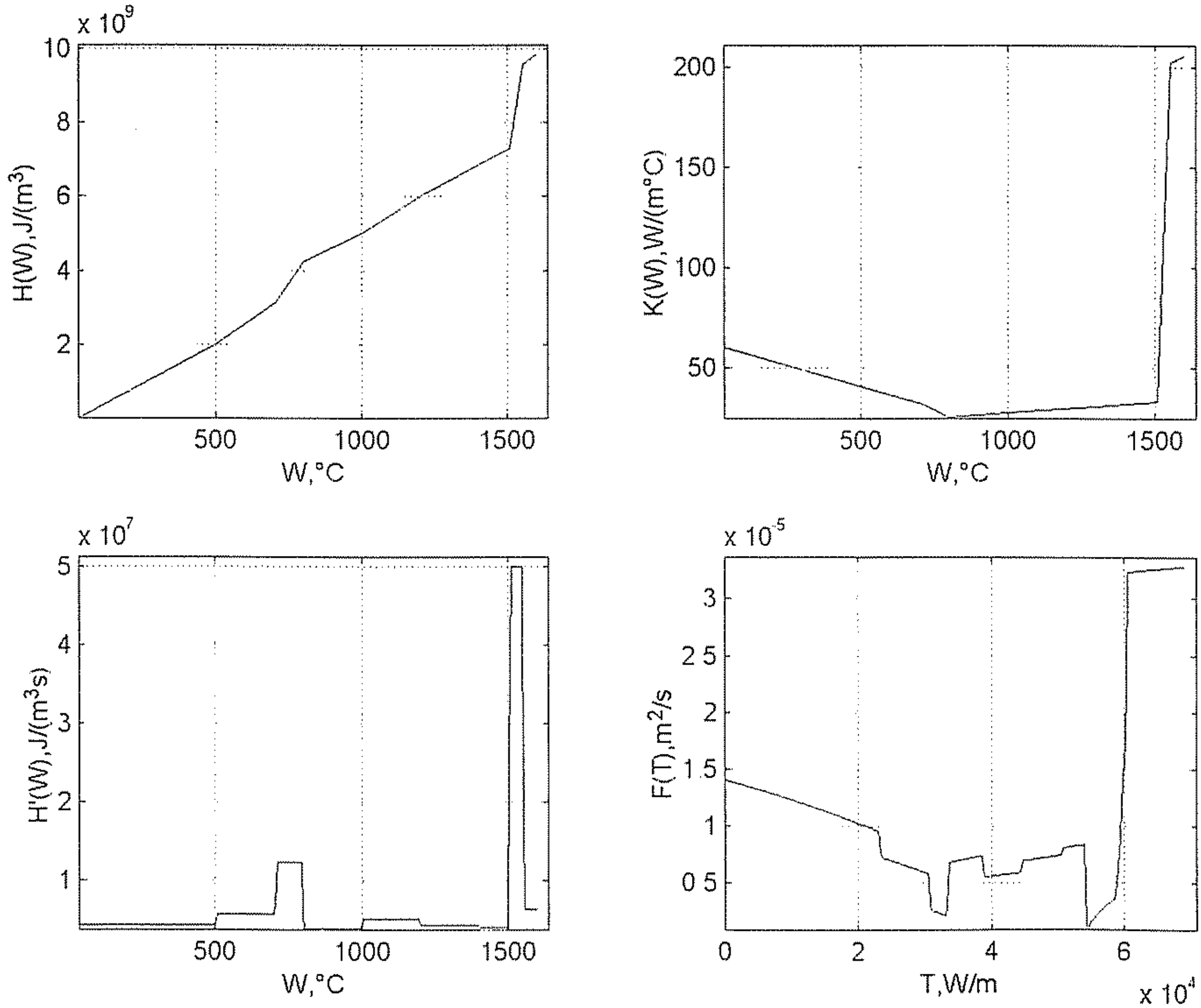

Figure 2. The functions $H=H(W)$, $H'=\partial H/\partial W$, $K=K(W)$, and $F=F(T)$

u_1, u_2 can be considered as the control inputs in the secondary cooling zone. It can be seen that the transformed equation (7) is equivalent to equation (1) but with a simpler structure which makes the system easier to study, both theoretically and numerically.

The semi-discrete approximation

In this section, the spatial variables in equation (7) are discretized while system time is kept continuous, i.e. the so-called semi discrete approximation in numerical analysis of partial differential equations. First of all, given an m n partition of the region $\Omega_0 = \{(x,y)|0 < x < a/2,\ 0 < y < b/2\}$:

$$0 < \Delta x < 2\Delta x < \cdots < m\Delta x = a/2,$$
$$0 < \Delta y < 2\Delta y < \cdots < m\Delta y = b/2.$$

Denoting $T_{ij} = T(i\Delta x, j\Delta y, t)$ results in:

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \Omega(\Delta x^2),$$
$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} + \Omega(\Delta y^2)$$

The resulting approximation equation becomes

$$\frac{dT_{ij}}{dt} = F(T_{ij})\left[\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}\right], \quad i = 0,1,\ldots,m,\ j = 0,1,\ldots,n \tag{9}$$

with corresponding discrete boundary conditions:

$$\frac{\partial T}{\partial x}(0,j,t) \approx \frac{T_{1j} - T_{-1,j}}{2\Delta x} = u_{2j}(t),$$
$$\frac{\partial T}{\partial y}(i,0,t) = \frac{T_{i1} - T_{i,-1}}{2\Delta y} = u_{1i}(t),$$
$$\frac{\partial T}{\partial x}(a/2,j,t) \approx \frac{T_{m+1,j} - T_{m-1,j}}{2\Delta x} = 0,$$
$$\frac{\partial T}{\partial y}(i,b/2,t) \approx \frac{T_{i,n+1} - T_{i,n-1}}{2\Delta y} = 0$$

that is,

$$\begin{cases} T_{-1,j} = T_{1j} - 2\Delta x u_{2j}(t), \\ T_{i,-1} = T_{i1} - 2\Delta y u_{1i}(t), \\ T_{m+1,j} = T_{m-1,j},\ T_{i,n+1} = T_{i,n-1} \end{cases} \tag{10}$$

Equations (9) - (10) are called the steady-state semi-discrete model of the continuous casting process. In order to formulate the process in a more lumped form, different cases should be treated separately by distinguishing the boundary and inner nodal points:

$$(i)\ \frac{dT_{i0}}{dt} = F(T_{i0})$$
$$\left(\frac{1}{\Delta x^2}, -2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right), \frac{2}{\Delta y^2}, \frac{1}{\Delta x^2}\right)\begin{bmatrix} T_{i-1,0} \\ T_{i0} \\ T_{i1} \\ T_{i+1,0} \end{bmatrix}$$
$$-F(T_{i0})\frac{2}{\Delta y}u_{1i}(t).$$

$$(ii)\ \frac{dT_{0j}}{dt} = F(T_{0j})$$
$$\left(\frac{1}{\Delta y^2}, -2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right), \frac{2}{\Delta x^2}, \frac{1}{\Delta y^2}\right)\begin{bmatrix} T_{0,j-1} \\ T_{0,j} \\ T_{1,j} \\ T_{0,j+1} \end{bmatrix}$$
$$-F(T_{0j})\frac{2}{\Delta x}u_{2j}(t).$$

$$(iii)\ \frac{dT_{mj}}{dt} = F(T_{mj})$$
$$\left(\frac{2}{\Delta x^2}, \frac{1}{\Delta y^2} - 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right), \frac{2}{\Delta y^2}\right)\begin{bmatrix} T_{m-1,j} \\ T_{m,j-1} \\ T_{m,j} \\ T_{m,j+1} \end{bmatrix}$$

$$(iv)\ \frac{dT_{in}}{dt} = F(T_{in})$$
$$\left(\frac{2}{\Delta y^2}, \frac{1}{\Delta x^2} - 2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right), \frac{1}{\Delta x^2}\right)\begin{bmatrix} T_{i,n-1} \\ T_{i-1,n} \\ T_{in} \\ T_{i+1,n} \end{bmatrix}$$

$$(v)\ \frac{dT_{mn}}{dt} = F(T_{mn})$$
$$\left(\frac{2}{\Delta x^2}, -2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right), \frac{2}{\Delta y^2}\right)\begin{bmatrix} T_{m-1,n} \\ T_{mn} \\ T_{m,n-1} \end{bmatrix}$$

$$(vi)\ \frac{dT_{m0}}{dt} = F(T_{m0})$$
$$\left(\frac{2}{\Delta x^2}, -2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right), \frac{2}{\Delta y^2}\right)\begin{bmatrix} T_{m-1,0} \\ T_{m0} \\ T_{m1} \end{bmatrix}$$
$$-F(T_{m0})\frac{2}{\Delta y}u_{1m}(t).$$

$$(vii)\ \frac{dT_{0n}}{dt} = F(T_{0n})$$

$$\left(\frac{2}{\Delta y^2}, -2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right), \frac{2}{\Delta x^2}\right)\begin{bmatrix} T_{0,n-1} \\ T_{0n} \\ T_{1n} \end{bmatrix}$$

$$-F(T_{0n})\frac{2}{\Delta x}u_{2n}(t)$$

$$(viii)\ \frac{dT_{00}}{dt} = F(T_{00})$$

$$\left(\frac{2}{\Delta y^2}, -2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right), \frac{2}{\Delta x^2}\right)\begin{bmatrix} T_{01} \\ T_{00} \\ T_{10} \end{bmatrix}$$

$$-F(T_{00})\left(-\frac{2}{\Delta y}, -\frac{2}{\Delta x}\right)\begin{bmatrix} u_{01}(t) \\ u_{02}(t) \end{bmatrix}$$

$$(ix)\ \frac{dT_{ij}}{dt} = F(T_{ij})$$

$$\left(\frac{1}{\Delta x^2}, \frac{1}{\Delta y^2}, -2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right), \frac{1}{\Delta x^2}, \frac{1}{\Delta y^2}\right)\begin{bmatrix} T_{i-1,j} \\ T_{i,j-1} \\ T_{ij} \\ T_{i+1,j} \\ T_{i,j+1} \end{bmatrix}$$

Separation layer formulation

In order to illustrate the heat transfer process, equations (9) - (10) are further formulated in terms of separation layers. It is assumed without loss of generality that $m \le n$. The first layer consists of boundary with control, that is,

$$\mathbf{T}_0 = (T_{m0}, T_{m-1,0}, \cdots, T_{00}, \cdots, T_{0n})^T, \qquad (11)$$

and

$$\mathbf{U} = (u_{1m}, \cdots, u_{10}, u_{20}, \cdots, u_{2n})^T \qquad (12)$$

The j-th ($1 \le j \le n-1$) layer is given by:

$$\mathbf{T}_j = (T_{mj}, T_{m-1,j}, \cdots, T_{jj}, \cdots, T_{jn})^T \qquad (13)$$

And the final layer is described by:

$$T_n = (T_{mn}, T_{m-1,n}, \cdots, T_{nn})^T \qquad (14)$$

Based on separation layers, the semi-discrete model, in matrix form, can be formulated as follows:

$$\begin{cases} \dfrac{d\mathbf{T}_0}{dt} = F(\mathbf{T}_0)[A_0\mathbf{T}_0 + C_0\mathbf{T}_1 + B\mathbf{U}], \\ \dfrac{d\mathbf{T}_j}{dt} = F(\mathbf{T}_j)[A_j\mathbf{T}_j + B_j\mathbf{T}_{j-1} + \\ C_j\mathbf{T}_{j+1}],\ j = 1,2,\cdots,n-1. \\ \dfrac{d\mathbf{T}_n}{dt} = F(\mathbf{T}_n)[A_n\mathbf{T}_n + B_n\mathbf{T}_{n-1}] \end{cases} \qquad (15)$$

where $F(\mathbf{T}j) = diag\{F(Tij)\}$ are diagonal matrices, and for $1 \le j \le n-1$,

$$A_j = \left(-\frac{2}{\Delta x^2}, -\frac{2}{\Delta y^2}\right)I_{m+n-2j+1} + [A_{j1}, A_{j2}],$$

$$A_{j1} = \begin{bmatrix} 0 & \frac{2}{\Delta x^2} & 0 & \cdots & 0 & 0_{1,m-j+1} \\ \frac{1}{\Delta x^2} & 0 & \frac{1}{\Delta x^2} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0_{m-j+1,1} & 0 & 0 & \cdots & \frac{1}{\Delta x^2} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \frac{1}{\Delta y^2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$A_{j2} = \begin{bmatrix} 0_{1,m-j+2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{\Delta y^2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{\Delta y^2} & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \frac{2}{\Delta y^2} & 0 \end{bmatrix},$$

$$A_n = -\left(\frac{2}{\Delta x^2}+\frac{2}{\Delta y^2}\right)I_{m-n+1} +$$

$$\begin{bmatrix} 0 & \frac{2}{\Delta x^2} & 0 & \cdots & 0 & 0 & 0 \\ \frac{2}{\Delta x^2} & 0 & \frac{2}{\Delta x^2} & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \frac{2}{\Delta x^2} & 0 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} \frac{2}{\Delta x^2}I_{m-1} & 0 & 0 \\ 0 & \frac{2}{\Delta y^2} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{2}{\Delta x^2} & 0 \\ 0 & 0 & \frac{2}{\Delta x^2}I_{n-1} \end{bmatrix},$$

$$C_j = \begin{bmatrix} \frac{1}{\Delta y^2} I_{m-j-1} & 0 & 0 \\ 0 & \frac{1}{\Delta y^2} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\Delta x^2} & 0 \\ 0 & 0 & \frac{1}{\Delta x^2} I_{n-j-1} \end{bmatrix},$$

$$B_j = \begin{bmatrix} \frac{1}{\Delta y^2} I_{m-j} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\Delta y^2} & 0 & \frac{1}{\Delta x^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\Delta x^2} I_{n-j} \end{bmatrix},$$

$$B_n = \begin{bmatrix} \frac{2}{\Delta y^2} I_{m-n} & 0 & 0 & 0 \\ 0 & \frac{2}{\Delta y^2} & 0 & \frac{2}{\Delta x^2} \end{bmatrix},$$

$$B = \begin{bmatrix} -\frac{2}{\Delta y} I_{m+1} & 0 \\ 0 & \frac{2}{\Delta x} I_n \end{bmatrix}$$

where $0_{i,j}$ denotes the entry 0 at the position (i,j) in the matrices, I_k denotes $k \times k$ identity matrix.

The separation layer formulation (15) clearly shows the heat transfer process: $\mathbf{T}_0$ depends only on $\mathbf{T}_1$, $\mathbf{T}_j$ $(1 < j < n)$ depends on $\mathbf{T}_{j-1}$ and $\mathbf{T}_{j+1}$. The final layer $\mathbf{T}_n$ depends on $\mathbf{T}_{n-1}$ only. That is, system (15) is a typical control system with balanced realisation. More precisely, if the time-dependent temperature distribution $y(t) = \mathbf{T}_0(t)$ of the outer layer is observed, then the whole temperature distribution of the strand can be recovered from:

$$\begin{cases} \mathbf{T}_1 = C_0^- F(T_0)^{-1}\{\frac{d\mathbf{T}_0}{dt} - F(\mathbf{T}_0)[A_0\mathbf{T}_0 + B\mathbf{U}]\}, \\ \mathbf{T}_{j+1} = C_j^- F(T_j)^{-1}\{\frac{d\mathbf{T}_j}{dt} - F(\mathbf{T}_j)[A_j\mathbf{T}_j + \\ B_j\mathbf{T}_{j-1}]\},\ j = 1,2,\cdots,n-1 \end{cases} \quad (16)$$

where

$$C_j^- = \begin{bmatrix} \Delta y^2 I_{m-j} & 0 & 0 & 0 & 0 \\ 0 & \frac{\Delta y^2}{2} & 0 & \frac{\Delta x^2}{2} & 0 \\ 0 & 0 & 0 & 0 & \Delta x^2 I_{n-j} \end{bmatrix}.$$

In addition to (16), if the temperature distribution $\mathbf{Y}(t^*) = (\mathbf{T}_0(t^*), \mathbf{T}_j(t^*), \mathbf{T}_n(t^*))^T$ of the last section is observed, then by letting $\mathbf{Y}_j(t^*-t) = \mathbf{T}_j(t)$ and solving the following initial value problem:

$$\begin{cases} \frac{d\mathbf{Y}_0}{dt} = -F(\mathbf{Y}_0)[A_0\mathbf{Y}_0 + C_0\mathbf{Y}_1 + B\mathbf{U}], \\ \frac{d\mathbf{Y}_j}{dt} = -F(\mathbf{Y}_j)[A_j\mathbf{Y}_j + B_j\mathbf{Y}_{j-1} + \\ C_j\mathbf{Y}_{j+1}],\ j = 1,2,\cdots,n-1, \\ \frac{d\mathbf{Y}_n}{dt} = -F(\mathbf{Y}_n)[A_n\mathbf{Y}_n + B_n\mathbf{Y}_{n-1}], \\ \mathbf{Y}_i(0) = \mathbf{T}_i(t^*), i = 0,1,\cdots,n \end{cases} \quad (17)$$

the whole temperature distribution of the strand can be obtained

Identification of system parameters

In equation (15), $F(T_{ij})$ are the system parameters. Once the temperature distribution and water spray profiles are known, the system parameters can be identified by means of, e.g. application of the least square method.
Now, write

$$\begin{cases} \mathbf{F}_0 = (F(T_{m0}),\cdots,F(T_{00}),\cdots,F(T_{0n}))^T, \\ \mathbf{F}_j = (F(T_{mj}),\cdots,F(T_{jj}),\cdots,F(T_{jn}))^T, \\ \mathbf{F}_n = (F(T_{mn}),\cdots,F(T_{nn}))^T, \\ \mathbf{F} = (F_0, F_1,\cdots,F_n)^T \end{cases}$$

Then $\mathbf{F}$ satisfies

$$\hat{e}(\mathbf{T},\mathbf{U})\mathbf{F} = \mathbf{Y}(\mathbf{T}) \quad (18)$$

where

$$\hat{e}(\mathbf{T},\mathbf{U}) = \{\mathbf{a}_{ij}\}, \mathbf{a}_{ij} = 0, \text{if } i \neq j,$$
$$\mathbf{a}_{11} = A_0\mathbf{Y}_0 + C_0\mathbf{Y}_1 + B\mathbf{U},$$
$$\mathbf{a}_{jj} = A_j\mathbf{Y}_j + B_j\mathbf{Y}_{j-1}C_j\mathbf{Y}_{j+1},$$
$$\mathbf{a}_{nn} = A_n\mathbf{Y}_n + B_n\mathbf{Y}_{n-1}$$

All the vectors in the expressions of $\hat{e}$ are presented as diagonal matrices by putting the i-th component in original vector as the (i,i)-th entry of the corresponding diagonal matrix:

$$\mathbf{Y}(T) = (\frac{d\mathbf{T}_0}{dt}, \frac{d\mathbf{T}_1}{dt}, \cdots, \frac{d\mathbf{T}_n}{dt})^T$$

The least square solution $\mathbf{F}^*$ of (18) satisfies

$$\hat{e}^T(\mathbf{T},\mathbf{U})\hat{e}(\mathbf{T},\mathbf{U})\mathbf{F}^* = \hat{e}^T(\mathbf{T},\mathbf{U})\mathbf{Y}(\mathbf{T}). \quad (8)$$

Local controllability in the secondary cooling zone

Local controllability of the system (15) in the secondary cooling zone is considered here. Given any state $\mathbf{T}^* = (\mathbf{T}_0^*, \mathbf{T}_j^*, \mathbf{T}_n^*)$, the linearization of the system (15) about $\mathbf{T}^*$ is:

$$\begin{cases} \frac{d\mathbf{T}_0}{dt} = F(\mathbf{T}_0^*)[A_0\mathbf{T}_0 + C_0\mathbf{T}_1 + B\mathbf{U}], \\ \frac{d\mathbf{T}_j}{dt} = F(\mathbf{T}_j^*)[A_j\mathbf{T}_j + B_j\mathbf{T}_{j-1} + \\ C_j\mathbf{T}_{j+1}],\ j = 1,2,\cdots,n-1, \\ \frac{d\mathbf{T}_n}{dt} = F(\mathbf{T}_n^*)[A_n\mathbf{T}_n + B_n\mathbf{T}_{n-1}] \end{cases} \quad (20)$$

where all $F(T_0^*)$, $F(T_j^*)$, $F(T_n^*)$ are diagonal matrices and their nonzero entries are positive. It may be assumed without loss of generality that all these matrices are identity matrices. Now (20) becomes:

$$\begin{cases} \frac{d\mathbf{T}_0}{dt} = A_0\mathbf{T}_0 + C_0\mathbf{T}_1 + B\mathbf{U}, \\ \frac{d\mathbf{T}_j}{dt} = A_j\mathbf{T}_j + B_j\mathbf{T}_{j-1} + C_j\mathbf{T}_{j+1}, \\ j = 1,2,\cdots,n-1, \\ \frac{d\mathbf{T}_n}{dt} = A_n\mathbf{T}_n + B_n\mathbf{T}_{n-1} \end{cases} \quad (21)$$

Let $T = (T_0, T_1, \cdots, T_n)T$

$$\mathrm{A} = \begin{bmatrix} A_0 & C_0 & 0 & \cdots & 0 & 0 & 0 \\ B_1 & A_1 & C_1 & \cdots & 0 & 0 & 0 \\ & & & \cdots & & & \\ 0 & 0 & 0 & \cdots & B_{n-1} & A_{n-1} & C_{n-1} \\ 0 & 0 & 0 & \cdots & 0 & B_n & A_n \end{bmatrix},$$

$$\mathrm{B}^{\mathrm{T}} = (B \quad 0 \quad \cdots \quad 0).$$

Then (21) can be written as the standard lumped-parameter control system:

$$\frac{d\mathbf{T}}{dt} = \mathrm{A}\mathbf{T} + \mathrm{B}\mathbf{U} \quad (22)$$

Theorem 1. The linear system (22) is completely controllable and hence system (15) is locally controllable for any given state $\mathbf{T}^*$

Proof. The following relation can be obtained from mathematical induction:

$$\mathrm{A}^j = [A^{j1}, A^{j2}],$$

$$A^{j1} = \begin{bmatrix} * & * & * & \cdots \\ * & * & \cdots & \cdots \\ B_jB_{j-1}\cdots B_1 & * & \cdots & \cdots \\ 0 & B_{j=1}B_j\cdots B_2 & \cdots & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix},$$

$$A^{j2} = \begin{bmatrix} * & * & \cdots & * \\ & & \cdots & \\ B_nB_{n-1}\cdots B_{n-j+1} & * & \cdots & * \end{bmatrix}$$

Therefore,

$$(\mathrm{B}, \mathrm{AB}, \cdots, \mathrm{A}^n\mathrm{B}) = \begin{bmatrix} B & * & * & \cdots & * \\ 0 & B_1B & * & \cdots & * \\ 0 & 0 & B_2B_1B & \cdots & * \\ & & & \cdots & \\ 0 & 0 & 0 & \cdots & B_nB_{n-1}\cdots B_1B \end{bmatrix}$$

Again by induction, it can be shown that

$$B_jB_{j-1}\cdots B_1B = \begin{bmatrix} \otimes I_{m-j} & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \otimes & 0 & \cdots & \otimes & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \otimes I_{n-2j-1} \end{bmatrix}$$

where $\otimes \neq 0$. From the above structure, it follows that

$$Rank(\mathrm{B}, \mathrm{AB}, \cdots, \mathrm{A}^n\mathrm{B}) = \text{dimension of state } \mathbf{T}$$

which, in particular, implies that system (22) is completely controllable.

The analysis of controllability stated above demonstrates how the water sprays affect the temperature distribution from the surface to the middle face which was first indicated in [9]. In fact, applying further a difference scheme to equation (21) yields:

$$\begin{cases} \mathbf{T}_0(m+1) = (I_0 + A_0)\mathbf{T}_0(m) \\ + C_0\mathbf{T}_1(m) + B\mathbf{U}(m), \\ \mathbf{T}_j(m+1) = (I_j + A_j)\mathbf{T}_j(m) + B_j\mathbf{T}_{j-1}(m) \\ + C_j\mathbf{T}_{j+1}(m),\ j = 1,\cdots,n-1 \\ \mathbf{T}_n(m+1) = (I_n + A_n)\mathbf{T}_n(m) \\ + B_n\mathbf{T}_{n-1}(m),\ m = 0,1,\cdots \end{cases} \quad (23)$$

where I_i denotes the appropriate size identical matrix which is clear from the context. Suppose $1 < 2/\Delta x + 2/\Delta y$. Denoting by A_d the matrix A where $A_i, I = 0,1..n$ are substituted by $I_i + A_i$ which has the same structure as A_i, then (21) can be written as:

$$\mathbf{T}(m+1) = \mathrm{A}_d\mathbf{T}(m) + \mathrm{B}\mathbf{U}(m),\ m = 0,1,\cdots \quad (24)$$

The process of controllability analysis for system (24) is exactly the same as that for system (22). It follows from (24) that

$$T(m+1) = A_d^n T(m) + \sum_{i=1}^{n} A_d^{n-i} B U(m+i-1).$$

By the structure of A, B, one can see that $U(m+n-1)$ affects directly the components $T_0(m+n)$ of $T(m+n)$, i.e., the temperature on the surface; and $U(m)$ affects the middle temperature; and $U(m+i-1)$ affects the temperature on the ith-layer between the surface and the middle face. So the effect of the water sprays goes from the surface to the middle surface. This is a well-known fact in practice.

Conclusion

The heat transfer of a continuous casting process with water-spray cooling control is discussed. A non-linear heat equation with boundary control which describes this process is transformed to a simplified non-linear heat equation with boundary control by using a non-linear state and feedback control transformation. The semi-discrete approximation is applied to the transformed equation to get a standard lumped control system with balanced realisation. The following conclusions are drawn:

- the approach of feedback linearization in the study of finite non-linear control systems can be potentially applied to partial differential equation system control;
- the partial differential equation models of continuous casting can be approximated by a standard lumped control system through a semi-discrete approximation method;
- the continuous casting system is locally controllable;
- numerical methods for the solution of ordinary differential equations can be used for the numerical study of continuous casting process.

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